Problem 1 (Submit): Consider the ODE:

\[(1 + x^3)y'' - 6xy = 0.\]  \hspace{1cm} (1)

(a) Compute the first 3 nonzero terms of power series expansion about \(x = 0\) for two linearly independent solutions.

(b) Use the ratio test to determine the radius of convergence of the series. Could your result have been predicted by inspection?

Problem 2 (Submit): Consider the Differential equation

\[(1 + x^2)y'' + xy' - y = 0.\]  \hspace{1cm} (2)

(a) Find the first 3 nonzero terms of the power series expansion of the general solution about \(x = 0\).

(b) Use the ratio test to determine the radius of convergence of the series. What can you say about the radius of convergence without solving the ODE?

(c) Determine the solution that satisfies the initial conditions \(y(0) = 1\) and \(y'(0) = 0\).

Problem 3 (Do not submit): Compute the first 3 nonzero terms of the power series expansion about \(x = 0\) of two linearly independent solutions of the ODE: \(y'' - \sin(x)y = 0\).

Problem 4 (Do not submit): Find the power series expansion about \(x = 0\) of two linearly independent solutions of the ODE \(y'' + x^2y = 0\). What can you say about the radius of convergence of this power series?

Problem 5 (Do not submit): Find all the singular points of the following ODEs and determine whether each one is regular or irregular. If the singular point is regular, determine the indicial equation and determine a lower bound for the radius of convergence of the Frobenius series. You must not try to compute the series solution itself.

a. \((x^2 - 1)y'' + x^2y' + \cot(x)y = 0.\)

b. \(\sinh(x)y'' + xy' + y = 0.\)

c. \((x^2 + 4)y'' + xy' + y = 0.\)

d. \(y'' + x^{1/2}y' - y = 0.\)