MATH 105 – Written Homework #1

For this assignment, you are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence of your solutions. You are encouraged to work together, but your solutions must be written ***independently****. Please write your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together. This assignment is due at* ***1:00pm on Friday, July 14****. Late assignments will not be accepted.*

1. (a) Use a *midpoint* Riemann Sum to approximate the area between the curves $y = e^x$ and $y = x$ from $x = 0$ to $x = 2$ with $n = 4$. (Hint: draw a picture!)

   (b) Find a bound for the *absolute* error in the previous part.

   (c) How many rectangles would we need to use if we wanted to ensure the error was smaller than 0.001 ?

2. (a) Use a *left-hand* Riemann Sum to approximate the area under the curve $y = t^2+1$ from $t = 0$ to $t = 3$ with $n = 4$. Is it an underestimate or an overestimate? Justify your answer.

   (b) Repeat the previous question using now a *right-hand* Riemann Sum.

   (c) And now the same using the *midpoint* Riemann Sum. Give a bound for the error in this case.

3. (a) Find the value of a ($a < 3$) that maximizes the integral

$$\int_a^3 \left(e^{-(s-2)^2} - \frac{1}{e} \right) ds.$$

   (b) Provide a *geometric* argument to explain why your solution in the previous part makes sense.

4. **Bonus.** A function $f(x)$ is *periodic* with *period* a if $f(x + a) = f(x)$ for all x. For example, $f(x) = \sin(x)$ is periodic with period 2π. If $f(x)$ is periodic with period a and continuous on $[0, a]$, show that

$$\int_0^a f(x)dx = \int_b^{a+b} f(x)dx.$$