MATH 105 – Review Problems

*Some of these problems will be solved at the **Review Session**, and the solutions to all of them will appear right after. The Review Session is on Tuesday, July 18th, at 11.30am, in Math 100*

Short Answer

1. Evaluate the following integrals.

   (a) $\int u(u+4)(2u+1)du$

   (b) $\int_3^4 \frac{1}{y^2-4y-12}dy$

   (c) $\int_0^{\frac{a\sqrt{2}}{2}} \frac{1}{\sqrt{a^2-x^2}}dx$

   (d) $\int \cos^2 x \tan^3 x dx$

   (e) $\int \cos^4(\theta)d\theta$

   (f) $\int \frac{x^2+2}{x+2}dx$

   (g) $\int \sin x \cos(\cos x)dx$

   (h) $\int \frac{\cos(\frac{\pi}{x})}{x^2}dx$ (Hint: try a substitution)

   (i) $\int t\cos(t^2)dt$

   (j) $\int x^{3/2}\ln(x)dx$

   (k) $\int \ln(x)dx$ (Hint: Think of $\ln(x)$ as $1 \cdot \ln(x)$.)

2. If $f(x)$ is a function such that $f'(x)$ is continuous, $f(1) = 12$ and $\int_1^4 f'(x)dx = 17$, find $f(4)$.

3. If $\int_2^5 f(x)dx = -30$, find $\int_1^2 7f(3x-1)dx$.

4. A car is traveling on a straight line with velocity $v(t) = t(t-2) m/s$. What is the *total distance* traveled between $t = 0$ and $t = 4$ seconds?

5. Solve the following initial value problem

$$(x-2)\frac{dy}{dx} = \frac{1}{2y(x^2+1)}, \quad y(0) = 1.$$

Long Answer

1. Find the equation of the line tangent to the curve given by $F(x) = \int_{\sin(x)}^{\cos(x)} (1-t^2)dt$ at $x = \frac{\pi}{4}$.

2. Find the derivative of the function $F(x) = \int_0^x xe^t dt$. (Hint: x does not depend on t.)

3. Find the area between the curves $y = \sin(\pi t)$ and $y = \cos(\pi t)$ from $t = 0$ to $t = \frac{1}{2}$.

4. For $a > 0$, use a method of integration to show that the area of the ellipse $x^2 + \left(\frac{y}{a}\right)^2 = 1$ is πa.

   Hint: By symmetry, it suffices to show that the area of the portion of the ellipse lying in the first quadrant is $\frac{1}{4}\pi a$.

5. (a) Write down the Midpoint Riemann Sum approximation of the area under the curve $y = e^{1/t}$ between $t = 1$ and $t = 2$ with $n = 3$. Leave your answer in "calculator-ready" form.

   (b) Now use sigma notation to write down a Midpoint Riemann Sum approximation of the area under the curve $y = e^{1/t}$ between $t = 1$ and $t = 2$ with $n = 10$. Do not evaluate the Riemann sum.

   (c) Use the error formula

$$E_M \leq \frac{M(b-a)^3}{24n^2}$$

   to find a bound for the absolute error in the approximation in (b).

6. The *Gompertz equation*

$$\frac{dP}{dt} = -27\ln\left(\frac{P}{120}\right)P,$$

   is a differential equation used to model limited populations of phytoplankton.

   (a) If the initial population of phytoplankton in a pond is $P(0) = 60$, and if the population of the phytoplankton satisfies the Goempertz equation, find population as a function of t (i.e. find $P(t)$).

   (b) What will happen to the population of phytoplankton in the long run?