Problem 1 (Do not submit) Fourier series - Determine whether the following functions are odd, even or neither:

(a) \( f(x) = x^2 + |x| \)  \( (b) \ f(x) = e^{\sin^2 x} \)  \( (c) \ f(x) = \cosh x + \sinh x \)

Problem 2 (Submit) Fourier series - For the following functions, sketch an odd extension of \( f(x) \) on \([-L, L]\), and find the Fourier sine series of \( f(x) \) assuming that the function has a period of \( 2L \).

(a) \( f(x) = \cos(\pi x/L), \ 0 \leq x \leq L \)

(b) \( f(x) = \begin{cases} 0 & \text{if } 0 \leq x < L/2 \\ x & \text{if } L/2 \leq x \leq L \end{cases} \)

Problem 3 (Do not submit) Fourier series - For the following functions, sketch an even extension of \( f(x) \) on \([-L, L]\), and find the Fourier cosine series of \( f(x) \) assuming that the function has a period of \( 2L \).

(a) \( f(x) = \sin(\pi x/L), \ 0 \leq x \leq L \)

(b) \( f(x) = \begin{cases} 0 & \text{if } 0 \leq x < L/2 \\ x & \text{if } L/2 \leq x \leq L \end{cases} \)

Problem 4 (Submit) Heat equation - Apply the method of separation of variables to solve the heat equation

\[ u_t = u_{xx} \quad \text{for} \quad t > 0, \quad -2 \leq x \leq 2, \]

with periodic boundary conditions

\[ u(-2, t) = u(2, t), \quad \text{and} \quad \frac{\partial u(-2, t)}{\partial x} = \frac{\partial u(2, t)}{\partial x} \]

and initial condition

\[ u(x, 0) = \cos(\pi x/2) + \sin(\pi x). \]

Problem 3 (Submit) Heat equation - Apply the method of separation of variables to solve the heat equation

\[ u_t = 3u_{xx} \quad \text{for} \quad t > 0, \quad 0 \leq x \leq \pi, \]

with boundary conditions

\[ u(0, t) = 0 = u_x(\pi, t), \]
and initial condition

\[ u(x, 0) = \sin(x) + \sin(3x/2). \]

**Problem 4 (Do not submit):** Apply the method of separation of variables to determine a solution to the one dimensional heat equation with homogeneous Neumann boundary conditions, i.e.

\[
\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}
\]

BC: \( \frac{\partial u(0, t)}{\partial x} = 0 \) and \( \frac{\partial u(\pi, t)}{\partial x} = 0 \)

IC: \( u(x, 0) = \cos \gamma x, \quad 0 \leq x \leq \pi \)

Distinguish between the cases in which \( \gamma \) is and is not an integer. Show by evaluating \( u(\pi, 0) \) that if \( \gamma \) is not an integer then:

\[
\cot \pi \gamma = \frac{1}{\pi} \left[ \frac{1}{\gamma} - \sum_{n=1}^{\infty} \frac{2\gamma}{n^2 - \gamma^2} \right]
\]

**Problem 5 (Do not submit):** Apply the method of separation of variables to find the temperature in a laterally insulated bar with length \( L \) and thermal diffusion coefficient \( \alpha^2 \) whose ends are kept at temperature zero and its temperature initially is

\[
u(x, 0) = f(x) = \begin{cases} x & \text{if } 0 \leq x \leq \frac{L}{2} \\ L - x & \text{if } \frac{L}{2} \leq x \leq L \end{cases}
\]