MATH 105 – Written Homework #3

For this assignment, you are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence of your solutions. You are encouraged to work together, but your solutions must be written ***independently****. Please write your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together. This assignment is due at* ***1:00pm on Friday, July 28****. Late assignments will not be accepted.*

1. Suppose the amount of time T that you wait in the self-serve kiosk at the grocery store has pdf

$$f(t) = \begin{cases} \frac{1}{7}e^{-t/7} & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$

   (a) How much time do you expect to wait in line?

   (b) Instead of going through the self-serve kiosk line, you also have the option to go through the cashier's line. Suppose the wait time for the cashier's line has pdf

$$g(t) = \frac{1}{\sqrt{2\pi}}e^{-(t-7)^2/2}.$$

   Given that the amount of time you'd expect to wait in either line is the same, and that $\displaystyle\int_{-\infty}^{\infty} t^2 \cdot g(t)dt = 50$, which of the two check-out options is more consistent? Justify your answer through calculations.

2. State whether the following are true or false. If true, provide a short justification. If false, provide a counterexample.

   (a) If $a_n \geq 0$ for all n and the sequence $\{a_n\}$ converges to 0, then the sequence $\{(-1)^n a_n\}$ converges.

   (b) If the sequence $\{a_n\}$ diverges, then $\displaystyle\lim_{n\to\infty} a_n = \pm\infty$.

   (c) If the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\sum_{n=1}^{\infty} \cos(a_n)$ converges.

3. The *Sierpinski carpet* is a two-dimensional fractal constructed as follows.

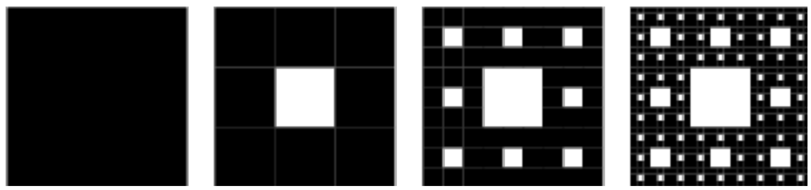
(i) Begin with a square whose sides have length 1, I_0.

(ii) Remove the centre one-ninth square to get I_1.

(iii) Remove the centre one-ninth square from the each of the remaining eight squares to get I_2.

(iv) Repeat this process ad infinitum (forever).

The figure shows the first four steps of this construction.



(a) By considering a suitable series, show that the total area removed from the original square is equal to 1.

(b) Explain, without using series, why you'd expect the total area removed to be finite.

(c) **[Bonus]** Let P_n be the sum of all of the perimeters of the squares removed in the nth step (only the nth step). For example, $P_1 = \frac{4}{3}$ and $P_2 = \frac{32}{9}$. Show that $\lim_{n\to\infty} P_n = \infty$.