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Innovation and the dynamics of global warming



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ABSTRACT

Global warming and the carbon cycle are a dynamic system with positive feedbacks. Fossil fuels are exhaustible resources. These two facts mean that innovation in clean energy technology, rather than mitigating global warming, can lead to a permanently higher temperature path. This paper explores the impact of innovation in the simplest model linking the economic theory of exhaustible resources with positive feedback dynamics in the carbon cycle.

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Introduction

The green paradox literature shows that seemingly obvious propositions about climate change policy are often wrong. Commitment to future green policies, whether these policies are carbon pricing or subsidies to the innovation of clean energy technology, tends to *raise* current carbon emissions. The logic of the paradox is familiar: the anticipation of green policies lowers the expected returns from the future sale of fossil fuels and therefore lowers the opportunity cost of selling the fossil fuel today. This reduces fossil fuel prices, increasing the consumption of fossil fuels. Carbon emissions therefore rise instead of falling.

The green paradox is generally framed in terms of the unintended impact of policies.¹ But the idea extends directly to the impact of innovation, whether innovation is influenced by policy or not (Hoel, 2008; van der Ploeg and Withagen, 2012). Consider, for example, a market for energy produced from oil in which the current price is 100 dollar per barrel and extraction costs are low. Suppose that a clean, inexhaustible energy substitute is discovered and becomes immediately available at a cost equivalent to 60 dollars per barrel of oil. The owner of any conventional fuel deposit would prefer to sell at 59.99 or less rather than share the energy market with the substitute. Oil from these deposits will be sold at a lower price and exhausted before clean energy captures any market share at all. With the drop in price, oil will thus be extracted more intensively and exhausted at an earlier date. The current flow of carbon emissions rises as a result of innovation as does the near-term stock of atmospheric green house gases.

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¹ Van der Werf and Di Maria (2012) review more than 20 papers about the climate change policy and the green paradox. These authors identify four different policy approaches that may induce a green paradox.

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In the central framework of the green paradox literature, however, innovation in the clean backstop must help the battle against global warming in the long run. The framework includes a market for exhaustible resources with heterogeneous extraction costs for fossil and a clean backstop technology. van der Ploeg and Withagen (2012) offer the most in-depth analysis. The discovery of a clean energy substitute at a cost equivalent of 60 dollars per barrel in the van der Ploeg–Withagen model would mean that any fossil fuel with extraction cost greater than this amount would be left in the ground instead of being extracted. While cumulative emissions rise in the short run with innovation because of the decline in oil prices, cumulative emissions necessarily fall in the long run. If the growth in atmospheric green house gases equals cumulative emissions, as it does in the van der Ploeg–Withagen model, innovation must eventually reduce atmospheric green house gases.² The message from the green paradox literature, in short, is that the paradox disappears in the long run.

This is far too optimistic a conclusion about the impact of clean-energy innovation. This paper shows that innovation – and any government policy that increases the rate of innovation – can not only raise temperatures in the short run but also set us on a *permanently* higher temperature path.

The proposition is not merely a point of disagreement between theoretical models. As carbon pricing has proved politically impossible in many countries (the U.S., in particular), innovation and development of clean energy sources such as wind and solar energy are emerging as the key strategies in the battle against global warming. The strategy has been supported in the U.S. across a wide range of the political spectrum.³ The green paradox literature supports this pro-innovation strategy in the sense that it predicts a long run benefit from clean-energy innovation. But in reality, clean-energy innovation – as a naked policy instrument, unsupported by carbon pricing – can lead to runaway global warming.

The argument is simple. I integrate the economic theory of exhaustible resource extraction with the simplest climate dynamics incorporating a fundamental feature of the carbon cycle: positive feedback effects. In reality, as greater atmospheric carbon raises the global temperature, reflective ice-field melt and methane gas is released from melting permafrost (to take just two examples), resulting in a higher *rate* of flow of carbon from the surface to the atmosphere. On the economic side, energy is produced with either carbon-emitting fossil fuels (of heterogeneous extraction costs) or perfectly clean energy. Innovation in the model consists of an exogenous shift in the per unit cost of clean energy from b_0 to $b_1 < b_0$. Innovation occurs once and for all, and the probability of innovation in a small time interval dt is ρdt , where ρ is an exogenous probability rate. The date of innovation is the only random variable.

These dynamics yield two possible stable steady-state levels of atmospheric carbon and temperature, with an intermediate tipping point. Exceeding the tipping point leads inexorably to the higher steady state temperature, interpreted as “runaway global warming.” Clean energy innovation accelerates carbon emissions through the green paradox effect, which can take the stock of atmospheric carbon above the tipping point. In this model, the long run impact of innovation depends on the innovation date.

The two main results of the theory are illustrated with the aid of Fig. 1. First, the set of innovation dates that lead to runaway global warming is in general not connected. If innovation is realized sufficiently late, in $\Psi_2 \equiv \{t | t \geq t_2\}$ in Fig. 1, then (for the parameter values underlying the figure) low-cost clean energy does not displace enough fossil fuel to avoid atmospheric carbon and temperature crossing the tipping point. Long run temperature converges to the higher steady state value. If innovation is realized early, in $\Psi_1 \equiv \{t | t \leq t_1\}$, then a large stock of remaining fossil fuel and *in situ* carbon is subject to the green paradox effect of lower prices and accelerated emissions. The rapid build up of carbon pushes the atmospheric temperature past the tipping point and again runaway global warming results. The effect of early *realized* innovation I label the ex post long run green paradox. In the intermediate range $t \in (t_1, t_2)$, innovation is late enough that most of the low-cost fossil fuel has already been extracted. The post-innovation price (which must cover extraction costs) therefore remains high and the green paradox effect is weak. Yet enough additional fossil fuel is displaced by the low-cost clean energy that the innovation has the effect of eliminating runaway global warming.

Our second main result is that for a sufficiently low probability rate ρ of innovation, the set Ψ_1 remains, but Ψ_2 can *disappear*. This result captures an ex ante green paradox. The non-empty Ψ_2 at higher values of ρ reflects the lower fossil fuel price that results from the *threat* of innovation. In short, either the early realization of innovation or the mere threat of innovation can lead to runaway global warming.

This is not an anti-clean-energy paper. The possibility of perverse effects of innovation does not mean that optimal policy should limit innovation. The potential negative social impact of innovation, through either the ex post or ex ante green paradox effect, is eliminated through the adjustment of optimal carbon pricing if carbon pricing is adopted. The policy message of this paper is that carbon taxes and innovation subsidies are not *substitute* instruments to battle global warming as is generally assumed. Instead, carbon pricing is even more important with innovation than without. The policy instruments are *complementary*: with carbon taxes, innovation is always valuable whereas without carbon taxes it may or may not be.

² The positive long run benefit of innovation in the van der Ploeg–Withagen model clearly generalizes to a more standard assumption of the literature: that green house gas dynamics are described by $\dot{g}^A = e - ag^A$, where g^A is the stock of atmospheric green house gases, e is emissions, and $a > 0$ is the rate of reabsorption of gases to the earth's surface.

³ Consider, for example, the recent joint call by the Brookings Institute and the American Enterprise Institute (a liberal think tank and conservative think tank, respectively) for an increase in clean energy investment from 4 billion to 25 billion annually (Hayward et al., 2010). In the popular press as well, clean energy subsidies have been touted as a superior instrument to carbon taxes (e.g., David Leonhardt, 2010. A climate proposal beyond cap and trade. New York Times, October 12).

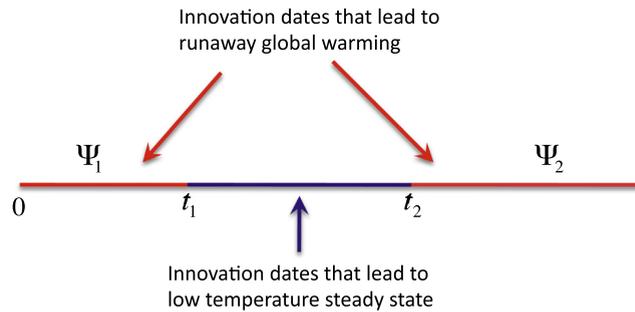


Fig. 1. Mapping of innovation (discovery of low-cost clean technology) into eventual steady state temperatures.

The next section of this paper sets out the model and analyzes the impact of exogenous, stochastic innovation on global warming dynamics in the absence of carbon pricing. The subsequent section moves from theory to reality. I ask whether the theoretical propositions of this paper have any empirical plausibility given available evidence on climate dynamics and fossil fuels reserves. In the conclusion, I connect the theme of this paper to the literature on integrated assessment models with a focus on the most prominent of these, Nordhaus' DICE model.

Model

Background: positive feedback effects

This paper integrates into the green paradox theory positive feedbacks in the carbon cycle. The term positive feedback refers here to an increasing rate of release of green house gases from the earth's surface to the atmosphere as temperature rises. Feedback mechanisms are a standard component of climate models. Among the feedback mechanisms identified by climate scientists, seven are particularly important. The melting of ice sheets reduces the cooling effect that the ice has in reflecting radiation away from the Earth. This means that higher temperatures lead to an increase in the rate of change of temperature, a positive-feedback process known as the ice-albedo effect. The death of vegetation in regions such as the Amazonian rainforests through reduced rainfall, leads to a second mechanism, the release of CO₂ to the atmosphere and to reductions in the absorption of CO₂ by plants. Cox et al. (2000) uncover a third positive-feedback mechanism: global warming can result in increased respiration from bacteria in the soil, releasing additional CO₂. A fourth positive-feedback mechanism is the release of GHG's (mainly methane) from the tundra in the arctic, mainly Eastern Siberia. A well-known study in 2007, led by the University of Alaska's International Arctic Research Centre and the Russian Academy of Sciences, concluded that this mechanism was of strong potential. The fifth mechanism is the release of methane from the oceans, in the form of methyl hydrates, potentially leading to the risk of a "runaway methane global warming". The sixth positive feedback is from the evaporation of water and the accumulation of water vapor in the upper atmosphere. Water vapor is accumulated in greater amounts in a warmer atmosphere, as we know from basic physics. This leads to a higher rate of temperature increase, water vapor being among the most powerful greenhouse gases. The seventh positive feedback is the reduced ability of oceans to absorb carbon dioxide as temperature rises and the oceans become more acidic. The amount of carbon in the oceans is huge and a significant disruption in the rate of absorption of carbon by the oceans could overwhelm all other aspects of carbon-cycle dynamics.⁴ The potential power of positive feedback mechanisms is enormous. Obviously, the uncertainty involved in any of these estimates is huge, given both the inherent uncertainty in predicting physical processes outside of our experience and the property of positive feedback mechanisms to magnify any uncertainty in dynamic systems.

Assumptions

I explore the implications of clean-energy innovation in the simplest framework that joins an economic model of exhaustible fossil fuels with a positive feedback in the carbon cycle. The economic model is based on the following five assumptions:

- (A1) A competitive market for energy. Energy can be supplied by either an existing clean energy technology, at cost b_0 , or by an extraction of a flow x of an exhaustible resource (fossil fuels) at a cost per unit x that depends upon the stock X of the resource already extracted. This unit cost, $c(X)$, satisfies $c'(X) > 0$ and $c''(X) > 0$.⁵ There are no taxes.
- (A2) Innovation takes the form of a once-and-for-all discovery of a new technology that allows the production of clean

⁴ Other positive feedback mechanisms in the carbon cycle include mechanisms on the demand side of the energy market: higher temperatures lead to greater demand for air-conditioning, greater fuel use and therefore a higher rate of carbon emissions.

⁵ This assumption builds into the model the property of competitive exhaustible resource markets so that lower extraction cost deposits are extracted first (Herfindahl, 1967).

energy at a cost $b_1 < b_0$. The probability of discovery in any small time interval dt , conditional upon no discovery to date, is ρdt .

- (A3) Demand for energy is given by a stationary demand function $q(p)$ that is derived from a quasi-linear utility function $u(q)$.⁶
- (A4) Each supplier and each demander of fossil fuels take as given the path of atmospheric carbon. That is, each individual ignores the consequences for global warming of their own transaction.
- (A5) Capital markets in the economy are perfect and the interest rate is a constant, r .

These assumptions generalize Dasgupta and Stiglitz (1981) to the case of heterogeneous extraction costs, a generalization that is necessary to capture the essential trade-off in the impact of innovation. The model also draws upon an earlier article, Long (1975), which considers an exhaustible resource market under the risk of expropriation, rather than displacement by an alternative fuel. As in a number of previous continuous-time models of innovation (see, e.g., Reinganum, 1989) the probability of discovery of the new technology is an exogenous rate ρdt ; but this probability can be interpreted as an instrument of government policy. Subsidizing innovation would increase ρ .⁷

To this economic model, I add the following assumptions on the dynamics of atmospheric carbon and global warming.

- (A6) The environment is represented by a single state variable: the amount of carbon and other green house gases (“carbon” for brevity) in the atmosphere, A .⁸
- (A7) Each unit of fossil fuels extracted adds a unit of carbon emissions to A . That is, fossil fuel is measured in units of carbon emitted.
- (A8) Temperature at any time is a strictly increasing function of A .
- (A9) In addition to emissions from fossil fuel consumption, carbon is released from the earth's surface to the atmosphere. This release is at a rate that is a function of temperature and therefore a function $f(A)$ of A . The function f is increasing, twice differentiable, strictly convex below a threshold concentration, strictly concave above this threshold, bounded, and satisfies $f(0) > 0$.
- (A10) Atmospheric carbon, A , is reabsorbed into the earth's surface at a constant rate a .

These assumptions allow the simplest possible integration of positive feedback effects into a model of innovation and global warming. The assumed convexity of f for low A in (A9) reflects the fact that the marginal impact of feedback effects such as the release of methane from the permafrost are minimal at low temperatures and become more significant as temperature rises. The assumption of an initially increasing marginal impact of temperature on feedback effects is not controversial. The assumption of concavity at higher values of A follows naturally from an assumed bound on f ; the bound in turn is justified by the physical limit on the concentration of carbon in the atmosphere, which implies a bound on the rate of release of carbon from the surface to the atmosphere.⁹ Alternatively, concavity and a bound on temperature can be justified by the fact that irradiation of energy from a black body (the earth) is proportional to the fourth power of temperature of the body, from the Stefan–Boltzmann law. In short, the assumed shape of f is natural given the physics of climate change.

Note that for our analysis in this paper of the purely positive implications of innovation feedback we need not set out any assumptions on social damages from global warming.

Market equilibrium

The equilibrium in this model is Markov with three state variables: X , A and I , where I is an exogenous indicator variable taking the value 0 if a new clean energy has not been discovered and 1 if it has (state variables are upper case; flow

⁶ That is, $u(q) + e$ is the utility of a representative consumer, where e is the consumer's expenditure on all commodities other than energy.

⁷ These assumptions set aside any consideration of the incentives for innovation. I share the assumption of exogenous innovation with van der Ploeg and Withagen (2012) and Hoel (2008) as well as Dasgupta and Stiglitz (1981), but it is important to point out that a substantial literature considers the incentives for innovation in an exhaustible resource context. Early contributions are Gallini et al. (1983), Dasgupta et al. (1983), and Olson (1993). An emerging literature endogenizes innovation in the context of climate change: Gerlagh et al. (2009), Gans (2012), Acemoglu et al. (2012), and Jaakkola (2012). Analyzing the feedback effects of market dynamics on the incentive for innovation in these papers allows one to address a different set of questions than those in this paper. Here innovation is exogenous.

⁸ An earlier version of this paper adopted two state variables, carbon in the atmosphere and carbon on the earth's surface. Richer climate models have more state variables, including heat and carbon in a third reservoir, the deep ocean.

⁹ Obviously, this set of assumptions abstracts completely from a host of economy–climate interactions and the resulting model is by design illustrative rather than realistic. (Nordhaus' model has 16 dynamic equations and 24 dynamic variables; our model will reduce climate dynamics to a single differential equation.) More state variables would yield a more realistic carbon cycle. These would include a deep ocean carbon reservoir; the release of surface carbon as dependent on the history of atmospheric temperature, not just current temperature; regionally variable temperature changes (the areas near the poles are forecast to experience a much higher rate of temperature increase); and lags in the adjustment of temperature to atmospheric green house gases. Energy exchanges across adjacent reservoirs, not just carbon exchanges, are incorporated in more complex climate models. The absorption of carbon from the atmosphere is more complex than the constant rate assumed here and in most other economic models. A more realistic theory would also treat separately different types of feedback effects: higher temperature causing release of additional GHG from the earth's surface; the reduction in the rate at which solar energy is reflected out of the atmosphere by ice-fields; and so on.

variables are lower case). The state equations corresponding to the two endogenous state variables are

$$\dot{A} = x + f(A) - aA \tag{1}$$

$$\dot{X} = x \tag{2}$$

The three-state variable Markov model is rendered tractable by a key separability: the equilibrium prices and quantities depend on the state variables X and I but are completely independent of the environmental state A . This follows from the assumption (A4) that carbon emissions are a pure externality. A competitive equilibrium in the energy exhaustible resource markets consists of a price of energy, $p(X, I)$; a rate of extraction $x(X, I)$ of fossil fuel; and a rate of production $y(X, I)$ of clean energy, that satisfy two conditions: market clearing and supply-side rationality. The market clearing condition, omitting arguments of the functions p, x and y , is

$$q(p) = x + y \tag{3}$$

The supply-side rationality condition is that the owner of each infinitesimal resource deposit of a particular extraction cost k is acting optimally in the following respect. The equilibrium assigns a date $\hat{t}(k)$ to that deposit in the event of no innovation up to date $\hat{t}(k)$ and date $\tilde{t}(k; \tau)$ in the event of innovation at date $\tau < \hat{t}(k)$. These extraction dates must maximize the expected present value of profits for the resource deposit owner. Rather than setting out this equilibrium condition in detail, I characterize the competitive equilibrium using a version of the first welfare theorem. Because the competitive equilibrium in the markets is independent of the environmental variables, the equilibrium quantities maximize the expected present value of market surplus ignoring any impact on the environment.

Post-innovation equilibrium: Following innovation at date τ , equilibrium quantities $x(X, 1)$ and $y(X, 1)$ solve the maximization problem on the right hand side of the following valuation equation, given $X(\tau)$

$$V(X, 1) = \max_{\tilde{x}(t), \tilde{y}(t)} \int_{\tau}^{\infty} e^{-r(t-\tau)} [u(\tilde{x}(t) + \tilde{y}(t)) - c(X(t))\tilde{x}(t) - b_1\tilde{y}(t)] dt$$

subject to

$$\begin{aligned} \dot{X}(t) &= \tilde{x}(t), & X(\hat{t}) &= X \\ \tilde{x}(t), \tilde{y}(t) &\geq 0 \end{aligned}$$

Pre-innovation equilibrium: Before innovation, the equilibrium quantities $x(X, 0)$ and $y(X, 0)$ solve the following problem:

$$V(X, 0) = \max_{\tilde{x}(t), \tilde{y}(t)} \int_{\hat{t}}^{\infty} e^{-r(t-\hat{t})} ([u(\tilde{x}(t) + \tilde{y}(t)) - c(X(t))\tilde{x}(t) - b_0\tilde{y}(t)](1 - \Omega_t) + \Pi_t V(X, 1)) dt \tag{4}$$

subject to

$$\begin{aligned} \dot{X}(t) &= \tilde{x}(t), & X(\hat{t}) &= X \\ \tilde{x}(t), \tilde{y}(t) &\geq 0 \end{aligned} \tag{5}$$

where $\Omega_t = 1 - e^{-\rho(t-\hat{t})}$ is the probability that no innovation happens before t and $\Pi_t = \rho e^{-\rho t}$ is the probability rate of the innovation being made at t . We can simplify the objective function to the following:

$$\max_{\tilde{x}, \tilde{y}} \int_0^{\infty} e^{-(r+\rho)t} [u(\tilde{x}(t) + \tilde{y}(t)) - c(X(t))\tilde{x}(t) - b_0\tilde{y}(t) + \rho V(X, 1)] dt \tag{6}$$

Applying the maximum principle yields characterizations in the following propositions of the post-innovation equilibrium and the pre-innovation equilibrium (proofs are in the appendix).

Proposition 1 (*Post-innovation*). *Following an innovation at a date τ for which $b_1 > c(X(\tau))$, the competitive equilibrium is characterized by two phases. The first is an extraction phase, in which energy is produced with only fossil fuels. This phase lasts until the cost of extraction $c(X(t))$ reaches b_1 . Then energy is produced only with the new clean technology at cost b_1 . Over the extraction phase, the price path $p_d(t; \tau)$ and rents $p_d(t; \tau) - c(X(t))$ satisfy*

$$\dot{p}_d = r[p_d - c(X(t))] \tag{7}$$

$$\frac{d}{dt} \left[\frac{p_d - c(X(t))}{p_d - c(X(t))} \right] = r - \frac{dc(X(t))/dt}{p_d - c(X(t))} = r - \frac{c'(X(t)) \cdot q(p_d)}{p_d - c(X(t))} \tag{8}$$

with terminal condition $p = c(X(t)) = b_1$. Following innovation at date τ for which $b_1 < c(X(\tau))$, energy is produced with only the new technology.

For the intuition behind Eq. (8), recall that with constant extraction costs a competitive supplier would be compensated by interest earned over an instant dt in order to be indifferent between extracting at the equilibrium date and waiting an instant. Under our assumption of heterogenous extraction costs, a firm assigned an extraction date t in equilibrium would earn *more* than the change in equilibrium rent by waiting dt , because it would face lower costs than firms supplying in

equilibrium at $t + dt$. Setting the opportunity cost of waiting an instant to the benefit of waiting gives the equilibrium condition (8), under which the equilibrium rate of increase in rents is reduced by the rate of change in extraction costs.

Proposition 2 (Pre-innovation). *Prior to innovation, the competitive equilibrium is again characterized by two phases: energy is produced only with fossil fuels, until the cost of extraction $c(X(t))$ reaches b_0 . Then only the existing clean-energy technology is used. The paths of price, $p_0(t)$, and rents, $p_0(t) - c(X(t))$, satisfy*

$$\dot{p}_0 = r(p_0 - c(X(t))) + \rho(p_0 - p_d(t; t)) \quad (9)$$

$$\frac{d}{dt} \left[\frac{p_0 - c(X(t))}{p_0 - c(X(t))} \right] = r - \frac{c'(X(t)) \cdot q(p_0)}{p_0 - c(X(t))} + \rho \left(\frac{p_0 - p_d(t; t)}{p_0 - c(X(t))} \right) \quad (10)$$

with terminal condition $p_0 = c(X(T_0)) = b_0$. Innovation at date τ for which $b_1 < c(X(\tau))$ results in a discontinuous drop in price to $p_d(t; t)$.

Comparing (10) with (8) (or with a market in which the probability of innovation is 0) we see an additional term in the rent dynamics. To satisfy the optimal timing of extraction for a competitive fossil fuel owner, i.e. to make the owner indifferent between extracting at t and at $t + dt$, for small dt , an additional component representing a new opportunity cost must be added. This component is the immediate risk of a capital loss as price falls to the post-innovation level if discovery is made. Moving backwards in time from the extraction termination date T_0 , at which $p = c(X(T_0))$, (10) shows that at any X , the price of energy is lower when $\rho > 0$ than when $\rho = 0$. The quantity demanded of fossil fuel energy is therefore greater at any X as a result of a positive ρ . The mere *threat* of innovation thus induces a more rapid rate of extraction and carbon emissions.

Corollary 1. *The pre-innovative cumulative extraction path for $\rho > 0$ lies above the equilibrium path for the case $\rho = 0$. Along the pre-innovation cumulative extraction path, let T_0 be defined by $c(X(T_0)) = b_0$ and \hat{T} be defined by $c(X(\hat{T})) = b_1$. If discovery is possible but not realized until after T_0 , the same amount of resource, $c^{-1}(b_0)$, is extracted as if innovation were impossible. But it is extracted earlier and at a more intensive rate at any given X . If discovery is made before date $\tau < \hat{T}$, extraction takes place at a higher rate and terminates even earlier, but the total resource extracted (and carbon emitted) is less than $c^{-1}(b_0)$.*

Realized innovation thus has two effects on the pattern of carbon emissions: an earlier release of any given amount of emissions, and a reduction in total emissions. To anticipate the discussion below, in a model with the optimistic assumption of no feedback effects (f being a constant) these effects both work in the same direction in terms of the long run impact of innovation on global warming. Both effects reduce the temperature path after some date \hat{T} . Under the assumption of no feedback effects innovation *must* help in the battle against global warming. This is not true with feedback effects. With feedback effects there may be a green paradox even in the long run.

Continuing with the characterization of the economic equilibrium, we can depict the equilibrium using both a phase diagram in p and X and examples of the time paths of prices and extraction. The phase diagram (Fig. 2) of $(p(t), X(t))$, is determined by the following equations, along with the obvious terminal conditions:

$$\text{(pre - innovation): } \dot{p}_0(t) = r(p_0(t) - c(X(t))) + \rho(p_0(t) - p_d(t, t))$$

$$\text{(post - innovation): } \dot{p}_d(t) = r(p_d(t) - c(X(t))) \quad \text{if } \tau < \hat{T}$$

$$p_d(t) = b_1 \quad \text{if } \tau \geq \hat{T}$$

$$\dot{X}(t) = \tilde{x}(t) = q(p(t))$$

The phase diagram illustrates the impact of two possible realizations of the random innovation date. The innovation can take place early, when $X(t)$ is low, or late at a greater value for $X(t)$. With early innovation, the current marginal cost of fossil fuel extraction is still lower than the new cost of clean energy, so the old energy is still in use until the marginal cost reaches b_1 . With later innovation, the marginal cost of extraction has already risen above than the cost of clean energy. In this case, fossil fuel extraction is terminated immediately after the innovation.

Fig. 3 compares three price paths over time: the price path when innovation is impossible (or $\rho = 0$); the price path when innovation is possible but not realized; and the price path for given discovery at some $\tau < \hat{T}$. (The dates at which the extraction phase is terminated are denoted by T , T_0 and $T^d(\tau)$.) Fig. 4 illustrates the extraction paths. This figure captures the two effects of realized innovation: the total amount of fuel extracted (and carbon emitted) is reduced by innovation; but the fuel is extracted earlier and at a more intensive rate.

Benchmark: innovation and climate dynamics without feedback effects

To highlight the importance of feedback effects for long run implications of innovation, I start by considering the case of innovation in an integrated model *without* feedback effects. For this case, we replace assumption (A9) with the assumption

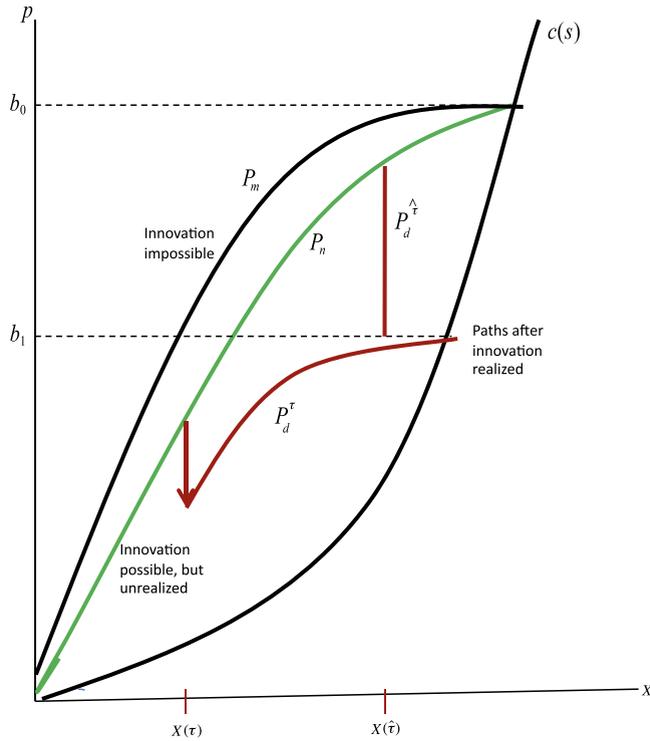


Fig. 2. Economic state space: early innovation at τ or late innovation at $\tilde{\tau}$. Note that this figure depicts the relationship that is traced between price and the state variable X as X grows. If innovation happens early, the price path drops and continues to rise with X ; if innovation happens late, the price drops to b_1 and X stops growing.

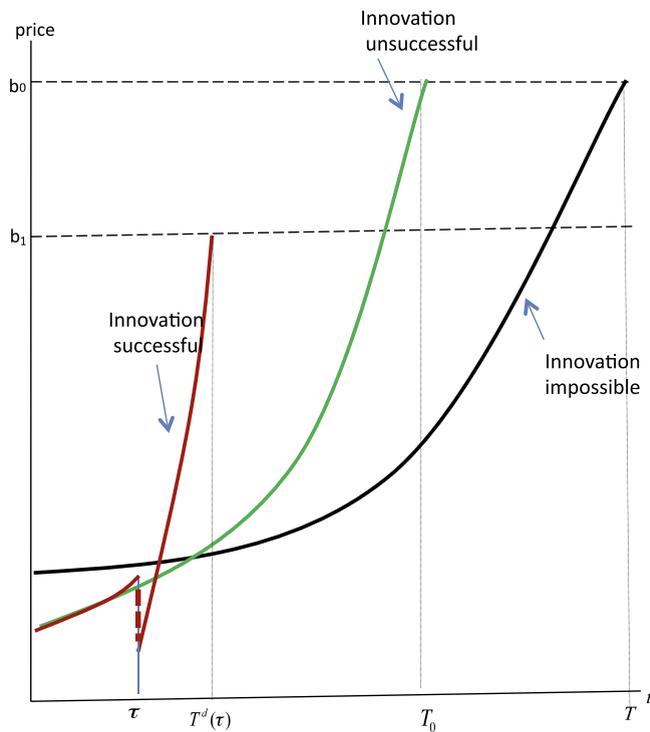


Fig. 3. Fossil fuel price paths for three cases: innovation impossible, innovation unsuccessful, and innovation successful at date τ .

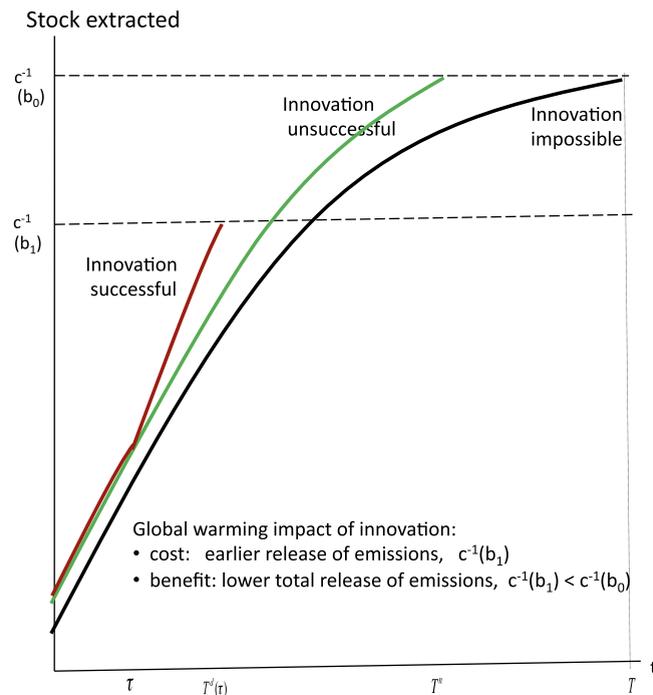


Fig. 4. Fossil fuel cumulative exhaustion paths: innovation impossible, innovation unsuccessful, and innovation successful at date τ .

that f is a constant:

$$(A9') \quad f(A) = h \quad \text{for some } h > 0.$$

Proposition 3 (No feedback effects). When assumption (A9) is replaced by (A9'):

- Ex ante long run impact of innovation: For any $t \geq T$, $A(t)$ is lower when innovation is possible but not realized, than when innovation is impossible.
- Ex post long run impact of innovation: For any date of discovery $\tau < \hat{T}$, there exists $T' < T_0$ such that for $t \geq T'$, $A(t)$ is lower with the innovation than with no innovation.
- As $t \rightarrow \infty$, $A(t)$ converges to a steady state temperature, $A^* = h/a$, that is independent of both the date of innovation and the ex ante probability of innovation.

This proposition shows that without feedback effects, innovation must *help* in its long run impact on temperature – and that it has no impact whatsoever in the steady state. While the effect of innovation is to increase emissions and temperature in the short run (the “weak” green paradox), eventually the impact must be beneficial, and the limiting steady state temperature is unaffected by innovation.

Innovation during the extraction phase of the economy has two effects on carbon emissions. First, it reduces the cumulative carbon emissions. This effect always reduces atmospheric carbon eventually (whether there are feedback effects or not). Second, innovation means that the carbon that is emitted is emitted earlier. In the benchmark case of zero feedback effects, the earlier emissions of carbon are also beneficial eventually because atmospheric carbon has had more time to decay, or settle back to the earth’s surface, at rate a .¹⁰ Both effects of innovation help the long run battle against global warming. No long run green paradox emerges without feedback effects.

The result that without feedback effects innovation helps in the long run is particularly simple in this model. But the prediction is shared by other models that integrate innovation and global warming. The most in-depth model of innovation and global warming, van der Ploeg and Withagen (2012), yields this result, as discussed in the Introduction. Another prominent contribution, Hoel and Kverndokk (1996, p. 118) analyzes optimal extraction of fossil fuels given a global warming externality. The paper adopts an assumption that “the preindustrial stock is assumed to be an equilibrium stock,

¹⁰ An emission of one unit of carbon at date t leads to an increase in concentration at date $\tilde{t} > t$ of $e^{-a(\tilde{t}-t)}$ under assumption (A10); the earlier is t , the lower is the increase in concentration at \tilde{t} .

meaning that the atmospheric stock will approach the preindustrial level in the long run ... when fossil fuels are exhausted," which implies that the long run steady state temperature and long run damages are unaffected by the timing of the emissions and even the total emissions. Yet another prominent contribution to the theory of economic policy and global warming, Acemoglu et al. (2012), arrives at an optimistic conclusion about the power of policy to correct global warming and innovation-related distortions by adopting an assumption that the environment can recover from any state no matter how disastrous.¹¹ All climate change damages can be reversed under this assumption. Even species rendered extinct by global warming can somehow be resurrected.¹² Feedback effects and the associated concept of *tipping points* are prominent in the climate literature¹³ and models with feedback effects are emerging in the integrated assessment literature.¹⁴ But feedback effects have not been incorporated in the most prominent models on the green paradox or the impact of innovation on global warming. These models yield the false implication that clean energy innovation is helpful or at worst neutral in the long run.

Innovation and climate dynamics with feedback effects

Climate dynamics in post-extraction phase

The starting point in considering the feedback effects is modeling the dynamics of the post-extraction phase of the market. Whichever market model is at work – a realized innovation, innovation-possible-but-not-realized, or innovation impossible – the market endows the environment with a stock of atmospheric carbon A at the end of the extraction phase. Climate dynamics are then characterized by the evolution of A . Setting $x(t)$ equal to zero in Eq. (1) yields a single differential equation governing A in the post-extraction phase, an evolution over which policy no longer has an influence:

$$\dot{A} = f(A) - aA \quad (11)$$

Steady states are values of A for which the right hand side of (11) are zero. In other words, these are values of A for which f intersects the ray from the origin with slope a in $A-\dot{A}$ space. The following has a simple proof, provided in the appendix.

Lemma 1. *For f satisfying assumption (A 9), there are, generically, either 1 or 3 steady states, i.e. solutions to $f(A) - aA = 0$. For a given f , the range of a for which there are three steady states is an interval, (\underline{a}, \bar{a}) .*

Fig. 5 depicts, for a given function f , rays with three different values of a . The ray with slope a_1 intersects f at a low temperature. Policy has no impact on the steady state in this case, because the rate of reabsorption of atmospheric carbon is so high that atmospheric carbon (and temperature) converges to low values. Global warming is not a long run problem. Where $a = a_3$, the rate of reabsorption is so low and feedback effects high that policy – even termination of fossil fuel use – cannot prevent convergence to a high temperature. Our interest is in the intermediate case, where policy *can* make a difference to the steady state. In this case, there are three steady states, A_L, A^* , and A_H , with the middle steady state being unstable. The steady state A_H is interpreted as the outcome of disastrous, runaway global warming. Depending on whether the endowment at the end of the exhaustion phase is above or below A^* , atmospheric carbon converges to either A_L or A_H .

Exceeding A^* is disastrous at any t , even during the fossil fuel exhaustion phase.

Proposition 4. *Suppose that parameter values for f and a yield three steady states, A_L, A^* and A_H . The A^* is a tipping point in the sense that if A^* is exceeded at any t , then $A(t)$ remains above A^* and A_H is the steady state.*

The proof is simple. Any emissions simply add to \dot{A} , which from (1) then remains positive.

Innovation and climate dynamics with feedback effects

With the ingredients of the equilibrium analysis of the energy market, the impact of innovation on carbon emissions, and climate dynamics with feedbacks, we can now illustrate the impact of innovation on the steady-state climate. I do so with a numerical example.

¹¹ Eq. (12) of Acemoglu et al. assumes that the dynamics of environmental quality, S_t in their notation, depends on the production of dirty inputs, Y_{dt} , according to

$$S_{t+1} = -\xi Y_{dt} + (1 + \delta)S_t$$

where δ is the rate of "environmental regeneration". The quality S_{t+1} is then truncated below by 0 and above by a level \bar{S} which is the maximum possible environmental quality (achieved with zero emissions). The implication is that no matter how low S_t gets, if fossil fuels are displaced by clean energy, the environment returns to the maximal, ideal state, \bar{S} .

¹² The 2007 IPCC reported as a consensus projection that "there is medium confidence that approximately 20–30 percent of species assessed so far are likely to be at an increased risk of extinction if increases in global average warming exceed 1.5–2.5 °C (relative to 1980–1999). As global average temperature increase exceeds about 3.5 °C, model projections suggest significant extinctions (40–70 percent of species assessed) around the globe." (IPCC: Climate Change 2007: Synthesis Report). More recent projections of species loss tend to be lower.

¹³ For example, United Nations Environmental Program, 2009, pp. 22 and 23, depicts 20 feedback mechanisms in the climate cycle.

¹⁴ See Lemoine and Traeger (2014), Cai et al. (2012) and van der Ploeg and De Zeeuw (2012). Lemoine and Traeger show that incorporating tipping points leads to an increase in the optimal near-term carbon tax of up to 45 percent in their base case specification. Cai et al. show the computational feasibility of incorporating feedback effects. Van der Ploeg and De Zeeuw develop a theoretical analysis of catastrophic global warming outcomes in a growth model.

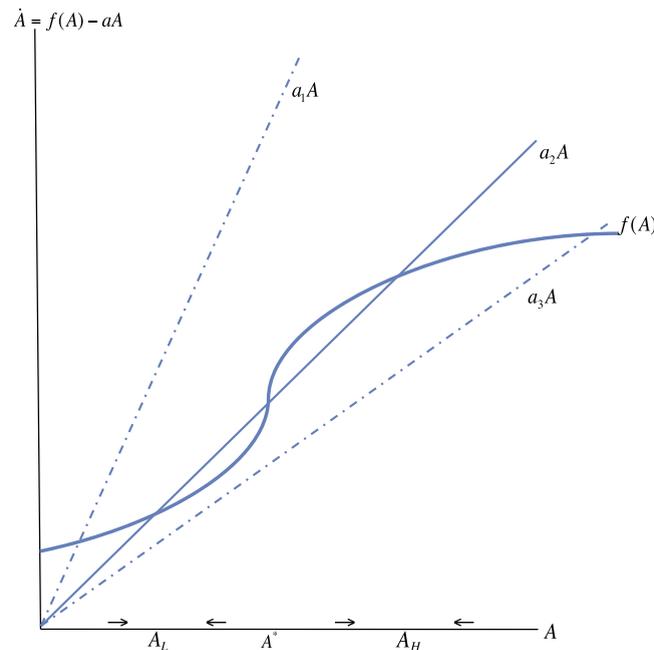


Fig. 5. Steady states for three values of a : values of A for which $f(A) - aA = 0$.

The example has the following elements: a linear demand, $q(p) = 1 - p$; a linear extraction cost, $C(X) = cX$; and a logistic function for f :

$$f(x) = v + \left(\frac{1}{1 + e^{-kx}} \right) (w - v)$$

This yields 10 exogenous parameters for the model: A_0 , a , c , b_0 , b_1 , ρ , v , w , k and the realized random date τ . The endogenous parameters are $p(t)$; $x(t)$ and its integral $X(t)$; $A(t)$; and steady states A_L , A^* and A_H .

Fig. 6 illustrates for a particular set of parameter values¹⁵ the paths of atmospheric gas under four scenarios:

- the probability of innovation, ρ , equals zero (the dark blue line);
- innovation is discovered early, at date 0.025 (green);
- innovation is discovered late, at date beyond 0.15 (light blue);
- innovation is discovered at an intermediate date, 0.11 (red).

The figure illustrates both the ex post and ex ante forms of the long run green paradox. To focus first on the ex post paradox, note that with an early discovery date for innovation, emissions accelerate the accumulation of green house gases with the result that the tipping point, $a^* = 0.39$ is passed. Atmospheric gases and temperatures then converge to the high steady state equilibrium of runaway global warming (the green line). With the late discovery of innovation, we get the conventional result that one would expect: innovation is discovered too late to help in the long run and again we get runaway global warming. When innovation is discovered in an intermediate range of dates (the red line), however, innovation is early enough to help battle global warming – but late enough that extraction costs have risen so much that a severe ex post green paradox does not kick in. Atmospheric gases and temperature converge to the lower steady state.

The ex ante green paradox is illustrated with a comparison of the dark blue line ($\rho = 0$, or innovation impossible) and the light blue line. The positive ex ante probability of innovation is enough to send A_t over the tipping point if innovation is not realized until late – even after the extraction phase, in which case the realization has no impact on carbon emissions. This is because the lower price of fossil fuel with $\rho > 0$ leads to a higher extraction rate and more rapid accumulation of atmospheric carbon.

By varying τ , the date of discovery, in the simulation, we can isolate the set \mathcal{Y} of all dates τ for which innovation at t leads to runaway global warming. The set \mathcal{Y} is disconnected, as illustrated in Fig. 1. For early innovation, $\tau < 0.419$ given the other parameter values generating Fig. 6, the ex post green paradox effects kick the dynamics into converging towards the high steady state temperature. For a late innovation date, $\tau > 0.647$, the power of the ex post green paradox has diminished but the new clean technology arrives too late to rescue the climate from runaway global warming. At the intermediate range of

¹⁵ These values are the following: $A_0 = 0$; $a = 12$; $c(s) = s$; $b_0 = 1$; $b_1 = 0.6$; $\rho = 3$; $v = 1$; $w = 20$; $k = 100$; S_τ (early τ) = 0.1; S_τ (late τ) = 0.5; $r = 1$; and a demand function: $q = 3/p$.

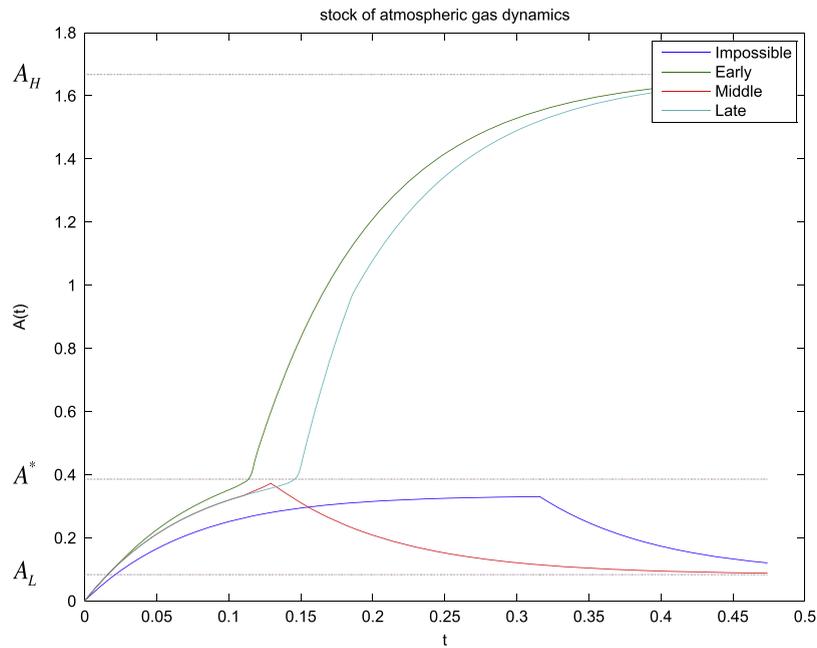


Fig. 6. Simulated time paths for A.

Scenario	Steady State Carbon
Innovation impossible	A_L
Innovation possible, late realization	A_H
Innovation possible, early realization	A_H
Innovation possible, intermediate realization	A_L

dates, $\tau \in (0.419, 0.647)$, the innovation occurs late enough to avoid the impact of the ex post green paradox on the long run steady state, but early enough to curtail sufficient emissions that the lower steady state is reached.

In sum, temperature reaches a low steady-state value in the example with the discovery of innovation at an intermediate date or if innovation is impossible. Runaway global warming results from a positive probability of innovation and either sufficiently early or sufficiently late realization of the innovation.

The impact of innovation in a world with optimal carbon pricing

I have developed the long run green paradox effects, ex ante and ex post, under the real-world condition of no carbon pricing. If carbon pricing, e.g. carbon taxes, is assumed to be available, then the first-best social optimum consumption of fossil fuel and clean energy can be implemented.¹⁶ The optimal carbon tax conditional upon state variables (X, A, θ) is set equal to the externality given by the marginal impact of additional emissions on the present value of damages.¹⁷ It follows immediately that innovation must be beneficial in a world where carbon taxes are optimal. Green Paradox phenomena are about a world without carbon taxes.

The possibility of negative consequences of innovation through green paradox effects might appear to justify cutbacks on government subsidy of clean energy innovation. The more legitimate interpretation is that carbon pricing is

¹⁶ This point can be made more precise by introducing a social planner with an objective function that includes damages assumed (as is standard in the literature) to be a function of current temperature and therefore of current carbon: $d(A)$. The objective function for the planner is the expected present value of $u(x_t + y_t) - c(X_t)x_t - b_0y_t - d(A_t)$ during the pre-innovation phase of the market and $u(x_t + y_t) - c(X_t)x_t - b_1y_t - d(A_t)$ after the (random) innovation date τ . The instrument of the planner is a tax as a function of current state variables: $t(A, X, I)$. It is straightforward to show that the optimal tax can implement that first-best allocation. Note that in this optimal carbon tax problem, the feedback effects introduce Skiba points or DNSS points. These are points at which a dynamic optimum steady state jumps discontinuously (Deckert and Nishimura, 1983; Sethi, 1977; Skiba, 1978).

¹⁷ In a full formulation of the optimal taxation problem, it would be important to relax the assumption in this paper that innovation is exogenous. A higher temperature can engender a higher carbon tax, which would increase R&D and the likelihood of a breakthrough in clean energy technology. Acemoglu et al. (2012) offer a simple model of climate change in which innovation is endogenous.

essential in the battle against global warming. The following proposition might seem intuitive: “if we cannot cut back on carbon emissions through carbon pricing because taxes are politically infeasible, then it is even more important to develop clean energy alternatives.” The proposition is wrong. Clean energy innovation and carbon pricing are complements, not substitutes.

Empirical perspectives

Plausibility of the long run green paradox

The central proposition of this paper is that clean-energy innovation, in the absence of carbon taxes, can lead to a permanently higher temperature path, notwithstanding the decrease in total emissions entailed by innovation in the extraction phase of the fossil fuel market. But this is a theoretical proposition – and only a “possibility theorem” at that. Does available evidence suggest that such a proposition is plausible, rather than merely theoretical? I point to available estimates from climate models and the distribution of remaining fossil fuels to suggest that the proposition is indeed plausible. The evidence suggests that there is a very substantial period over which even a dramatic clean-energy innovation would lead to higher emissions and that feedback effects are potentially strong over the range of temperatures that will likely prevail in the absence of a carbon tax.

Let us take as a horizon 2115, about a century from now. We start with a forecast of carbon concentration in the absence of carbon taxes, and then move to a forecast of temperature. A recent version of the Nordhaus DICE model, which is widely used as a most prominent model endogenizing economic growth, emissions and climate change, forecasts that in 2115 the atmospheric carbon concentration will be 865 parts per million (ppm) in its “base case”, i.e. without carbon taxes.¹⁸ The IPCC documents that in a recent year (2004) the percentage of non-carbon greenhouse gas emissions was 23 percent of total anthropogenic emissions in CO₂-equivalent units.¹⁹ Current CO₂ is about 400 ppm, with non-CO₂ GHG's adding to this for a total GHG concentration of approximately 470 ppm. Pre-industrial CO₂ (in 1750) was 280 ppm. The DICE model incorporates non-CO₂ GHG's directly through exogenous additions to radiative forcing rather than endogenizing them as cumulative CO₂-equivalents. Nonetheless, it is helpful in arriving at a summary of the 2115 base case to calculate the effective total GHG's. Assuming that non-CO₂ GHG's will be 15 percent of the total GHG's in 2115 – a conservative assumption, since non-CO₂ gases have been increasing as a proportion of total GHG's – gives a total forecast GHG concentration of about 1020 ppm CO₂e's.

The implications for temperature depend upon many parameters in climate models, one of the most critical being *climate sensitivity*, which is measured by climate scientists as the impact on temperature of a doubling of the stabilization level of GHG's since the pre-industrial level of 280 ppm. (That is, if GHG's were to stabilize at 560 ppm permanently, what would be the resulting increase in steady state temperature since the pre-industrial date of 1750?²⁰) Meinshausen et al. (2009) offer a meta-analysis of 19 studies of the climate sensitivity parameter, arriving at a 90 percent confidence interval for this parameter of 2.1–7.1 °C²¹; the value used in the 2010 DICE model is 3.2 °C.

The estimated temperature increase over the next century in the DICE model base case (the business-as-usual case, without carbon pricing) is 4.1 °C above pre-industrial levels. This projection must be considered highly uncertain as suggested by the wide confidence interval in even a single parameter, climate sensitivity, and the incorporation of many additional highly uncertain parameters. Among climate scientists, however, it is fair to say that the combination of 4.1 °C temperature change and a GHG concentration of over 1000 would, as a consensus, put us well within the range of a substantial probability of severe feedback effects, beyond the point at which even drastic reductions in CO₂ emissions would resolve the global warming problem.²² Allen and Frame (2007), in a recent *Science* commentary, state that “Once the world has warmed by 4 °C, conditions will be so different from anything we can observe today ... that it is inherently hard to say when the warming will stop.”²³ To consider just one feedback mechanism, methane trapped in permafrost and especially in

¹⁸ The Nordhaus model is available online at Professor Nordhaus' web page <http://nordhaus.econ.yale.edu/RICEmodels.htm>. I have used the beta version of the DICE 2010 model (DICE2010_082710d.xlsx) (accessed June 25, 2012). The value 865 is taken from cell M113 of the “base” spreadsheet.

¹⁹ Figure SMP.3, page 5, of IPCC (2007). Note that greenhouse gases vary greatly in their atmospheric lifetimes, from 12 years for methane to 114 years for nitrous oxide. (The lifetime is variable for CO₂.) The figures quoted are CO₂-equivalents on the basis of 100-year Global Warming Potentials. The 23 percent is made up of 14 percent CH₄, 8 percent N₂O, and 1 percent fluoride gases.

²⁰ Full adjustment of temperature to a change in radiative forcing brought about by an increase in carbon dioxide would take place through heat exchange with the ocean surface and deep ocean heat and carbon reservoirs and would take thousands of years. Climate scientists use the concept of *transient climate response*, which is defined in terms of a 20-year period around a CO₂ doubling via a growth rate in the concentration of 1 percent per year (for about 70 years). The climate change sensitivity parameter varies of course with the climate model adopted. Note that the climate change parameter incorporates whatever feedback processes are incorporated in the particular climate change model adopted.

²¹ While these authors offer the frequentist confidence interval, they take a Bayesian approach to determining the likelihood of various emission targets for limiting global warming to 2 °C.

²² Further context is provided for the projected temperature increase for 2115 of 4.1 °C and projected GHG concentrations of about 1000 ppm is obtained by applying the DICE model's climate sensitivity parameter of 3.2 to the increase in GHG. Even ignoring nonlinearities in the temperature sensitivity, the projected long run equilibrium temperature consistent with a stabilized GHG concentration in that range is 3.2 (1000–280)/280=8.2 °C.

²³ Another prominent climate scientist James Hansen formerly of NASA suggests that a CO₂ concentration in the order of only 450 ppm or greater, if long maintained, would push the Earth toward an ice-free state and that “such a CO₂ level likely would cause the passing of climate tipping points and initial dynamic responses that could be out of humanity's control” (Hansen et al., 2008). Hansen is regarded by some as an extremist.

methane hydrate (methane clathrate) deposits in the ocean are thousands of times larger in volume than the total methane in the atmosphere. Methane in the atmosphere is about 14 percent of total GHG's on a CO₂-equivalent basis. Melting permafrost and release of methane from the ocean carry by themselves (without incorporating other feedback mechanisms) a substantial risk of runaway global warming.

My aim in this very brief discussion is not to add to the voices calling for serious global warming policy. My purpose is much more modest. Could a dramatic innovation in a clean substitute, in the absence of carbon pricing, plausibly make global warming worse by accelerating feedback mechanisms? The answer is yes. The discussion above makes it clear that global warming must be addressed within the next few decades if tipping points and irreversible damage are to be avoided with confidence.

Consider the impact of clean energy innovation against this background. Let us take as an example an innovation of a clean energy that substituted for oil perfectly at a cost equivalent to 80 dollars per barrel. The [International Energy Association \(2010\)](#) estimates (with obvious imprecision) that there are about 150 years of current consumption of oil at extraction costs of 80 dollars or less. This means that applying our simple model directly, the global warming costs of the innovation through the green-paradox-induced increase in emissions would be manifest for many decades, possibly a century, before the benefits of substitution were realized.²⁴ Estimates of the long-run elasticity of energy range up to 0.7 ([Energy Forum 1980](#)). This implies that if the price of oil dropped to a value well below 80 over the next century rather than rising from the current value of about 100 dollars, substantially greater oil would be used over the next century, and substantially greater CO₂ would be emitted from this consumption of oil.

Oil and gas are of course not the only fossil fuels, but account for the 70 percent of UNEP projected fossil fuel CO₂ emissions for 2020 ([Rekacewicz, 2005](#)).²⁵ It is very plausible that following a substantial innovation in clean energy emissions over the next century would increase significantly. And it is our control of emissions over the next century that will determine whether the global warming problem is resolved. The available estimates therefore point to a conclusion that innovation in clean energy, if carbon pricing is not implemented, may very well exacerbate the problem of global warming. The Innovation Green Paradox is not just a theoretical proposition. Without carbon pricing, the net benefits of clean-energy innovation may well be negative.

Conclusion

The problem of global warming cannot be resolved with a policy of encouraging innovation in clean energy. Carbon pricing is essential. Without carbon pricing, innovation can easily make the global problem worse even in the long run, through its impact on endogenous prices of fossil fuels and the interaction with feedback effects in the carbon cycle. The perverse warming effect of innovation works through the greater intensity of fossil fuel consumption and earlier and more intensive release of carbon emissions. When feedback effects in the carbon-temperature dynamics are strong and the wait for beneficial displacement effects is long, the perverse effect will dominate. The long run green paradox is very plausible given current evidence on climate dynamics and oil and gas reserves.

Neither of the building blocks behind the long run green paradox, Hotelling rents ([Hotelling, 1931](#)) and climate feedback effects, is incorporated in the most prominent integrated assessment model, Nordhaus's DICE model ([Nordhaus, 2008](#)). Nordhaus aggregates all fossil fuels, coal, oil and natural gas. Because an estimated 900 years of coal is available at current consumption levels, Hotelling rents are close to zero in the first century of Nordhaus' model. The possibility of perverse consequences of innovation or of rising tax rates is ignored because this perverse impact operates through the negative impact on Hotelling rents. Nordhaus' optimal carbon tax starts at about 25–30 dollars per ton of carbon ([Nordhaus, 2008, 2012](#)), which is only about 8 cents per gallon of gasoline or about 2 percent of price. The estimated optimal tax then rises rapidly. Ignoring the Hotelling-effect consequences of innovation or rapidly rising tax rates would be acceptable if energy production through coal dominated the sources of carbon emissions, but it does not.

Feedback effects are also absent in the DICE model. The rate of flow of carbon from the earth's surface to the atmosphere is taken to be a constant.²⁶ And feedback effects for non-CO₂ gases, methane in particular, are absent in the model because these other gases enter exogenously into the radiative forcing Eq. (16) of the DICE model, rather than endogenously.²⁷

²⁴ Of course in reality the substitution between fossil fuels and clean energy varies across uses, but the point remains valid. Emissions would increase for decades before falling.

²⁵ Coal accounts for the remaining 30 percent. Since there are approximately 900 years of coal left at current consumption levels, the Hotelling rent on coal is approximately zero; the green paradox theories do not apply to coal.

²⁶ Eq. (13) of the DICE model ([Nordhaus, 2008, p. 206](#)) is

$$M_{AT}(t) = E(t) + \varphi_{11}M_{AT}(t-1) + \varphi_{21}M_{UP}(t-1)$$

Here M_{AT} is the mass of carbon in the atmosphere and M_{UP} is the mass on the earth's surface, mainly oceans. (The deep ocean is a separate reservoir of carbon and energy in the DICE model.) The coefficient φ_{21} is a constant, rather than an increasing function of atmospheric temperature.

²⁷ The absence of feedback effects in the DICE model is shared by integrated assessment models in general, leading [Pindyck \(2013 at 860\)](#) to conclude that these models tell us very little and are "close to useless tools for policy analysis".

A critical element of any integrated assessment model is its treatment of uncertainty. The DICE model of optimal carbon taxes adopts what Nordhaus labels a “certainty-equivalence” approach, which sets aside all uncertainty and treats each parameter as given without error by the mean of estimates from scientific studies.²⁸ Setting aside uncertainty would be acceptable under either of the conditions. The first is that all functions transforming parameter inputs to the final objective are linear. In the DICE model, these functional relationships are many, and include damages as a function of the parameter representing the temperature response to increased atmospheric carbon. In fact, the nonlinearity in even this one relationship is substantial, once feedback effects are recognized.²⁹ In testing for the second condition, whether uncertainty in is large enough to matter significantly, Nordhaus adopts as a measure of uncertainty the variance in the posterior mean estimates of the parameter from 16 different scientific studies.³⁰ Even taking a classical-statistics approach the standard error in a meta-estimate that combined the mean estimates of the separate studies would understate the standard error. Taking a Bayesian approach, and recognizing that scientists are looking largely at the same data, [Meinshausen et al. \(2009\)](#) arrive at a more reasonable estimate (using an updated set of 20 studies). Their estimate is a 68 percent confidence interval of (2.3, 4.5) and a 90 percent confidence interval of (2.1, 7.1) for TSP. This corresponds to a standard error of more than twice the Nordhaus standard error. And even this understates the confidence interval to the extent that there is uncertainty in the specifications of the models underlying the estimates. Nordhaus’ certainty-equivalence approach merely *appears* to be robust to uncertainty because in testing for robustness, Nordhaus vastly understates the standard error of at least one parameter, TSP, and omits the potential feedback consequences of very high temperatures.³¹

The available empirical evidence suggests that the long run green paradox effect of clean energy innovation is not just a theoretical possibility, but quite plausible. More generally, the Hotelling effects on oil and gas from innovation or a pattern of increasing carbon tax rates must be considered in any estimate of optimal policy. Feedback effects must also be incorporated. The most important and influential model of optimal carbon pricing, however, is one that incorporates neither Hotelling rents nor feedback effects, and is suspect in its treatment of uncertainty.

Important aspects of the interaction of innovation and climate change dynamics have been set aside in the theory offered in this paper. The carbon cycle is more complex than the risibly simple dynamics assumed here. In terms of economics, the full range of relevant technologies and innovation include not just clean energy, but technologies to mitigate carbon emissions (such as carbon sequestration or cleaner automobile engines); demand-side innovation to make more efficient use of energy; and even technology to directly mitigate global warming (such as injecting reflective particles in the atmosphere). Much remains to be explored in the theory of economic and climate dynamics.

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Appendix A. Proofs of propositions

Proposition 1 (*Post-innovation*). *Following an innovation at a date τ for which $b_1 > c(X(\tau))$, the competitive equilibrium is characterized by two phases. The first is an extraction phase, in which energy is produced with only fossil fuels. This phase lasts*

²⁸ The term “certainty equivalence” as used by [Nordhaus \(2008, p. 62\)](#) is unrelated to the definition of that concept in decision theory.

²⁹ Let us evaluate the nonlinearity of damages as a function of a single input, the climate sensitivity parameter, which maps from atmospheric carbon to temperature. Nordhaus refers to this parameter as the temperature-sensitivity parameter, or TSP. The 2010 DICE model uses a value of 3.2° for the impact on temperature of each 280 ppm increase in GHG above the pre-industrial value of 280. To evaluate linearity, we can ask: is the loss in social welfare from a 1° (per 280 ppm) downward error in TSP equal to the loss from an upward error of the same magnitude? Suppose that policy were based on a value for TSP of 3.2 but the truth were $s=4.2$. Note that at the DICE optimum based on a value of 3.2, the estimated GHG’s reach a maximum of about 720 ppm CO₂e. At the value of TSP = 4.2, this yields a temperature increase relative to pre-industrial levels of $4.2 \times (720 - 280) / (560 - 280) = 6.6^\circ$. The possible consequences of only a 1° error involve facing an environment of 6.6° higher temperature rather than the DICE-optimum predicted values of 3° higher. These consequences are enormous, as the summary of evidence in the previous section of this paper indicates, especially given feedback effects that are omitted in the DICE model. The cost of 1° upward error, on the other hand, planning for $s=3.2$ when the truth is $s=2.2$ can be calculated from DICE-2010 as approximately 560 Billion dollars. This is about 3 days worth of World GDP. The loss function over errors in the TSP parameter, in short, is nowhere near symmetric. The relationship nowhere near linear.

³⁰ [Nordhaus \(2007, p. 126\)](#) further reduces the estimate of uncertainty in s by putting 50 percent weight on an estimate generated by the DICE specification with historical data on CO₂ and other variables. He then (reasonably) doubles the resulting standard error “based on the presumption that the models and empirical estimates are likely to underestimate the uncertainty.” Nordhaus’ final estimate of the standard error, or uncertainty, in s is 0.5° (per 280 ppm).

³¹ More recently and in a non-technical overview, [Nordhaus \(2013\)](#) devotes substantial discussion to uncertainty (even in the title of the book) and to tipping points arising from feedback effects. Recent formal literature, e.g. [Lemoine and Traeger \(2014\)](#), incorporates feedback effects and uncertainty but these effects are not in estimated models that have had significant impact on policy.

until the cost of extraction $c(X(t))$ reaches b_1 . Then energy is produced only with the new clean technology at cost b_1 . Over the extraction phase, the price path $p_d(t; \tau)$ and rents $p_d(t; \tau) - c(X(t))$ satisfy

$$\dot{p} = r[p - c(X(t))] \tag{12}$$

$$\frac{d}{dt} \left[\frac{p - c(X(t))}{p - c(X(t))} \right] = r - \frac{dc(X(t))/dt}{p - c(X(t))} = r - \frac{c'(X(t)) \cdot q(p)}{p - c(X(t))} \tag{13}$$

with terminal condition $p = c(X(t)) = b_1$. Following innovation at date τ for which $b_1 < c(X(\tau))$, energy is produced with only the new technology.

Proof. The social planner's problem, maximizing net benefits from the energy market without regard to externalities, is the following:

$$\max_{x(t), y(t)} \int_{\tau}^{\infty} e^{-rt} [u(x(t) + y(t)) - c(X(t))x(t) - b_1 y(t)] dt$$

subject to

$$\dot{X}(t) = x(t); \quad x(t), y(t) \geq 0$$

The current value Hamiltonian and first-order conditions are (omitting time arguments)

$$H = u(x + y) - c(X)x - b_0 y + \lambda x + \nu x + \omega y(t) \tag{14}$$

$$H_x = u'(x + y) - c(X) + \lambda \leq 0 \quad (= 0 \text{ for } x > 0) \tag{14}$$

$$H_y = u'(x(t) + y(t)) - b_0 \leq 0 \quad (= 0 \text{ for } y > 0) \tag{15}$$

$$-H_X = \dot{\lambda} - r\lambda = c'(X)x \tag{16}$$

and the transversality condition

$$\lim_{t \rightarrow \infty} e^{-rt} \lambda = 0$$

Setting price $p = u'(x + y)$ to implement the optimum with a competitive market, Eq. (14) then implies $p = c(X) + \lambda$ when $x > 0$. Taking the time derivative of this equation yields $\dot{p} = dc(X(t))/dt + \dot{\lambda}$. This equation, plus $dc(X(t))/dt = c'(X(t))X(t) = c'(X(t))x(t)$, plus (16), yields (12). Eq. (13) follows.

Proposition 2 (Pre-innovation). *Prior to innovation, the competitive equilibrium is again characterized by two phases: energy is produced only with fossil fuels, until the cost of extraction $c(X(t))$ reaches b_0 . The path of price, $p_0(t)$, and rents, $p_0(t) - c(X(t))$, satisfy*

$$\dot{p} = r(p - c(X(t))) + \rho(p - p_d(t; t)) \tag{17}$$

$$\frac{d}{dt} \left[\frac{p - c(X(t))}{p - c(X(t))} \right] = r - \frac{c'(X(t)) \cdot q(p)}{p - c(X(t))} + \rho \left(\frac{p - p_d(t; t)}{p - c(X(t))} \right) \tag{18}$$

with terminal condition $p = c(X(T_0)) = b_0$. Innovation at date τ for which $b_1 < c(X(\tau))$ results in a discontinuous drop in price to $p_d(t; t)$.

Proof. Maximizing (6) subject to (5) yields the following current value Hamiltonian of this problem (omitting time arguments):

$$H = u(x + y) - c(X)x - b_0 y + \rho V(X, 1) + \lambda x + \nu x + \omega y$$

The first-order conditions are

$$H_x = u'(x + y) - c(X) + \lambda \leq 0 \quad (= 0 \text{ for } x > 0) \tag{19}$$

$$H_y = u'(x + y) - b_0 \leq 0 \quad (= 0 \text{ for } y > 0) \tag{20}$$

$$-H_X = \dot{\lambda} - (r + \rho)\lambda = c'(X)x - \rho V_X(X, 1) \tag{21}$$

and the transversality condition

$$\lim_{t \rightarrow \infty} e^{-(r+\rho)t} \lambda = 0$$

Set the price $p_t = u'(x + y)$, when $x > 0$, $\nu = 0$. Then (19) implies

$$\dot{\lambda} = -[\dot{p} - c(\dot{X})]$$

Together with $V_X(X, 1) = -[p_d - c(X)]$ and (21), this implies

$$\dot{p} = r(p - c(X)) + \rho(p - p^d) \quad \square$$

This proves the first equation of the proposition. The second follows directly.

Proposition 3 (No feedback effects). When assumption (A9) is replaced by (A9'):

- Ex ante long run impact of innovation: For any $t \geq T$, $A(t)$ is lower when innovation is possible but not realized, than when innovation is impossible.
- Ex post long run impact of innovation: For any date of discovery $\tau < \hat{T}$, there exists $T' < T_0$ such that for $t \geq T'$, $A(t)$ is lower with the innovation than with no innovation.
- As $t \rightarrow \infty$, $A(t)$ converges to a steady-state temperature, $A^* = h/a$, that is independent of both the date of innovation and the ex ante probability of innovation.

Proof. To prove part (a), let $\mathbf{X}(t; \rho)$ represent the ex ante cumulative extraction path, when the probability of innovation is ρ . That is, $\mathbf{X}(t; \rho)$ is the solution to the set of equations in the proof of Proposition 2. It follows from the monotonicity of $\mathbf{X}(t; \rho)$ in t that the inverse function $\mathbf{t}(X; \rho) \equiv \mathbf{X}^{-1}(t; \rho)$ is well-defined and monotonically increasing in X over the extraction phase. The function $\mathbf{t}(X; \rho)$ is the date at which X has been extracted. It follows from (17) and the monotonicity of $q(p)$ that $\mathbf{t}(X; \rho)$ is strictly decreasing in ρ . Now, in the post-extraction phase, $A(t)$ is given by the depreciated (at rate a) emissions from dates 0 to the termination date T_0 :

$$A(t) = \int_0^{T_0} x(\tilde{t}) e^{-a(t-\tilde{t})} d\tilde{t} \quad (22)$$

We implement a change in variables, in the integration, from t to X , using $dX/dt = x(X)$ which implies $dt = dX/x(X)$. Substituting this into (22) yields the following. Let \bar{X} , defined by $c(\bar{X}) = b_0$, be the total amount extracted when there is no innovation until after T_0

$$A(t) = \int_0^{\bar{X}} e^{-a[t-\mathbf{t}(X; \rho)]} dX = \int_0^{\bar{X}} \frac{e^{a\mathbf{t}(X; \rho)}}{e^{at}} dX \quad (23)$$

From this equation, an increase in ρ from 0 to a positive value reduces $A(t)$ since $\mathbf{t}(X; \rho)$ is strictly decreasing in ρ . The more rapid rate of emission of the same stock, \bar{X} , when innovation is possible means that each unit of emissions has a longer time to decay (settle back to the earth's surface) between the time of emission and the date t . \square

The proof of (b) is parallel. Part (c) follows directly from setting $\dot{A}(t) = x(t) + h - aA = 0$ and, in the post-extraction phase, $x(t) = 0$.

Lemma 2. For f satisfying assumption (A 9), there are, generically, either 1 or 3 steady states, i.e. solutions to $f(A) - aA = 0$. For a given f , the range of a for which there are three steady states is an interval, (\underline{a}, \bar{a}) .

Proof. Let \tilde{A} be the value of A below which f is strictly convex and above which f is strictly concave. The function $g(A) = f(A) - aA$ is also strictly convex below \tilde{A} and strictly concave above \tilde{A} , since $g''(A) = f''(A)$. Suppose that there were more than two distinct roots of g in $(0, \tilde{A})$. Let these roots be A_1, A_2, A_3, \dots . Then $g(A_1) = g(A_2) = g(A_3) = 0$ would imply, by Rolle's theorem, that there exist distinct values $\hat{A}_1 \in (A_1, A_2)$ and $\hat{A}_2 \in (A_2, A_3)$ satisfying $g'(\hat{A}_1) = g'(\hat{A}_2) = 0$. This contradicts $g' > 0$ from the strict convexity of g in $(0, \tilde{A})$. Thus there are at most two roots of g in $(0, \tilde{A}]$. By a parallel argument, there are at most two roots of g in (\tilde{A}, ∞) . This means that the number of roots of g is in the set $\{0, 1, 2, 3, 4\}$. But $g(0) = f(0) > 0$ and $g(A) < 0$ for A sufficiently large since f is bounded. This implies that the number of roots of g is, generically in (f, a) , odd. (Generically here means that for a given f , the property holds for a full Lebesgue measure of a .) This rules out, generically, 0, 2, and 4, leaving 1 and 3 as the possible numbers of roots of g . \square

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