Opl Res. Q., Pergamon Press 1977. Vol. 28, 2, i, pp. 293 to 304. Printed in Great Britain

A Markov Analysis of Per Capita State and Local Police Expenditures and the Allocation Problem of Federal Aid

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A Markov chain model is applied to analyze changes in real annual state and local police expenditures in the United States, where transition probabilities are functions of crime, demographic and economic variables. Of these variables, the inflation rate for state and local government expenditures and federal aid to state and local governments are assumed controllable by public policies. Then a mathematical programming problem is formulated to allocate a fixed amount of federal aid among the North Central states, in which the competing objectives are to maximize the sum of the weighted probabilities so that real per capita police expenditures will be increased in each of the North Central states in the following year while holding the rate of inflation in the state and local government sector to the present rate.

INTRODUCTION

CONSIDERABLE scholarly and public interest has been expressed in how federal expenditure and taxation policies might be used to bring about a general increase or equalization in per capita state and local expenditures for public services such as police protection or education.[1–6] Federal aid to state and local governments is of particular interest in this context, since the federal government has wide latitude in determining both the form and amount of these allocations to different states and regions. Owing to the separation of powers between the various levels of government in the United States, the impact of federal aid on state and local government expenditure patterns is probabilistic in nature. The federal government may tempt, but cannot force lower level governments to accept tied aid. Nor can the federal government place conditions on aid offered to state and local governments which might infringe improperly on the areas of jurisdiction of these governments. Secondly, any realistic allocation problem would probably involve multiple and competing or divergent objectives.

In this paper, a finite Markov chain is used to describe observed changes in real state and local police expenditures in the North Central states (Ohio, Indiana, Illinois, Michigan, Wisconsin, Minnesota, Iowa, Missouri, North Dakota, South Dakota, Nebraska and Kansas) for 1959–69 in terms of observed changes in uncontrollable economic and demographic variables, federal aid to state and local governments, and the rate of inflation in the state and local government sector. We have also estimated a highly simplified model describing the impact of changes in federal aid to state and local governments on observed changes in the rate of inflation in the state and local government sector. These estimated relationships are then used to analyze a hypothetical problem of allocating a fixed amount of federal aid among the 12 North Central states. In choosing among various possible allocations, the objectives are to maximize the sum of the weighted probabilities that real per capita police expenditures will increase in the North Central states, while not allowing the rate of inflation in the state and local government sector to rise above the present rate. The weights in the objective function have been introduced to reflect differences in the urgency which federal policy makers attach to bringing about the desired increase in police expenditures in each of the North Central states.

FINITE MARKOV CHAINS AND MATHEMATICAL PROGRAMMING PROBLEMS OF RESOURCE ALLOCATION

Finite Markov chains are often used as a model to describe probabilistic structures in, for instance, health,[7] marketing,[8,9] educational,[6,10] agricultural,[11] economic,[12,16] and socioeconomic systems.[17,17a] Many of these applications do not involve decision processes which might affect the transition probabilities of a system over time. Markovian and Semi-Markovian decision models are common, however.[18–24] Algorithms for the Markovian and more general Semi-Markovian decision models with an infinite-time planning horizon are available to find optimal transition probabilities which minimize either the expected total discounted cost for the case with a discounting factor for costs, or the expected average cost per unit time (and possibly a bias term) for the case without discounting.[18,19,21–24] Algorithms are also available for problems with a finite-time planning horizon.[20] These models assume the availability of known alternative transition probabilities. Frequently, however, these transition probabilities are not known and must be estimated. This is generally the case in public sector planning problems.

We are concerned with a system which attains one of n possible states $(1, 2, \ldots, n)$ in period t $(t = 0, 1, 2, \ldots, T)$. The transition probability that the system will attain State j in period t given that it was in State i in period $t-1$ $(i, j = 1, 2, \ldots, n)$ is assumed to behave according to a relationship of the following form:

$$p_{ij}(x) = \sum_{k=0}^{K} a_k^{ij} x_k, \quad i, j = 1, 2, \ldots, n, \tag{1}$$

where $x_0 \equiv 1$, $x = (x_1, x_2, \ldots, x_K)$ and $x_1, x_2, \ldots, x_K$ are explanatory variables some of which may be controllable. The constants, a_k^{ij} for all k, i and j, may be estimated from past data subject to the provisions that sufficient data is available and that the estimates obtained can be regarded as reasonable over the (finite or infinite) planning horizon. Without a loss of generality we can assume that $x_1, x_2, \ldots, x_J$ ($J \leq K$) are controllable explanatory variables subject to certain constraints. (Telser[9] used a linear specification similar to (1). Although functional forms used in probit analysis[25] might appear more convenient than linear forms, Goldfeld and Quandt[26] found some Monte-Carlo evidence that for less than two hundred observations a linear model works at least as well as a probit model even if the true generating process is probit.)

Let $p_i(t)$ be the probability that the system will be found in State i in period t. Then

$$p_i(t+1) = \sum_{j=1}^{n} p_j(t) p_{ji}(x), \quad i = 1, 2, \ldots, n, \ t = 0, 1, 2, \ldots, T-1, \tag{2}$$

where the values of $p_i(0)$ are assumed given such that

$$\sum_{i=1}^{n} p_i(0) = 1, \quad p_i(0) \geq 0. \tag{3}$$

Suppose that values are known or predicted over T periods for the uncontrollable explanatory variables $x_{J+1}(t), x_{J+2}(t), \ldots, x_K(t)$, $t = 0, 1, 2, \ldots, T-1$. Suppose further that the controllable explanatory variables $x_1(t), x_2(t), \ldots, x_J(t)$, $t = 0, 1, 2, \ldots, T-1$, must satisfy the following constraints:

$$\bar{x}(t) \in \Omega_t, \quad t = 0, 1, \ldots, T-1, \tag{4}$$

and

$$(\bar{x}(0), \bar{x}(1), \ldots, \bar{x}(T-1)) \in \Omega^* \tag{5}$$

where $\bar{x}(t) = (x_1(t), x_2(t), \ldots, x_J(t)), \Omega_t \subset R^J, and\ \Omega^* \subset R^{JT}$.

Let $c_i(t)$ be an index of benefit (or cost) assigned to State i in period t. Then an index of the total expected benefit (or cost) over T periods is given by

$$\sum_{t=1}^{T} \sum_{i=1}^{n} c_i(t) p_i(t). \tag{6}$$

Thus our problem can be restated as follows:

Problem I. Maximize (or minimize) expression (6) subject to (1), (2), (4) and (5).

We note that by (1) the $p_j(t)$, $t = 1, 2, \ldots, T$, are linear in $x_k(t-1)$, since

$$p_j(t) = \sum_{i=1}^{n} p_i(t-1) \sum_{k=0}^{K} a_k^{ij} x_k(t-1). \tag{7}$$

The $p_j(t)$, however, are not linear in $x_k(0)$, $x_k(1), \ldots, x_k(t-2)$ when $T > 1$. Our police expenditure example is formulated as a single period problem ($T = 1$) and the objective function is linear in $x_k(0)$.

When we are interested in analyzing a system over an infinite planning horizon it is appropriate to use stationary probabilities, where the $x_{J+i}(t)$ are assumed to be constant over time for $i = 1, 2, \ldots, K - J$. Let $\pi_i(x)$ be the stationary probability that the system is found in State i ($i = 1, 2, \ldots, n$) given the vector of controllable explanatory variables $\bar{x}$ which must satisfy

$$\bar{x} \subset \Omega \tag{8}$$

where $\Omega \in R^J$. We assume that the Markov chain is ergodic for all $\bar{x}$ in Ω. Then the $\pi_i(x)$ ($i = 1, 2, \ldots, n$) are uniquely determined by

$$\pi_j(x) = \sum_{i=1}^{n} \pi_i(x) p_{ij}(x), \tag{9}$$

where

$$\sum_{i=1}^{n} \pi_i(x) = 1, \; \pi_j(x) \geq 0, j = 1, 2, \ldots, n,$$

and where the first J elements of x are identical to those of $\bar{x}$ and the last $K - J$ elements of x are assigned either given or predicted values. Let c_i be an index of the benefit (or cost) a policy maker associates with State i ($i = 1, 2, \ldots, n$). Then the problem for the stationary case is as follows.

Problem II. Maximize (or minimize) an expression for expected benefit (or cost) of the form

$$\sum_{i=1}^{n} c_i \pi_i(x) \tag{10}$$

subject to (8) and (9).

Note that $\pi_i(x)$ is a ratio of two determinants which are polynomials in x. When $n = 2$, however, Problem II reduces to a linear fractional program if Ω is defined by linear inequalities as is shown in Nakamura and Nakamura,[27] and can be solved by linear programming.[28]

ANALYSIS OF STATE AND LOCAL POLICE EXPENDITURES AND ALLOCATION OF FEDERAL AID

In this section we consider the hypothetical problem of allocating a fixed amount of federal aid among the 12 North Central states so as to maximize the weighted probability that these states will increase their real per capita police expenditures in 1970.

Let PC_t denote real per capita state and local police expenditures in period t. Then the first difference is defined by

$$\Delta PC_t = PC_t - PC_{t-1}, \tag{11}$$

where Δ denotes the first difference operator. Two States of the Markov chain are defined in terms of ΔPC_t as follows: (1) $\Delta PC_t \leq 0$ in State 1, and (2) $\Delta PC_t > 0$ in State 2. Pooled data[27] was collected for the 12 North Central states for 1958–68 for the following explanatory variables:

PCM	Murders/100,000 population
PCRB	Robberies/100,000 population
PCAS	Assaults/100,000 population
PCBG	Burglaries/100,000 population
PCLR	Larcenies/100,000 population
PCAT	Auto thefts/100,000 population
TD	Traffic deaths/100,000 population
POP	Population
PCY	Real per capita income
RW	Average real weekly earnings of production workers
DNS	Population density
GNPD	Implicit GNP deflator for state and local government purchases
FA	Real per capital federal aid to state and local governments

Also pooled data was collected for real per capita state and local police expenditure, PC, for these same 12 North Central states for 1959–69.

Our 10 first difference observations for each of our 13 variables for all 12 North Central states were then grouped into two samples according to the following rule (s denotes a North Central state):

$$\text{Sample 1} \quad \Delta PC_{s,t-1} \leq 0$$
$$\text{Sample 2} \quad \Delta PC_{s,t-1} > 0.$$

Thus Sample 1 consists of all observations on all 13 variables such that the state in question was in State 1 in the preceding year, and Sample 2 consists of all observations such that the state in question was in State 2 in the preceding year.

Transition probabilities p_{12} and p_{22} were estimated using multiple regression in the following manner (see Nakamura and Nakamura[27] for a general discussion of estimating transition probabilities from different types of data):

(i) p_{12} was estimated from Sample 1, where we set the dummy dependent variable for p_{12} equal to 1 if $\Delta PC_t > 0$ and equal to 0 otherwise.

(ii) p_{22} was estimated from Sample 2, where we set the dummy dependent variable for p_{22} equal to 1 if $\Delta PC_t > 0$ and equal to 0 otherwise.

The results of these two regressions are:

$$\begin{aligned}\hat{p}_{12} = {} & .8158 + \underset{(.1517)}{.1858}\,\Delta PCM - \underset{(.0198)}{.0310}\,\Delta PCRB \\ & - \underset{(.0200)}{.0244}\,\Delta PCAS - \underset{(.0045)}{.0048}\,\Delta PCBG - \underset{(.0082)}{.0001}\,\Delta PCLR \\ & + \underset{(.0103)}{.0199}\,\Delta PCAT + \underset{(.0006)}{.0001}\,\Delta TD + \underset{(.0069)}{.0127}\,\Delta POP \\ & - \underset{(.0014)}{.0010}\,\Delta PCY + \underset{(.0490)}{.0587}\,\Delta RW - \underset{(.3132)}{.6257}\,\Delta DNS \\ & - \underset{(.0630)}{.0218}\,\Delta GNPD + \underset{(.0216)}{.0168}\,\Delta FA \end{aligned} \tag{12}$$

$$R^2 = .5176$$
$$\text{S.E.} = .3804$$

$$\begin{aligned}\hat{p}_{22} = {} & .2850 - \underset{(.0771)}{.1574}\,\Delta PCM + \underset{(.0091)}{.0062}\,\Delta PCRB \\ & + \underset{(.0065)}{.0113}\,\Delta PCAS - \underset{(.0020)}{.0014}\,\Delta PCBG + \underset{(.0033)}{.0009}\,\Delta PCLR \\ & + \underset{(.0031)}{.0004}\,\Delta PCAT - \underset{(.0004)}{.0003}\,\Delta TD - \underset{(.0043)}{.0013}\,\Delta POP \\ & + \underset{(.0005)}{.0012}\,\Delta PCY + \underset{(.0245)}{.0035}\,\Delta RW + \underset{(.2079)}{.1029}\,\Delta DNS \\ & + \underset{(.0299)}{.0454}\,\Delta GNPD + \underset{(.0100)}{.0183}\,\Delta FA \end{aligned} \tag{13}$$

$$R^2 = .2814$$
$$\text{S.E.} = .4267$$

where all the explanatory variables are lagged one year because police budgets are generally set in the year preceding when the money is to be spent on the basis of whatever information is available to citizens and decision makers at that time. The numbers in parentheses below the coefficients are the standard errors of the coefficients.

The signs of the coefficients of the crime variables and the traffic deaths variable in both regressions are thought to reflect the feelings of citizens and state and local government decision makers about the relative efficiency of police versus private and other public measures for decreasing the expected losses of citizens from crime and traffic accidents. The signs of the coefficients for ΔPOP in each regression are believed to reflect the elasticity of total real state and local police expenditures with respect to changes in the number of people living in each state. Real per capita income is included both as a measure of the ability of citizens to pay for state and local government services, and because as personal incomes rise private security measures may tend to become more attractive as alternatives to public security measures.

Average real weekly earnings of production workers is included to control for regional price differences. Population density is included because the efficiency of public versus private security measures is believed to rise with increasing density. The coefficients of $\Delta GNPD$ in both regressions, like the coefficients of ΔRW, are thought to reflect the elasticity of real per capita state and local police expenditures with respect to changes in the price of a given amount and quality of police services. During the period spanned by this study, very little federal aid to state and local governments was earmarked for expenditure on police services. This aid thus artificially lowered the price of supplying other state and local services relative to the price of supplying police service. On the other hand, this aid also served to relax state and local budget constraints. Thus the coefficients of ΔFA in both regressions are believed to represent the net effects of a substitution and an income effect.

The results of the regressions are moderately satisfactory. Autocorrelation is not a serious problem. Nor are the somewhat low R^2's obtained for both regressions necessarily an indication that either is incorrectly specified.[27] (See also Morrison[29] for the properties of R^2's for regressions based on micro data.) The low levels of significance associated with several of the parameter estimates are attributed to both the weakness of some of the causal relationships, and measurement and aggregation errors. Since the major emphasis in this paper is on the approach rather than our numerical results, we have not attempted to improve the empirical results. However, in any actual application verifying and measuring the strength of the relevant causal relationships would be crucial to a successful solution of the allocation problem.

Assuming now that (12) and (13) are correctly specified, and that the coefficient estimates obtained are reasonable estimates of the parameters of these relationships, we will now use these results to analyze how federal aid might be allocated among the North Central states to achieve certain hypothetical objectives.

ΔFA for each state s is assumed to be subject to federal control over some range

$$F_1^s \leq \Delta FA_s \leq F_2^s \tag{14}$$

where the lower and upper bounds on changes in federal aid to state s result from budgetary circumstances, political considerations, and so forth. In the present example, the bounds used for each state are the lowest and highest values attained by ΔFA_s for that state over the sample period. We have also assumed that increases in ΔFA in one year contribute to increases in $\Delta GNPD$ in the following year[2] and hence that $\Delta GNPD$ is subject to indirect federal control over some range

$$G_1 \leq \Delta GNPD \leq G_2. \tag{15}$$

In the present example, the bounds used are 2.6 and 11.4 where these values are the highest and lowest values attained by $\Delta GNPD$ over the sample period.

Finally we have assumed that as part of an anti-inflation policy one of the government's multiple objectives is to maintain $\Delta GNPD_{t+1} \leq \Delta GNPD_t$. If

$$\Delta GNPD_{t+1} = a\Delta FA_{s,t}, \tag{16}$$

then we must have

$$a\Delta FA_{s,t} \leq \Delta GNPD_t. \tag{17}$$

Using our entire data base for the 12 North Central states, the estimate obtained for a is 0.56146. Relationship (16) is clearly a gross simplification of reality. We have introduced relationships (16) and (17) to illustrate how competing objectives of decision-makers might be accommodated within this type of a model.

Finally we have introduced a budget constraint on total monetary federal aid to all state and local governments in the North Central states. Congress, for instance, might decide on regional objectives and ceilings for federal aid to state and local governments. The planning problem would then be to achieve an optimal allocation of federal aid within each region, given these regional objectives and expenditure ceilings. In our example, allocation of federal aid between the 12 North Central states must satisfy the relationship

$$(GNPD_{t-1} + \Delta GNPD_t) \sum_{s=1}^{12} POP_{s,t}(FA_{s,t-1} + \Delta FA_{s,t}) \leq B, \tag{18}$$

where $POP_{s,t}$ is the projected population for state s in period t, and B is the given expenditure ceiling. In this example, actual 1969 population figures and 1968 values for $GNPD_{t-1}$ and $FA_{s,t-1}$ are used.

We will now use our solution to Problem I for $T = 1$ to select optimal values of ΔFA_s for the 12 North Central states, and indirectly to select an optimal value for $\Delta GNPD$ for the period of 1968–69. Let d_s denote an index of benefit assigned to State 2 of the Markov chain for state s. Then our objective function becomes

$$\sum_{s=1}^{12} (p_1^s(1) + d_s p_2^s(1)) \tag{19}$$

where $p_i^s(1)$ is the probability that state s will be found in State i of the Markov Chain in the next period ($s = 1, 2, \ldots, 12$; $i = 1, 2$). Substituting known or predicted 1968–69 values for ΔPCM_s through ΔDNS_s into (12) and (13), the probabilities of a state moving from State 1 to State 2, or of remaining in State 2 of the Markov Chain can be analyzed as functions of $\Delta GNPD$ and ΔFA_s for 1968–69 using expression (2). In this example we have used known values for ΔPCM_s through ΔDNS_s. Thus our problem becomes to maximize (19) with respect to ΔFA_s and $\Delta GNPD$ subject to (14), (15), (17), (18) and

$$p_j^s(1) = p_1^s(0)\, p_{1j}^s + p_2^s(0)\, p_{2j}^s, \quad j = 1, 2, \tag{20}$$

where p_{12}^s and p_{22}^s are given by (12) and (13) with 1968–69 values substituted for all variables except ΔFA_s and $\Delta GNPD$ (clearly $p_{11}^s = 1 - p_{12}^s$ and $p_{21}^s = 1 - p_{22}^s$, and the subscript for $\Delta GNPD$ and the second subscript for $\Delta FA_{s,t}$ will be dropped hereafter).

This problem would be a linear program except for constraint (18). Since the range for $\Delta GNPD$, however, is given by (15) ($2.6 \leq \Delta GNPD \leq 11.4$ in our case), it is possible to approximately solve the problem by discretizing the range of values for $\Delta GNPD$ and by solving the resulting linear program for fixed values of $\Delta GNPD$.

Substituting into (12) and (13) the 1968–69 values for $\Delta PCM_s - \Delta DNS_s$ we have

$$p_{12}^s = a_s + 0.0168\, \Delta FA_s - 0.0218\, \Delta GNPD \tag{21}$$

and

$$p_{22}^s = b_s + 0.0183\, \Delta FA_s + 0.0454\, \Delta GNPD, \tag{22}$$

where a_s and b_s are constant terms. Values for F_1^s, F_2^s, $p_1^s(0)$ and $p_2^s(0)$ are given in Table 1, where $p_i^s(0) = 1$ if state s was found in State i for 1967–68 and $p_i^s(0) = 0$ otherwise ($i = 1, 2$). Also included in Table 1 are values for real per capital federal aid to state s in 1968 (in dollars).

It is seen from the table that the seventh state ($s = 7$), Iowa, is the only state for which real federal aid decreased from 1967–1968. Thus we will consider two possible sets of weights for States 1 and 2 of the Markov Chain:

Case 1.
$$\begin{aligned} d_s &= 1.5 \\ d_7 &= 3.0 \end{aligned} \quad (s \neq 7)$$

TABLE 1. UPPER AND LOWER F_1^s, F_2^s; REAL FEDERAL AID FA_s: INITIAL PROBABILITIES $p_1^s(0)$, $p_2^s(0)$

s	1	2	3	4	5	6	7	8	9	10	11	12
F_1^s	−3.73	−2.27	−1.87	−0.49	−2.95	−2.70	−8.64	−3.89	−11.44	−15.50	−6.78	−3.48
F_2^s	8.38	7.08	10.36	7.94	8.69	11.36	8.93	11.62	18.26	15.15	11.37	16.03
FA_s for 1968 (in dollars)	40.03	40.97	52.20	50.85	49.73	69.86	56.88	53.51	80.29	86.77	56.24	51.00
$P_1^s(0)$	0	0	0	0	0	0	1	0	0	0	0	0
$P_2^s(0)$	1	1	1	1	1	1	0	1	1	1	1	1

Case 2. $d_s = 2$ for all s.

Then the objective function (omitting the constant term) is seen to be the following:

Case 1. $$0{\cdot}0084 \sum_{\substack{s=1\\s\neq 7}}^{12} \Delta FA_s + 0{\cdot}0366\,\Delta FA_7 - 0.0291\,\Delta GNPD$$

Case 2. $$0.0168 \sum_{\substack{s=1\\s\neq 7}}^{12} \Delta FA_s + 0.0183\,\Delta FA_7 - 0.1944\,\Delta GNPD.$$

Also expression (18) implies that

$$\sum_{s=1}^{12} POP_{s,1968}\,\Delta FA_s \leq B/(1.447 + \Delta GNPD/100) - 2878.47$$

where

$$144.7 = GNPD_{1968}, 2878.47 = \sum_{s=1}^{12} POP_{s,1969} FA_{s,1968}$$

and POP and B are in terms of millions of people and millions of dollars respectively.

Actual federal aid to North-Central state and local governments in 1968 was 4,081 (in millions of dollars). Suppose congress set the budget for federal aid to the state and local governments of this region for 1969 at $B = 4{,}500$ (in millions of dollars). Then the optimal values of the objective function for both Case 1 and Case 2 for three hypothetical values of $\Delta GNPD$ (3, 5 and 11) are given in Table 2. Also included in Table 2 are the optimal values of the objective function for $B = 5{,}000$. Optimal values for ΔFA_s are given in Table 3. In the case of a tight budget (i.e. for smaller values of B) optimal values for ΔFA_s can be negative (subject to constraints). This might correspond to some sort of increased taxation in particular states, or perhaps to federal borrowing or revenue sharing transfers from certain state and local governments.

CONCLUSION

In this paper a finite Markov Chain model is estimated, and then used to determine an optimal allocation of federal aid to North-Central state and

TABLE 2. OPTIMAL VALUES OF THE OBJECTIVE FUNCTION FOR FIXED VALUES OF $\Delta GNPD$ (B IN MILLIONS OF DOLLARS)

	$B = 4{,}500$		$B = 5{,}000$	
$\Delta GNPD \div 100$	Case 1	Case 2	Case 1	Case 2
0.03	0.58	0.87	0.68	1.08
0.07	0.82	1.16	1.19	1.90
0.11	0.75	1.01	1.21	1.95

TABLE 3. OPTIMAL VALUES FOR ΔFA_s FOR FIXED VALUES OF $\Delta GNPD$ (ΔFA_s IN MILLIONS OF DOLLARS) AND FOR CASE 2

	$B = 4{,}500$			$B = 5{,}000$		
ΔFA_s	$\Delta GNPD = 3$	$\Delta GNPD = 7$	$\Delta GNPD = 11$	$\Delta GNPD = 3$	$\Delta GNPD = 7$	$\Delta GNPD = 11$
$s = 1$	0.25	−3.73	−3.73	5.34	8.38	0.93
2	5.34	−2.27	−2.27	5.34	7.08	7.08
3	−1.87	−1.87	−1.87	5.34	−0.19	−1.87
4	5.34	−0.49	−0.49	5.34	7.94	7.94
5	5.34	8.69	−2.95	5.34	8.69	8.69
6	5.34	11.36	5.27	5.34	11.36	11.36
7	5.34	8.93	8.93	5.34	8.93	8.93
8	5.34	−0.73	−3.89	5.34	11.62	11.62
9	5.34	12.46	18.26	5.34	12.46	18.26
10	5.34	12.46	15.15	5.34	12.46	15.15
11	5.34	11.37	11.37	5.34	11.37	11.37
12	5.34	12.46	16.03	5.34	12.46	16.03

local governments given a regional expenditure ceiling and competing objectives. Markov models for finite and infinite planning horizons and estimation of transition probabilities are also discussed. Our policy expenditure example is intended to be purely illustrative since in practice policy makers would surely have to take into account the impact of federal aid on other aspects of state and local expenditure decisions.

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