Math 257/316, midterm 2, section 202
March 20, 2019, Duration: 50 min, Total marks: 100, Number of questions: 2

First Name: ___________________________ Last Name: ___________________________

SID: ___________________________ Section: ___________________________ Signature: ___________________________

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Instructions:

- Notes, calculators, phones, computers and your cheat sheets are not allowed.
- The formula sheet is on the last page of the exam booklet.
- Show all your work. A correct answer without the intermediate steps will not receive credit.

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**Student Conduct during Examinations**

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UoC card for identification.

2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.

3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.

4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be administered by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.

5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   (i) speaking or communicating with other examination candidates, unless otherwise authorised;
   (ii) purposely exposing written papers to the view of other examination candidates or imaging devices;
   (iii) purposely viewing the written papers of other examination candidates;
   (iv) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and
   (v) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s) (electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.

7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.

8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).
1. (60 marks) Consider the following inhomogeneous initial-boundary value problem for the heat equation

\[ u_t = u_{xx} + e^{-t} \cos\left(\frac{3x}{2}\right) + 1 \quad t > 0, \quad 0 \leq x \leq \pi, \]

BC: \quad u_x(0,t) = 0, \quad \text{and} \quad u(\pi, t) = t

IC: \quad u(x,0) = \cos\left(\frac{7x}{2}\right)

Solve this problem by using an expansion in terms of the appropriate eigenfunctions, which correspond to the homogeneous form of the stated boundary conditions. (Note: Only show cases that give non-trivial solutions when solving the eigenvalue problems.)
Question 1 (continued):
Question 1 (continued):
2. (40 marks) Consider the motion of a string on an elastic foundation with a stiffness \( \gamma \) \((\gamma \geq 0)\) that satisfies the following initial-boundary value problem:

\[
\begin{align*}
  u_{tt} &= u_{xx} - \gamma u \quad t > 0, \quad 0 < x < 1, \\
  BC: \quad u(0, t) &= 0, \quad \text{and} \quad u(1, t) = 0 \\
  IC: \quad u(x, 0) &= 0, \quad \text{and} \quad u_t(x, 0) = x
\end{align*}
\]

(a) Use the method of separation of variables to solve this problem. \((Note: \) Only show cases that give non-trivial solutions when solving the eigenvalue problems.\)

(b) For \( \gamma = 0 \) and also for \( \gamma = 8\pi^2 \) sketch the solution to the fundamental mode of vibration at \( t = 1/2 \). How does increasing the stiffness \( \gamma \) influence the amplitude of the vibrations for this mode?
Question 2 (continued):