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# Inventory Dynamics and Supply Chain Coordination

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This paper extends the theory of supply chain incentive contracts from the static newsvendor framework of the existing literature to the simplest dynamic setting. A manufacturer distributes a product through retailers who compete on both price and fill rates. We show that inventory durability is the key factor in determining the underlying nature of incentive distortions and their contractual resolutions. When the product is highly perishable, retailers are biased toward excessive price competition and inadequate inventories. Vertical price *floors* or inventory buybacks (subsidies for unsold inventory) can coordinate incentives in both pricing and inventory decisions. When the product is less perishable, the distortion is reversed and vertical price *ceilings* or inventory penalties can coordinate incentives.

*Key words*: supply chain coordination; inventory dynamics; price and inventory competition; contracts
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## 1. Introduction

The coordination of price and inventory incentives along a supply chain is both vital in practice and a central theme in the management science literature. The literature on this topic is built almost entirely on the assumption that inventory is completely perishable and that the traditional, static, newsvendor model applies (see Cachon 2003 for a review). The paradigmatic framework of this literature has a manufacturer distributing through downstream retailers who face demand uncertainty. This paper extends the theory of supply chain incentives by incorporating two features into this framework: the durability of inventory (and therefore inventory dynamics), and competition at the retail level on both price and fill rates.[1]

We find that incentive distortions along the supply chain and their contractual resolutions vary with changes in inventory durability. If the product is highly perishable, downstream retailers are biased toward excessive price competition and away from competition on fill rates. A vertical price floor or a subsidy for holding inventory can eliminate this incentive distortion: The floor prevents excessive price competition; alternatively, the subsidy encourages investment in inventory. When the product is less perishable, however, the distortion in retailers' competitive strategies can be reversed and opposite instruments are optimally applied.

Our insights into the impact of dynamics on optimal contracts can be traced ultimately to a single effect. As shown in Krishnan and Winter (2007), the static model that has been at the core of the supply chain coordination literature does not allow for perhaps the most fundamental of vertical incentive distortions: the double mark-up effect on prices. Under a wholesale price or two-part pricing contract, once inventory is sold, the upstream manufacturer has no interest in the retail price because its revenues are already determined. Recognizing the dynamics of real-world inventory decisions resurrects the double mark-up effect on prices; with the carryover of inventory, the manufacturer now has a direct interest in the retailer's price decision because this decision influences how much the retailer buys in future periods. A dynamic theory is therefore not only realistic but essential in understanding the incentive issues that arise in supply chains.

The literature reviewed by Cachon (2003) and the economics literature on vertical restraints (see Katz 1989) analyze price and inventory incentives in a static setting; see Deneckere et al. (1996, 1997), Butz (1997), Dana and Spier (2001), Bernstein and

[1] Fill-rate competition reflects the fact that inventory often plays a *strategic* role by attracting customers who value the greater probability of finding their preferred product in stock. As Dana (2001, p. 497) points out, "car dealers, video rental chains, department stores, mail order suppliers, and appliance stores" regularly advertise availability. For example, in 1998, the video rental chain Blockbuster launched the successful "Go Home Happy" marketing campaign highlighting the increased availability of popular movies in its stores (Dana and Spier 2001). Ioannou et al. (2009) estimate the effect of inventory levels on rentals in the U.S. video rental industry. The retail store Casual Male, which sells clothing to men who wear larger sizes, has advertised greater availability of popular lines of clothing (Speer 2006). Bernstein and Federgruen (2004) provide additional examples.

Federgruen (2005), Narayanan et al. (2005), and Krishnan and Winter (2007). In a dynamic setting, Bernstein and Federgruen (2004) analyze oligopoly price and inventory competition but do not analyze the vertical coordination of downstream decisions. Bernstein and Federgruen (2007) do analyze coordination, and show that a wholesale price and "backlogging penalty" achieves efficiency. None of these papers have identified the role that product durability plays in shaping competing retailers' incentives. A related and recent literature incorporates our second feature, the strategic effect of inventory in attracting demand (Balakrishnan et al. 2004; Dana and Petruzzi 2001; Deneckere and Peck 1995; Dana 2001; Bernstein and Federgruen 2004, 2007), but these papers do not consider the impact of inventory durability and dynamics on incentive coordination.

## 2. Model

The theories of incentive coordination and dynamic inventory management can each be very complex. In developing a theory combining both, it is therefore essential to adopt the simplest possible model. We consider a monopolist distributing a single product, in discrete time, through two competing retailers. The only link between periods is the inventory that is carried over. In combination with stationarity in demand, this will yield a recursive structure in which optimal contracts can be characterized as the solutions to simple, myopic problems.

Demand in each period is uncertain and depends on the price and inventory level chosen by each retailer $i$. In each period $t$, at retail prices $\mathbf{p}_t = (p_{1t}, p_{2t})$ and inventory levels $\mathbf{y}_t = (y_{1t}, y_{2t})$, the demand at retailer $i$ is $q_{it}(\mathbf{p}_t, \mathbf{y}_t, \boldsymbol{\phi}_t)$, where $\boldsymbol{\phi}_t$ is a real-valued random variable representing demand uncertainty. The number of *transactions* by firm 1 in period $t$ is $T_{it}(\mathbf{p}_t, \mathbf{y}_t, \boldsymbol{\phi}_t) = \min(q_{it}(\mathbf{p}_t, \mathbf{y}_t, \boldsymbol{\phi}_t), y_{it})$. The parameter $\boldsymbol{\phi}_t$ has a joint distribution for the two retailers, with a continuous density, and is independent and identically distributed over time.

Consumers base their shopping decisions on the price and expected fill rate at each store. In any period, consumers shop at only one outlet and unsatisfied demand in each period is lost.[2] We assume that in every period consumers observe the price and inventory choices of both retailers and can therefore infer fill rates: we thus follow the literature in abstracting completely from time delays in learning about inventories.

Note that the decision of a consumer to shop at store $i$ reduces the fill rate for all other consumers at that store. In this respect, the demand side is analogous to demand in a market with congestion externalities and the demand functions $q_{it}$ are themselves determined as the outcome of a subgame among consumers. With enough structure on the model, which we provide in the expanded working paper version of this article (Krishnan and Winter 2009), the existence of an equilibrium in this demand-side game is assured.

Throughout this article we assume that a firm's demand function, and therefore its transaction function, is nonincreasing in its own price and nondecreasing in the price at the other outlet.[3] Similarly, demand and transactions are nondecreasing in a firm's inventory level and nonincreasing in the inventory level at the other outlet. In Krishnan and Winter (2009), we also make assumptions to guarantee the existence of a unique and symmetric solution for the centralized firm, and a symmetric pure strategy Nash equilibrium in the game between the retailers. In short, we ensure that the transaction function $T_{it}$ is well behaved. For purposes of this article, we simply assume the existence of a unique, symmetric, centralized solution, and a symmetric pure strategy Nash equilibrium in the decentralized game.

### 2.1. Centralized Firm

The manufacturer's marginal production cost is $c$ per unit. Consider the payoffs of a centralized (vertically integrated) firm that chooses price and inventory to maximize total profits. Start with a single period. The profit of the centralized firm in this *static* model (hence the superscript $s$) is obtained by choosing $(\mathbf{p}, \mathbf{y})$ to maximize

$$E\Pi^s(\mathbf{p}, \mathbf{y}, \boldsymbol{\phi}) = p_1 ET_1(\mathbf{p}, \mathbf{y}, \boldsymbol{\phi}) + p_2 ET_2(\mathbf{p}, \mathbf{y}, \boldsymbol{\phi}) - cy_1 - cy_2. \quad (1)$$

In the dynamic model, however, excess inventory is carried forward. Define $(S_1, S_2)$ as the inventory available at each outlet at the beginning of any period. In our model, this inventory summarizes the history relevant for current payoffs, i.e., $(S_1, S_2)$ is the state in any period. Let $EV_t(S_1, S_2)$ denote the steady-state expected discounted profits in period $t$ given the state $(S_1, S_2)$. Let $\delta$ be the discount rate and $\rho$ be

[2] In principle, our model can be extended to incorporate backlogging, i.e., satisfying consumer demand with a (costly) delay.

[3] In Krishnan and Winter (2007), an increase in a rival's price has two impacts on a firm's demand: a direct effect of the price increase, and an indirect effect that operates through inventory spillovers. The indirect effect may be negative. The contractual evidence in Krishnan and Winter indicates that the direct effect dominates. We focus here on the presumptive case where the direct effect dominates, but our analytical framework can accommodate the scenario where the indirect effect dominates.

the exponential rate of inventory decay.[4] The dynamic programming recursive formulation is

$$EV_t(S_1, S_2) = \max_{\mathbf{p}_t \geq 0;\, y_{1t} \geq S_1;\, y_{2t} \geq S_2} E\Pi^s(\mathbf{p}_t, \mathbf{y}_t, \boldsymbol{\phi}_t) + c\sum_{i=1}^{2} S_i + \delta EV_{t+1}[(1-\rho)(y_{1t} - T_{1t}(\mathbf{p}, \mathbf{y}, \boldsymbol{\phi}_t)), (1-\rho)(y_{2t} - T_{2t}(\mathbf{p}, \mathbf{y}, \boldsymbol{\phi}_t))]. \quad (2)$$

In (2), $c\sum_{i=1}^{2} S_i$ represents the cost saved from inventory carryover into the current period.[5] The current-period payoff, therefore, is $E\Pi^s + c\sum_{i=1}^{2} S_i$. Rather than accounting for the value of inventory carried over into the current period, we can account for the value of inventory left over at the end of the previous period. Specifically, one unit of inventory carried over into the current period is worth $c$, or equivalently, one unit of inventory left over at the end of the previous period is worth $\delta(1-\rho)c$ in the current period. Let $\gamma \equiv \delta(1-\rho)$; we refer to $\gamma$ as the "durability factor" as a high value of $\gamma$ indicates that the product maintains its value well into the future. We now define

$$E\Pi^d(\mathbf{p}, \mathbf{y}, \boldsymbol{\phi}) \equiv E\Pi^s(\mathbf{p}, \mathbf{y}, \boldsymbol{\phi}) + \gamma c[y_i - ET_i(\mathbf{p}, \mathbf{y}, \boldsymbol{\phi}) + y_j - ET_j(\mathbf{p}, \mathbf{y}, \boldsymbol{\phi})]. \quad (3)$$

We have assumed that $E\Pi^s$, and therefore $E\Pi^d$ are well defined. It is clear that the dynamic program (2) satisfies the four sufficient conditions for the existence of a myopic optimum (see Heyman and Sobel 2004, pp. 83–85). The myopically optimal stationary solution is found by choosing $(\mathbf{p}, \mathbf{y})$ to maximize the modified single-period payoff function $E\Pi^d$.

The aim of contractual solutions to incentive problems is (to the extent possible) to duplicate this joint profit solution. We next analyze the fully decentralized solution in order to identify incentive distortions. Because we are interested in the feasibility of decentralizing downstream price and inventory decisions, we set aside the complete contract where the manufacturer simply specifies both the price and inventory decisions downstream.

### 2.2. Decentralized Firms

In any period the manufacturer charges the retailers a per-unit wholesale price $w$ and a fixed fee $F$. In a stationary dynamic model, optimal $w$ and $F$ are functions of the state variable. In our model, however, constant marginal production cost yields optimal $w$ and $F$ as constants. We will later allow the manufacturer to specify various other contractual terms at the outset, such as vertical price restraints (floors or ceilings) or payments tied to retail inventory levels (subsidies or penalties).

The best-response payoff function for each retailer can be defined using arguments similar to that used for the centralized firm. In the static model, the profit of retailer $i$, gross of the fixed fee $F$, is given by choosing $(p_i, y_i)$ to maximize

$$E\pi_i^s(p_i, y_i, \boldsymbol{\phi}; (p_j, y_j)) = p_i ET_i(\mathbf{p}, \mathbf{y}, \boldsymbol{\phi}) - wy_i. \quad (4)$$

In the dynamic case, retailer $i$'s best-response payoff function is

$$Ev_{it}(S_i) = \max_{p_{it} \geq 0;\, y_{it} \geq S_i} E\pi_i^s(\mathbf{p}_t, \mathbf{y}_t, \boldsymbol{\phi}_t) + cS_i + \delta Ev_{i,\,t+1}[(1-\rho)(y_{it} - T_{it}(\mathbf{p}_t, \mathbf{y}_t, \boldsymbol{\phi}_t))]. \quad (5)$$

This function also satisfies the conditions for a myopic optimum (using an argument analogous to the centralized case). Retailer $i$'s myopically optimal best-response function, again gross of the per-period fixed fee $F$, is given by

$$E\pi_i^d(p_i, y_i, \boldsymbol{\phi}; (p_j, y_j)) = E\pi_i^s(\mathbf{p}, \mathbf{y}, \boldsymbol{\phi}) + \gamma w[y_i - ET_i(\mathbf{p}, \mathbf{y}, \boldsymbol{\phi})]. \quad (6)$$

Note that the excess inventory is valued by the firm simply as a reduction in the necessary expenditure on inputs next period. For the decentralized retailer, this reduction in future costs depends on the wholesale price. For the centralized firm, the cost savings depend on the savings in the production cost. This difference in "salvage value" of unsold inventory (between the centralized and decentralized firms) will play a critical role in the dynamics of the incentive distortions.

## 3. Incentive Distortions

The starting point to any contract design problem is to identify why agents' incentives may be distorted relative to collectively optimal decisions. At the joint profit maximizing decision (where the centralized firm's first-order conditions are satisfied), why might a decentralized retailer have a marginal incentive to deviate from the collective optimum?

We extend the methodology of Krishnan and Winter (2007) to the dynamic problem considered here. This involves the comparison of first-order conditions for the individual retailer's optimum with those of the collective optimum in order to characterize incentive distortions. Two kinds of distortions

[4] The "exponential decay" model of inventory perishability is often used in the literature (Nahmias 1982).

[5] As long as the optimal order-up-to levels, $y_{it}$, in each period exceeds the starting inventories, $S_{it}$, the full cost savings are a benefit. We will show that this condition holds in our stationary framework.

appear. The first is the *vertical externality*: the retailer ignores the impact on the upstream manufacturer's profits of a change in price or inventory decisions. The second is the *horizontal externality*: the retailer ignores the impact on profits earned by its rival retailer (and the upstream manufacturer) from sales to the rival retailer.[6]

For the dynamic problem, using (6) and (3), we can solve for the difference in first-order conditions and decompose this difference as follows:

$$\frac{\partial E\pi_i^d}{\partial y_i} = \frac{\partial E\Pi^d}{\partial y_i} - \overbrace{(w-c)\left(1-\gamma\left(1-\frac{\partial ET_i}{\partial y_i}\right)\right)}^{\text{vertical externality}} - \overbrace{(p_j-\gamma c)}^{\text{horizontal externality}}\frac{\partial ET_j}{\partial y_i}, \tag{7}$$

$$\frac{\partial E\pi_i^d}{\partial p_i} = \frac{\partial E\Pi^d}{\partial p_i} - \overbrace{\gamma(w-c)\frac{\partial ET_i}{\partial p_i}}^{\text{vertical externality}} - \overbrace{(p_j-\gamma c)\frac{\partial ET_j}{\partial p_i}}^{\text{horizontal externality}}. \tag{8}$$

In (7), the first externality term represents the marginal flow of profits upstream from an additional unit of $y_i$ (the vertical externality); the second externality term represents the marginal flow of profits to both the rival retailer and the upstream firm through the impact of $y_i$ on the transactions at outlet $j$ (the horizontal externality). Similarly for the price externalities in (8).

Note that for each decision, $y_i$ and $p_i$, the two vertical and horizontal externalities are opposite in sign. The purchase of an additional unit of inventory, for example, increases upstream profits directly but reduces the rival's profit and upstream profits resulting from the impact on the rival's sales. Note, in addition, that the vertical externality is increasing in $w$ but the horizontal externality is independent of $w$. This means that for each action, inventory and price, there is a particular $w$ that will leave the externalities exactly offsetting and the retailer's incentives optimal, conditional upon the level of the other action.

Can a contract that specifies only a wholesale price $w$ and a fixed fee $F$ achieve the first best price and inventory in a decentralized setting? This requires that the same $w$ leaves the externalities offsetting in both equations. (Because of the availability of fixed fees, the wholesale price $w$ is an instrument for eliciting the right incentives; it need not be used to collect rents.)

If the pairs of externalities are not offsetting at the same $w$, then retailers will be distorted toward either excessive price competition or excessive inventory competition. For example, suppose setting $w$ equal to some value $w_{\text{inv}}$ renders the pair of externalities in the inventory equation identical (and similarly define $w_{\text{price}}$). If, at $w = w_{\text{inv}}$, the horizontal externality in the price equation exceeds (in absolute value) the vertical externality in that equation, then the retailer's incentive is distorted toward setting the price too low, i.e., toward excessive price competition. Similarly, if the horizontal price externality is lower than the vertical price externality, then the retailer's incentive is to price too high, i.e., toward insufficient price competition. Eliciting the correct value of price would involve setting $w = w_{\text{price}} < w_{\text{inv}}$ and this would lead to excessive inventories.

**A Special Case: Complete Perishability.** Consider the single-period model where inventory perishes at the end of the period. In other words, the inventory carried over is always zero. Equivalently, $\gamma = 0$. From Equations (7) and (8), setting $\gamma = 0$ yields the following:

$$\frac{\partial E\pi_i^s}{\partial y_i} = \frac{\partial E\Pi^s}{\partial y_i} - \overbrace{(w-c)}^{\text{vertical externality}} - \overbrace{p_j\frac{\partial ET_j}{\partial y_i}}^{\text{horizontal externality}}, \tag{9}$$

$$\frac{\partial E\pi_i^s}{\partial p_i} = \frac{\partial E\Pi^s}{\partial p_i} - \overbrace{p_j\frac{\partial ET_j}{\partial p_i}}^{\text{horizontal externality}}. \tag{10}$$

Equation (10) captures the main feature of the decentralization of price and inventory decisions in the static model. For a fixed level of inventory, *pricing decisions are not subject to a vertical externality*. In other words, given the inventory choice of an outlet, the manufacturer has no *direct* interest in the price at which the inventory is resold. However, the manufacturer is not indifferent to the pricing decision. Rather, from the manufacturer's perspective the downstream pricing decision is *always* distorted. The horizontal pricing externality invariably depresses prices, and because there is no offsetting vertical externality, decentralized pricing is biased toward a price that is too low.

**The General Case.** Consider the case where inventory is carried forward and $\gamma \in (0, 1)$. Pricing decisions are no longer free of the vertical externality: the manufacturer has a direct interest in the retail price because it affects the retailer's purchases in future periods (see Equation (8)).

Under what conditions will the same wholesale price correct the incentive distortions in both the inventory and price decisions? The answer depends entirely on the durability factor. For a general $\gamma$, a single wholesale price that cancels out horizontal

[6] The manufacturer cares about the horizontal externality because it reduces the fixed fee charged up front.

and vertical externalities in both price and inventory decisions may not exist. However, there may exist a particular value of $\gamma$, $\tilde{\gamma}$, at which a wholesale price $\tilde{w}$ alone can coordinate incentives. If $\tilde{\gamma}$ does exist, we can solve for it and $\tilde{w}$ by setting the externality terms in (8) and (7) to zero (when evaluated at $(p^*, y^*)$). Eliminating $\tilde{w}$ we get

$$\tilde{\gamma}=\frac{\partial ET_j/\partial p_i}{(\partial ET_j/\partial y_i)(\partial ET_i/\partial p_i)+\partial ET_j/\partial p_i-(\partial ET_j/\partial p_i)(\partial ET_i/\partial y_i)}\bigg|_{(p^*,y^*)}. \tag{11}$$

That $\tilde{\gamma} \geq 0$ is easily confirmed. Denote $p_i$ as the price charged by a decentralized retailer, and $p_c$ as the price charged by a centralized firm, and recall the assumption of a symmetric equilibrium ($p_i = p_j$; $ET_i = ET_j$, etc.). Define the price elasticity of individual outlet transactions as $\epsilon^T_{p_i} = (p_i/ET_i)(\partial ET_j/\partial p_i)$, and $\epsilon^T_{p_c}$ as the price elasticity of combined (total) transactions, both evaluated at $(p^*, y^*)$. The inventory elasticities, $\epsilon^T_{y_i}$ and $\epsilon^T_{y_c}$, are defined similarly. Using (11), we arrive at the following sufficient condition for $\tilde{\gamma} < 1$:

$$\frac{\epsilon^T_{p_i}}{\epsilon^T_{p_c}} < \frac{\epsilon^T_{y_i}}{\epsilon^T_{y_c}} \iff \epsilon^T_{p_i} < \frac{\epsilon^T_{y_i}}{\epsilon^T_{y_c}}. \tag{12}$$

The equivalence condition in (12) follows from a fundamental property of the newsvendor model with endogenous pricing: The price elasticity of transactions for the centralized firm, evaluated at the optimal centralized decisions, must equal 1. Once inventory is set, the optimal price for either decision maker must maximize revenue, and at a revenue-maximizing price the elasticity is always 1.[7]

If (12) is satisfied, and if $\gamma = \tilde{\gamma}$, then the wholesale price $w = \tilde{w}$ and a fixed fee can elicit the optimal price and inventory decision, and can transfer rents between the two firms. But if $\gamma \neq \tilde{\gamma}$, then the same wholesale price cannot correct the incentive distortions in both price and inventory. If we choose $w$ to elicit the optimal inventory decision, and if $\gamma < \tilde{\gamma}$, then the retail outlets choose a price that is too low. (The last two terms in (8), $-\gamma(w-c)\partial ET_i/\partial p_i - (p_j-\gamma c)\partial ET_j/\partial p_i$, will sum to a negative value.) On the other hand, if $\gamma > \tilde{\gamma}$, then the retail outlets choose a price that is too high. We summarize these arguments in the following proposition.

Proposition 1.
(a) *If* $\epsilon^T_{p_i} < \epsilon^T_{y_i}/\epsilon^T_{y_c}$, *then* $\tilde{\gamma} < 1$:
(i) *If* $\gamma = \tilde{\gamma}$, *then a wholesale price and fixed fee contract can elicit the optimal price and inventory decision.*
(ii) *If* $\gamma < \tilde{\gamma}$, *the outlets are excessively oriented toward price competition.*
(iii) *If* $\gamma > \tilde{\gamma}$, *the outlets are excessively oriented toward inventory competition.*
(b) *If* $\epsilon^T_{p_i} \geq \epsilon^T_{y_i}/\epsilon^T_{y_c}$, *then* $\tilde{\gamma} \geq 1$ *and the outlets are excessively oriented toward price competition for all values of* $\gamma \in (0, 1)$.

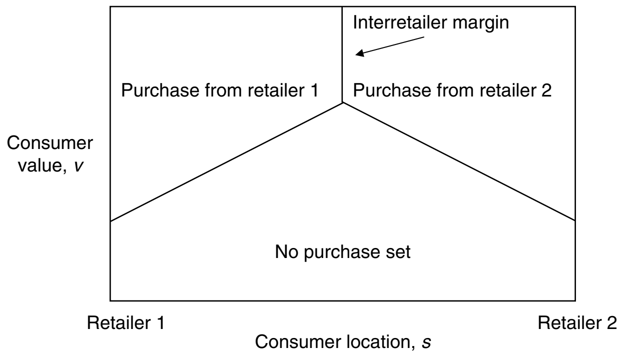
This proposition captures the impact of dynamics—i.e., inventory durability—on supply chain incentive distortions. As we move from the centralized firm to the individual decentralized firm, elasticities of demand with respect to price or inventory increase. According to Proposition 1(a), if price elasticity increases proportionately more, then retailers are always biased toward price competition and away from inventory competition relative to the collective optimum. If the inventory elasticity increases proportionately more, then retailers may be biased away from price competition when the durability factor $\gamma$ is sufficiently high. If inventories are highly perishable, then the vertical externality on price is not strong enough to offset the horizontal externality of price and retailers are biased toward price competition. (In the extreme, static case, the vertical externality on price is "missing.") As inventories become more durable, however, the vertical externality on price can be strong enough to more than offset the horizontal price externality.

In Krishnan and Winter (2009), we provide a structured model that yields condition (12) and therefore the full range of possible incentive distortions. The structured model is an address model, in which a consumer type is indexed by a location $s$ between two firms (which are located at the ends of a unit line segment) as well as a value $v$ for the product, which varies between two bounds. Consumers incur a travel cost per unit distance traveled and make only one shopping trip. The space of consumers is thus a rectangle. A consumer $(s, v)$ located higher in the rectangle cares relatively more about fill rates, because forgoing consumption is more costly for her in terms of forgone surplus. Demand is random, and a realization of demand is itself a density of consumers across this space of consumer types. Given decisions $(\mathbf{p}, \mathbf{y})$ on the part of the two firms, the "demand-side subgame" determines a partition of the consumer space as illustrated in Figure 1 (with proportional rationing among consumers in the event of a stockout, as in Tirole 1988, p. 213).

For perishable products, we have argued that the "missing" vertical externality on price means that retailers are biased toward price competition. Figure 1 is useful in understanding how this is overturned for durable products. For any changes in price and inventory, the marginal impact on the centralized firm profits is determined entirely through the tastes of consumers on the *product margin*: the consumers on the border of the "no purchase set." Because these consumers have relatively low valuation for the

[7] The relationship between optimal price and elasticity is described by the Lerner index; see Tirole (1988, p. 66).

**Figure 1 Spatial Model**




product, the cost to them of encountering a stock out is small and the marginal ex ante value that they attach to greater inventory is therefore low. The aggregate profits are maximized at a combination of low inventory and low price. A retail outlet, on the other hand, chooses price and inventory to accommodate the tastes of consumers on *its* margin. This includes part of the product margin but also the *interretailer margin*. The consumers on the interretailer margin have a higher valuation on average and therefore attach higher ex ante value to inventories. When inventory is not perishable, i.e., it retains its value well into the future, the analysis therefore points to a distortion toward excessive inventory competition and high pricing in a decentralized retail system. This distortion is traced directly to consumer heterogeneity in product valuation.

## 4. Coordinating Contracts

In the previous section, we showed that a wholesale price alone can elicit optimal price and inventory decisions only if $\gamma = \tilde{\gamma}$ and $\tilde{\gamma} < 1$. Otherwise, the price system alone fails to convey optimal retailer incentives and more complex contracts are predicted. We now outline the set of contracts that can be explained as a response to the incentive incompatibility of the simple two-part price contract. We focus on three types of contracts: price restraints (floors or ceilings), and contracts that subsidize or penalize retail inventories.[8]

Suppose retailers are biased toward excessive price competition. If the retailer's payoffs are quasiconcave, the manufacturer can correct the pricing distortion simply by setting a price floor at $p^*$, then lowering $w$ enough can elicit the optimal inventory. In other words, once a price floor at $p^*$ is set, the retailers' first-order condition for optimal inventory can be met at the optimal $y^*$ through the appropriate choice of the instrument $w$. By a similar argument, a price ceiling is an optimal response to an incentive distortion toward excessive inventory competition. This reinforces the insight from Deneckere et al. (1996) and Krishnan and Winter (2007) that price floors help provide incentives to hold inventory in markets where inventories are "important," i.e., where inventories are relatively perishable. Price ceilings coordinate the channel when inventory is less important—which mirrors the well-known optimality of price ceilings in particular price-only situations (e.g., to fix the double mark-up problem).

Inventory subsidies or penalties work as an alternative instrument that, in conjunction with the wholesale price $w$, can elicit the optimal values of the two targets. It is straightforward to incorporate an inventory subsidy $b$ into the two retailer first-order conditions. These two conditions then provide a mapping from the two instruments, $(w, b)$, into the two "targets," $(p, y)$. The mapping is invertible at $(p^*, y^*)$, showing that inventory subsidies (or penalties, if at the optimum $b < 0$) can resolve the incentive problem in inventory and pricing decisions. The optimal subsidy is positive in the case $\gamma < \tilde{\gamma}$, where retailers are inclined toward attracting consumers by relying to an excessive extent on low prices rather than higher inventories. The economic role of the subsidy in encouraging inventory is clear. Less obvious, perhaps, is the fact that an inventory subsidy creates a vertical externality even in the "missing externality" case of complete perishability—thus allowing the balancing of externalities that is necessary for optimal incentives. If $\gamma > \tilde{\gamma}$, a penalty discourages excessive inventory competition.

### 4.1. Practical Considerations and Evidence in the Choice of Contracts

An appropriately designed price restraint, inventory subsidy, or inventory penalty can coordinate price and inventory decisions in our theory. This raises the practical question: Which of these contracts should a manufacturer choose? The choice between alternative coordinating contracts is outside the formal theory and depends on the practical considerations in implementing each contract. The contracts considered in this paper require monitoring of retail prices or inventories. Monitoring is not costless, of course, and the costs depend on the specific product and industry.

The propositions in this paper provide testable implications. For perishable products (including products with a high risk of obsolescence) we predict price floors or inventory subsidies. For durable products, on the other hand, we predict price ceilings or inventory penalties.

The available evidence is that price floors have been popular in the distribution of perishable products. As noted in Krishnan and Winter (2007), inventory

[8] Again, formal details are offered in Krishnan and Winter (2009).

perishability (or limited shelf life) may be the result of (1) seasonality in demand (e.g., greeting cards, toys and other holiday gifts, sports equipment); (2) physical depreciation (e.g., pharmaceuticals); (3) product obsolescence (e.g., magazines, newspapers, etc.); and (4) fashion goods with a limited period of popularity (e.g., clothing). When legal, resale price maintenance has been popular in all these industries.[9] The evidence also suggests that buybacks (inventory subsidies) are used in many of these markets. Evidence to support the link between price ceilings, or inventory penalties, and the durability of inventory remains an open question.

## 5. Conclusion

This paper integrates inventory dynamics into the analysis of price and inventory incentives in a supply chain. We characterize the incentive distortions and optimal contractual responses. The central insight is the sensitivity of optimal contracts to product durability.

In terms of extensions, the key feature of our model that should be relaxed is stationarity. Stationarity is adopted by much of the inventory literature and is, we believe, valuable as a first step in integrating dynamics into the theory of supply chain coordination. Many aspects of inventory policy beyond a stationary structure are important in reality, however, and are addressed in the traditional literature on optimal inventory for a single firm. These include nonstationary demand, inventory with shelf life, and allowing a firm's fill rate to be learned via a process of reputation building rather than instantaneously. Incorporating these extensions into the characterization of supply chain contracts remains an open and challenging task. The decomposition of incentives distortions into horizontal and vertical externalities would continue to be at the foundation of the analysis and inventory durability would remain the key parameter affecting both the incentive distortions and their contractual resolutions.

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[9] In June 2007 the U.S. Supreme Court overturned a nearly century old rule that price floors are per se illegal (in *Leegin Creative Leather Products, Inc. v. PSKS, Inc.*, 551 U.S. 877 (2007)). This is likely to cause a renewed managerial interest in the application of price floors in supply chains.

### References

Balakrishnan A., M. S. Pangburn, E. Stavrulaki. 2004. "Stack them high, Let 'em fly": Lot-sizing policies when inventories stimulate demand. *Management Sci.* **50**(5) 630–644.

Bernstein, F., A. Federgruen. 2004. A general equilibrium model for industries with price and service competition. *Oper. Res.* **52**(6) 868–886.

Bernstein, F., A. Federgruen. 2005. Decentralized supply chains with competing retailers under demand uncertainty. *Management Sci.* **51**(1) 18–29.

Bernstein, F., A. Federgruen. 2007. Coordination mechanisms for supply chains under price and service competition. *Manufacturing Service Oper. Management* **9**(3) 242–262.

Butz, D. A. 1997. Vertical price controls with uncertain demand. *J. Law Econom.* **40**(2) 433–459.

Cachon, G. P. 2003. Supply chain coordination with contracts. A. G. de Kok, S. C. Graves, eds. *Handbooks in Operations Research and Management Science: Supply Chain Management*. Chap. 6. Elsevier, Amsterdam, 229–340.

Dana, J. D., Jr. 2001. Competition in price and availability when availability is unobservable. *RAND J. Econom.* **32**(3) 497–513.

Dana, J. D., Jr., N. C. Petruzzi. 2001. Note: The newsvendor model with endogenous demand. *Management Sci.* **47**(11) 1488–1497.

Dana, J. D., Jr., K. E. Spier. 2001. Revenue sharing and vertical control in the video rental industry. *J. Indust. Econom.* **49**(3) 223–245.

Deneckere, R., J. Peck. 1995. Competition over price and service rate when demand is stochastic: A strategic analysis. *RAND J. Econom.* **26**(1) 148–162.

Deneckere, R., H. Marvel, J. Peck. 1996. Demand uncertainty, inventories, and resale price maintenance. *Quart. J. Econom.* **111**(3) 885–914.

Deneckere, R., H. Marvel, J. Peck. 1997. Demand uncertainty and price maintenance: Markdowns as destructive competition. *Amer. Econom. Rev.* **87**(4) 619–641.

Heyman, D. P., M. J. Sobel. 2004. *Stochastic Models in Operations Research*, Vol. II. Dover Publications, Mineola, NY.

Ioannou, I., J. H. Mortimer, R. Mortimer. 2009. The effects of capacity on sales under alternative vertical contracts. *J. Indust. Econom.* Forthcoming.

Katz, M. L. 1989. Vertical contractual relationships. R. Schmalensee, R. D. Willig, eds. *Handbook of Industrial Organization*. North-Holland, Amsterdam, 655–721.

Krishnan, H., R. A. Winter. 2007. Vertical control of price and inventory. *Amer. Econom. Rev.* **97**(5) 1840–1857.

Krishnan, H., R. A. Winter. 2009. Inventory dynamics and supply chain coordination. Expanded working paper, University of British Columbia, Vancouver, BC, http://csgb.ubc.ca/files/2009-01-krishnan.pdf.

Nahmias, S. 1982. Perishable inventory theory: A review. *Oper. Res.* **30**(4) 680–708.

Narayanan, V. G., A. Raman, J. Singh. 2005. Agency costs in a supply chain with demand uncertainty and price competition. *Management Sci.* **51**(1) 120–132.

Speer, J. K. 2006. The remaking of casual male. *Apparel Magazine* **47**(11) 6–8.

Tirole, J. 1988, *The Theory of Industrial Organization*. MIT Press, Cambridge, MA.