Midterm-Retest Duration: 40 minutes

This test has ***2 questions*** *on* ***4 pages****, for a total of 24 points.*

- Read all the questions carefully before starting to work.
- Q1 is a short-answer question; all work has to be shown nevertheless.
- Q2 is a long-answer question; you should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, watches, cell phones, etc.)

First Name: Solutions Last Name: ________

Student-No: ________ Section: ________

Signature: ________

(Circle one) Option: 1 2

Question:	1	2	Total
Points:	18	6	24
Score:			

Short-Answer Questions. Question 1 is a short-answer question. Show all of your work. Each part is worth 3 marks, but not all parts are of equal difficulty.

18 marks 1. Evaluate the following integrals.

(a) $\int 2xe^{-x}dx.$

$u = 2x \quad dv = e^{-x}dx$

$du = 2dx \quad v = -e^{-x}$

$= -2xe^{-x} + \int +2e^{-x}\,dx$

$\boxed{= -2xe^{-x} - 2e^{-x} + C}$

(b) $\int -5x^3e^{x^4} + (x^2-1)x^3dx$ $= -5\int x^3e^{x^4}dx + \int x^5 - x^3 dx$

$= -5\underbrace{\int x^3e^{x^4}dx}_{} + \left(\frac{1}{6}x^6 - \frac{1}{4}x^4\right)$

$u = x^4$

$du = 4x^3dx$

$\frac{1}{4}du = x^3dx$

$= -\frac{5}{4}\int e^u du + \left(\frac{1}{6}x^6 - \frac{1}{4}x^4\right) = -\frac{5}{4}e^u + \left(\frac{1}{6}x^6 - \frac{1}{4}x^4\right) + C$

$= \boxed{-\frac{5}{4}e^{x^4} + \frac{1}{6}x^6 - \frac{1}{4}x^4 + C}$

(c) $\int \frac{8\ln x}{x^8}dx$

$u = 8\ln x \quad dv = x^{-8}dx$

$du = \frac{8}{x}dx \quad v = \frac{-1}{7}x^{-7}$

$= -\frac{8}{7}\frac{\ln x}{x^7} + \int +\frac{8}{7}\cdot\frac{1}{x^8}dx = \boxed{-\frac{8}{7}\frac{\ln x}{x^7} - \frac{8}{49x^7} + C}$

(d) $\int_0^{\pi/4} \cos^4 x dx$. You don't have to simplify your answer.

$$\cos^4 x = \left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)^2 = \frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{4}\cos^2 2x$$
$$= \frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{4}\left(\frac{1}{2}+\frac{1}{2}\cos 4x\right)$$
$$= \frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{8}+\frac{1}{8}\cos 4x$$

$$\therefore \int_0^{\pi/4}\cos^4 x\,dx = \int_0^{\pi/4}\left(\frac{3}{8}+\frac{1}{2}\cos 2x+\frac{1}{8}\cos 4x\right)dx$$
$$= \frac{3}{8}x+\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x\Big|_0^{\pi/4} = \frac{3\pi}{32}+\frac{1}{2}\sin\left(\frac{\pi}{2}\right)+\frac{1}{32}\sin(\pi$$
$$\boxed{= \frac{3\pi}{32}+\frac{1}{2}.}$$

(e) $\int \frac{x+5}{x^2+x-2}dx$

$$\frac{x+5}{x^2+x-2} = \frac{x+5}{(x+2)(x-1)} = \frac{A}{x+2}+\frac{B}{x-1} = \frac{-1}{x+2}+\frac{2}{x-1}.$$

$$\Rightarrow x+5 = A(x-1)+B(x+2).$$

$x=1 \Rightarrow \quad 6 = 0 + B\cdot 3 \Rightarrow B=2.$

$x=-2 \Rightarrow \quad 3 = A\cdot(-3)+0 \Rightarrow A=-1$

$$\therefore \int \frac{x+5}{x^2+x-2}dx \boxed{= -\ln|x+2|+2\ln|x-1|+C}$$

(f) Let $G(x) = \int_{1+x^3}^{2} \cos(e^{3t^2})dt$. Calculate $G'(1)$. You don't have to simplify your answer.

$$G(x) = -\int_2^{1+x^3}\cos\left(e^{3t^2}\right)dt$$

FTC

$$\Rightarrow G'(x) = -\cos\left(e^{3(1+x^3)^2}\right)\cdot(1+x^3)'$$
$$= -\cos\left(e^{3(1+x^3)^2}\right)\cdot 3x^2$$

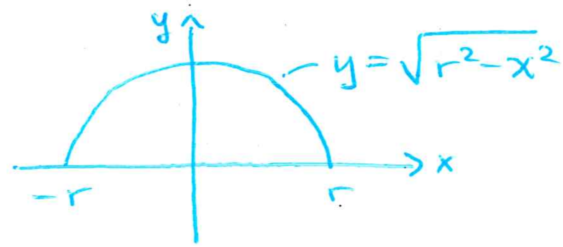
$$\therefore G'(1) = -\cos\left(e^{3\cdot(1+1)^2}\right)\cdot 3\cdot(1)^2 = \boxed{-3\cos\left(e^{12}\right)}$$

Long-Solution Problem. In this questions justify your answers and **show all of your work.** Unless otherwise indicated, **simplification of answers is not required in these questions.**

6 marks 2. The *arc length* (or simply the *length*) L of the curve $y = f(x)$ between $x = a$ and $x = b$ is defined to be

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\,dx.$$

Use this to show that the arc length of the upper semi-circle $\underbrace{y = \sqrt{r^2 - x^2}}$ is πr.



$$f(x) = \sqrt{r^2 - x^2}$$

$$\Rightarrow f'(x) = \frac{1}{2}(r^2 - x^2)^{-1/2} \cdot -2x = \frac{-x}{\sqrt{r^2 - x^2}}.$$

$$\Rightarrow 1 + [f'(x)]^2 = 1 + \frac{x^2}{r^2 - x^2} = \frac{r^2 - \cancel{x^2} + \cancel{x^2}}{r^2 - x^2} = \frac{r^2}{r^2 - x^2}.$$

$$\therefore L = \int_{-r}^{r} \sqrt{\frac{r^2}{r^2 - x^2}}\,dx = \int_{-r}^{r} \frac{r}{\sqrt{r^2 - x^2}}\,dx$$

Let $x = r\sin\theta$
$dx = r\cos\theta\,d\theta$.

$x = r \Rightarrow \sin\theta = 1 \Rightarrow \theta = \pi/2$.
$x = -r \Rightarrow \sin\theta = -1 \Rightarrow \theta = -\pi/2$.

$$= \int_{-\pi/2}^{\pi/2} \frac{r}{\sqrt{r^2 - r^2\sin^2\theta}} \cdot r\cos\theta\,d\theta = \int_{-\pi/2}^{\pi/2} \frac{r}{\cancel{r}\underbrace{\sqrt{1 - \sin^2\theta}}_{\cos^2\theta}} \cdot \cancel{r}\cos\theta\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \frac{r}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}\,d\theta = r\theta \Big|_{-\pi/2}^{\pi/2} = r\cdot\frac{\pi}{2} - r\cdot\left(\frac{-\pi}{2}\right)$$

$$= \pi r.$$