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On the role of revenue-sharing contracts in supply chains

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ABSTRACT

The supply chain coordinating role of revenue-sharing has, to date, been examined only in static models. With downstream competition, the central conclusion in these models is negative: revenue-sharing cannot, except in degenerate form, achieve coordination. Incorporating dynamics, by allowing inventory carryover in discrete time, this paper establishes a foundation for revenue-sharing contracts in aligning incentives.

1. Introduction

Revenue-sharing contracts play an important role in the management of supply chains. The economics and operations literatures have attempted to provide the foundations for the role of revenue-sharing contracts in aligning supply chain incentives by characterizing the optimal use of these contracts. To date, however, the role of revenue-sharing in eliciting optimal price and inventory decisions has been examined only in static models. And when these models incorporate downstream competition the central result is negative: only degenerate revenue-sharing contracts (in which each retailer pays all revenue as a royalty and in turn has all costs subsidized) resolve incentive distortions [5,4].

Under degenerate contracts, retailers retain no revenues and incur no costs and are therefore indifferent to *any* decisions on price and inventory. This is not a proposition that adds insight to the role of revenue-sharing contracts in supply chains. In reality, genuine (non-degenerate) revenue-sharing contracts are observed in video rental, franchising, online retailing and elsewhere, and downstream competition is important in all of these markets (see [16,2,18,3].)

This paper reexamines revenue-sharing contracts in a vertical supply chain incorporating downstream competition. We extend the traditional, static framework to incorporate dynamics, via the ability on the part of retailers to carryover inventory from one period to the next in a discrete time model. Surprisingly, this simple extension provides a theoretical foundation for revenue-sharing as a rational response to incentive distortions inherent in a supply chain whenever there is demand uncertainty. We characterize the revenue-sharing contracts that optimally coordinate price and inventory incentives along the supply chain.

2. Model and analysis

Our model and analysis builds on the framework of [14], which focuses on vertical restraints and does not incorporate revenue-sharing contracts. We consider one upstream manufacturer distributing a product through two competing retailers in an infinite horizon discrete time period model. The sole link between periods are the inventory amounts carried over by the retailers from one period to the next.

The demand for retailer i, in period t, is $q_{it}(p_{1t}, p_{2t}, \phi_t)$, where ϕ_t is a random variable representing demand uncertainty. The variable ϕ_t is independent and identically distributed over time and therefore demand is stationary. In each period t, retailer i chooses price p_i and inventory y_i prior to the realization of ϕ_t. The number of *transactions* or sales at retailer i in period t is then given by $T_{it}(p_{1t}, p_{2t}, y_{it}, \phi_t) = \min(q_{it}(p_{1t}, p_{2t}, \phi_t), y_{it})$. Note that inventories at retailer i do not impact sales at retailer j. The expression for transactions therefore incorporates the common assumption of no spillovers: the event of a stockout at one store has no effect on the demand at the other. We also assume that the firm's demand function, and therefore the transaction function, is non-increasing in the retailer's own price and is non-decreasing in the competing retailer's price. The partial derivatives $\partial ET_{it}/\partial p_{it}$, $\partial ET_{jt}/\partial p_{it}$, and $\partial ET_{it}/\partial y_{it}$ exist, an assumption that can be assured by adopting continuous probability distributions for the random variables.

A fraction ρ of unsold inventory perishes at the end of each period and inventory that does not perish is carried over into the next period, i.e. inventory perishes according to an "exponential decay" model. This simplifying assumption yields a stationary solution to the dynamic inventory problem, which allows a

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concise characterization of the impact of inventory perishability and durability on optimal contracts. We recognize that a large literature on perishable inventory management has proposed and analyzed alternative models of inventory perishability; see [17,8,10] for reviews. It is common in this literature to assume that items in inventory have a fixed or random lifetime; for instance, [6,15] assume that items in inventory has a two-period lifetime. But the optimal solution to the inventory problem in these models is non-stationary. As we discuss in [13], our analytical approach can be extended to non-stationary settings but the incentive distortions and optimal contracts will not be so simply characterized.

The marginal production cost is c; perishability and discounting are the only inventory costs. We assume that firms can transfer profits via a fixed fee. In the revenue-sharing contract, the retailer pays the manufacturer a fixed fee F (which can be negative), a share of revenue, $\mu \in [0, 1]$, and a per-unit wholesale price w (which may be less than the marginal production cost c).

Centralized firm:

Consider first the payoffs of a centralized (vertically-integrated) firm. Define S_{it} as the starting inventory available at retailer i in period t (inherited from period $t-1$) and let $EV_t(S_{1t}, S_{2t})$ denote the expected profits discounted to period t given the state variables (S_{1t}, S_{2t}). We let δ be the discount rate. As mentioned, ρ is the exponential rate of inventory decay. The dynamic programming recursion (omitting the arguments of the transaction functions) is:

$$\begin{aligned} EV_t(S_{1t}, S_{2t}) = \max_{p_{1t}\geq 0; p_{2t}\geq 0; y_{1t}\geq S_{1t}; y_{2t}\geq S_{2t}} & [p_{1t}ET_{1t} + p_{2t}ET_{2t} \\ & - cy_{1t} - cy_{2t} + cS_{1t} + cS_{2t} + \delta EV_{t+1} \\ & \times [(1-\rho)(y_{1t} - T_{1t}), (1-\rho)(y_{2t} - T_{2t})]]. \end{aligned} \tag{1}$$

Let $\gamma = \delta(1-\rho)$; this constant represents the fraction of its present value that a unit of unsold inventory carries over into the next period. When $\gamma = 0$, either the discount factor $\delta = 0$ or product perishability is complete, i.e. $\rho = 1$, and we get the traditional static model. Let $E\Pi(p_1, p_2, y_1, y_2)$ be the current period expected profits including the expected present value of cost-savings from inventory carryover to the next period. It is immediate that

$$\begin{aligned} E\Pi(p_1, p_2, y_1, y_2) \equiv\ & p_1ET_1 + p_2ET_2 - cy_1 - cy_2 \\ & + \gamma c(y_1 - ET_1) + \gamma c(y_2 - ET_2). \end{aligned} \tag{2}$$

Maximizing $E\Pi(p_1, p_2, y_1, y_2)$ yields the optimal stationary price and inventory, as long as the dynamic program (1) satisfies the four sufficient conditions for the existence of a stationary optimum (see [9], pp. 83–85). These conditions are satisfied as long as the starting inventories in all periods, S_t, are always less than the optimal inventory level; finally, this condition is in turn satisfied if we assume that the inventories at the outset of the game are zero.

The optimal prices, p_i^* and p_j^*, and inventories, y_i^* and y_j^*, satisfy the following necessary first-order conditions:

$$\frac{\partial E\Pi}{\partial p_i} = \frac{\partial[p_iET_i]}{\partial p_i} + p_j\frac{\partial ET_j}{\partial p_i} - \gamma c\frac{\partial ET_i}{\partial p_i} - \gamma c\frac{\partial ET_j}{\partial p_i} \tag{3}$$

$$\frac{\partial E\Pi}{\partial y_i} = p_i\frac{\partial ET_i}{\partial y_i} + p_j\frac{\partial ET_j}{\partial y_i} - c\left(1 - \gamma\left(1 - \frac{\partial ET_i}{\partial y_i}\right)\right). \tag{4}$$

Decentralized supply chain:

Can the efficient, centralized, solution be implemented in a decentralized supply chain? It is standard in the supply chain literature, as in the vertical restraints literature in economics, to analyze settings in which contracts *perfectly* resolve incentive problems. This method is adopted because it offers the clearest (if stark) explanation of the incentive-coordinating role of particular contracts.

We analyze the retailer's incentives under a revenue-sharing contract where the manufacturer charges the retailers a per-unit wholesale price w, a revenue share μ and a fixed fee F. Using arguments similar to the centralized case, the best-response payoff function for each retailer, $Ev_{it}(S_{it})$, can be defined as follows. Given w and μ, and gross of the fixed fee, the function is

$$\begin{aligned} Ev_{it}(S_{it}) = \max_{p_{it}\geq 0; y_{it}\geq S_{it}} & [(1-\mu)p_{it}ET_{it} - wy_{it} + wS_{it} \\ & + \delta Ev_{i,t+1}[(1-\rho)(y_{it} - T_{it})]]. \end{aligned} \tag{5}$$

Note that each retailer's payoff is a function of a state variable and current actions. In every period, therefore, the effect of the past on the current environment is captured entirely through the state variable S_{it}. Given this "Markovian" assumption, the appropriate equilibrium concept to apply in this dynamic game between retailers is the *Markov perfect equilibrium* defined "as a profile of Markov strategies that yields a Nash equilibrium in every proper subgame" (see [7]). (This equilibrium concept precludes any cooperation between the retailers that is enforced by punishment strategies, which can lead to a multiplicity of equilibria in infinitely repeated games because of the "folk theorem" (see [7]).)

Similar to the centralized case, the function $Ev_{it}(S_{it})$ also satisfies the conditions for a stationary optimum. The dynamic programming recursion in Eq. (5) can therefore be replaced by the following stationary expected payoff function (again gross of the fixed fee):

$$E\pi_i(p_1, p_2, y_i) \equiv (1-\mu)p_iET_i - wy_i + \gamma w(y_i - ET_i). \tag{6}$$

The π_i-maximizing price and inventory must satisfy the following first-order conditions:

$$\frac{\partial E\pi_i}{\partial p_i} = (1-\mu)\frac{\partial[p_iET_i]}{\partial p_i} - \gamma w\frac{\partial ET_i}{\partial p_i} \tag{7}$$

$$\frac{\partial E\pi_i}{\partial y_i} = (1-\mu)p_i\frac{\partial ET_i}{\partial y_i} - w\left(1 - \gamma\left(1 - \frac{\partial ET_i}{\partial y_i}\right)\right). \tag{8}$$

In our model, the transaction function, and therefore the payoff functions, are quite general. As is well known, existence of a Markov perfect equilibrium is not guaranteed (see [7,11]) and examples of non-existence of equilibrium are easily found. In the supply chain contracting literature, it is common to specify a set of assumptions and prove the existence (and sometimes uniqueness) of pure-strategy equilibria as a precursor to any analysis of the role of contracts (e.g. [1]). In our context, with not only downstream competition but also inventory-carryover in a dynamic setting, the game is complex and to assure existence and uniqueness of a pure-strategy equilibrium, one would need to adopt very specific and highly structured models.

As we demonstrate below, insights into the role of contracts can be developed clearly and directly in the more general setting. Additional structure necessary to guarantee the existence of a unique, pure-strategy, equilibrium can always be imposed on the model, but this does not alter the basic insights. In [12,14], we develop our insights under the general framework and then present a structured model (starting with consumer preferences) where the existence and uniqueness of the equilibrium is guaranteed. Accordingly, in our analysis of revenue-sharing contracts in this paper, we adopt the general model and simply note that the model can be restricted to develop structured examples where existence of a unique, symmetric pure strategy state-space Nash equilibrium is assured. Our formal characterization of the optimal role of revenue-sharing contracts must be qualified with the phrase "if a unique, symmetric, pure-strategy equilibrium exists for the decentralized game".

It is also worth noting here that the "no-spillover" assumption, which is common in the literature (e.g. [1]), is often made because incorporation of inventory spillovers can result in non-quasi-concave payoff functions, which can further complicate equilibrium analysis. Inventory spillovers can, however, be accommodated within a structured demand model that guarantees existence of a unique equilibrium. The static spatial model in [12], for example, incorporates inventory spillovers and generates payoff functions that, for some parameters, yield a unique equilibrium in the game between retailers. In this paper, however, the no-spillover assumption is made only for notational simplicity since we are already adopting the qualification "if an equilibrium exists".

We can now isolate the incentive distortions, i.e. externalities, by comparing the difference in first-order conditions between the retailer and full efficiency. From (3), (4), (7) and (8) we arrive at:

$$\frac{\partial E\pi_i}{\partial y_i} = \frac{\partial E\Pi}{\partial y_i} - \left[(w-c)\left(1-\gamma\left(1-\frac{\partial ET_i}{\partial y_i}\right)\right) + \mu p_i \frac{\partial ET_i}{\partial y_i}\right] \quad (9)$$

$$\frac{\partial E\pi_i}{\partial p_i} = \frac{\partial E\Pi}{\partial p_i} - \left[\gamma(w-c)\frac{\partial ET_i}{\partial p_i} + \mu\frac{\partial[p_i ET_i]}{\partial p_i}\right] - (p_j - \gamma c)\frac{\partial ET_j}{\partial p_i}. \quad (10)$$

The terms in square brackets in (9) and (10) represent the non-appropriated flow of profits upstream through the wholesale margin (the "vertical" externalities); the last term in (10) represents the "horizontal" externality flowing through the cross-elasticity of demand between retailers. At the π_i-optimizing price, note that (from setting RHS of (7) to zero) $\mu\frac{\partial[p_i ET_i]}{\partial p_i} = \frac{\gamma w \mu}{1-\mu}\frac{\partial ET_i}{\partial p_i}$. Therefore, Eq. (10) can be rewritten as:

$$\frac{\partial E\pi_i}{\partial p_i} = \frac{\partial E\Pi}{\partial p_i} - \gamma\left(\frac{w}{1-\mu} - c\right)\frac{\partial ET_i}{\partial p_i} - (p_j - \gamma c)\frac{\partial ET_j}{\partial p_i}. \quad (11)$$

The essence of supply chain coordination is to render the incentives for the independent retailer consistent with the efficiency of the system as a whole. This involves selecting contractual instruments that eliminate the externalities. Define w^* and μ^* as the values of w and μ that lead the vertical and horizontal externalities to sum to zero at (p^*, y^*). From Eqs. (9) and (10) it follows that:

$$w^* = (1-\mu^*)\left[c + \left(\frac{p^* - \gamma c}{\gamma}\right)\frac{\frac{\partial ET_j}{\partial p_i}}{-\frac{\partial ET_i}{\partial p_i}}\right]\Bigg|_{(p^*,y^*)} \quad (12)$$

$$\mu^* = \left[\frac{-\frac{\frac{\partial ET_j}{\partial p_i}}{-\frac{\partial ET_i}{\partial p_i}}\left(1-\gamma\left(1-\frac{\partial ET_i}{\partial y_i}\right)\right)}{\gamma\frac{\partial ET_i}{\partial y_i} - \frac{\frac{\partial ET_j}{\partial p_i}}{-\frac{\partial ET_i}{\partial p_i}}\left(1-\gamma\left(1-\frac{\partial ET_i}{\partial y_i}\right)\right) - \frac{\gamma}{p^*-\gamma c}(1-\gamma)c}\right]\Bigg|_{(p^*,y^*)}. \quad (13)$$

Note that when $\gamma = 0$, we get $\mu^* = 1$ and $w^* = 0$; the degenerate contract. For all $\gamma \in (0, 1)$, $\mu^* \in (0, 1)$ and is in this sense non-degenerate. Depending on the value of μ^*, $w^* > 0$ but may be greater than or less than c.

The following proposition, in summary, captures the ability of revenue-sharing contracts to coordinate incentives and characterizes the optimal such contracts.

Proposition 1. *Providing that symmetric equilibria exist for the centralized firm and the decentralized game, then in any such equilibria:*

(a) *When $\gamma = 0$, the optimal contract is degenerate: $w^* = 0$ and $\mu^* = 1$.*
(b) *When $\gamma \in (0, 1)$, w^* (defined in Eq. (12)) and μ^* (defined in Eq. (13) can also coordinate supply chain incentives).*

Part (a) of the proposition describes a degenerate contract in which the downstream retailer faces no costs and retains no revenue. This is the only such contract, demonstrating, in the case of complete perishability, the failure of non-trivial revenue-sharing contracts to coordinate incentives in the competitive setting. In the non-perishable case (b), however, genuine revenue-sharing achieves the first-best outcome.

To understand the intuition behind Proposition 1, observe that in Eq. (11) when $\gamma = 0$, the impact of both μ and w disappear. Because the final term in the RHS of (11) remains, whatever the value of w and μ, these instruments cannot, in this case, render $\frac{\partial E\pi_i}{\partial p_i} = \frac{\partial E\Pi}{\partial p_i}$. The revenue-sharing contract therefore cannot eliminate the impact of the horizontal externality that retailers' pricing decisions impose on each other, because there is no vertical externality that can offset, or balance, the horizontal externality. When $\gamma > 0$, however, the value of the inventory carried over into the next period *creates* a vertical externality on price because a higher retailer price today means that fewer units will be purchased from the upstream firm tomorrow. The revenue-sharing parameters can be set so that the horizontal and vertical externalities are exactly offsetting, eliminating the distortion in retailers' price incentives.

Note that the "purely vertical" channel, considered by [4], can be analyzed as a special case of our model. If there is a single downstream retailer, the centralized and decentralized decisions (in Eqs. (2) and (6)) can be aligned by simply setting $w = (1-\mu)c$. In the purely vertical channel, a revenue-sharing contract can therefore coordinate incentives for all $\gamma \in [0, 1]$. In fact, a fixed fee is not needed because the revenue (and cost) share μ can achieve any distribution of profits between the two firms.

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