Math 257/316 Assignment 7, 2019

Practice problems - not for submission

Problem 1: Solve the following heat conduction problem with heat loss and a distributed heat source

\[ u_t = u_{xx} - u + x, \quad 0 < x < 1, \quad t > 0 \]

BC: \( u_x(0, t) = 1, \quad u_x(1, t) = 2 \)

IC: \( u(x, 0) = x \)

Hint: First try to find a steady-state solution that satisfies the PDE and the inhomogeneous boundary conditions.

Problem 2: Solve the initial boundary value problem:

\[ u_t = u_{xx} + e^{-t} \cos \left( \frac{3\pi}{2} x \right) + 1, \quad 0 < x < 1, \quad t > 0 \]

BC: \( u_x(0, t) = 1, \quad u(1, t) = t \)

IC: \( u(x, 0) = 1 \)

Problem 3: Solve the inhomogeneous heat conduction problem subject to time dependent boundary conditions:

\[ u_t = \alpha^2 u_{xx} + 1 - xe^{-t}, \quad 0 < x < 1, \quad t > 0 \]

\( u(0, t) = e^{-t}, \) and \( u_x(1, t) = t \)

\( u(x, 0) = \sin \left( \frac{3\pi x}{2} \right) + 1. \)

Problem 4: Solve the inhomogeneous heat conduction problem subject to time dependent boundary conditions:

\[ u_t = u_{xx} + xt + 1, \quad 0 < x < 1, \quad t > 0 \]

\( u_x(0, t) = 0, \) and \( u(1, t) = t \)

\( u(x, 0) = 0. \)

Problem 5 : Solve the inhomogeneous heat conduction problem with heat loss, a time dependent source, and subject to time dependent boundary conditions:

\[ u_t = u_{xx} - u + e^{-t} \sin(x), \quad 0 < x < \frac{\pi}{2}, \quad t > 0 \]

\( u(0, t) = 0, \) and \( \frac{\partial u(\pi/2, t)}{\partial x} = e^{-t} \)

\( u(x, 0) = x. \)
Problem 6: The motion of a string on an elastic foundation with a stiffness $\gamma$ satisfies the following initial-boundary value problem:

$$u_{tt} = u_{xx} - \gamma u, \quad 0 < x < 1, \quad t > 0$$

$$u(0,t) = u(1,t) = 0$$

$$u(x,0) = 0, \quad u_t(x,0) = g(x)$$

(a) Solve (1) subject to the boundary conditions (2) using separation of variables.
(b) For $\gamma = 7\pi^2$ and $g(x) = \sin 3\pi x$, sketch the solution at $t = 3/8$. 