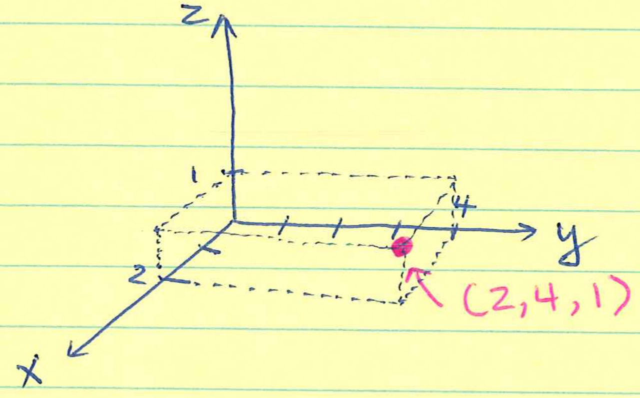
Lecture Examples – August 3rd

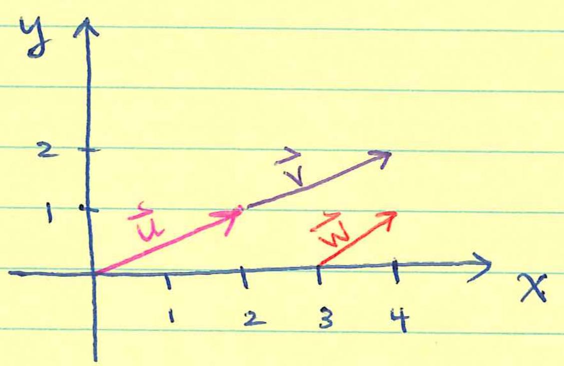
3-D Coordinate System

Example: Plot the point $(2, 4, 1)$.



Vectors

Example: Find the components of the vectors shown.



$\vec{u} = \langle 2, 1 \rangle$

$\vec{v} = \langle 4-2, 2-1 \rangle = \langle 2, 1 \rangle$ } same!

$\vec{w} = \langle 4-3, 1-0 \rangle = \langle 1, 1 \rangle$

Example: Find the length of the vector $\langle 1, 2, 1 \rangle$.

$$\| \langle 1, 2, 1 \rangle \| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$$

Example: If $\vec{u} = \langle 1,1,4 \rangle$ and $\vec{v} = \langle 2,0,-3 \rangle$, find $\vec{u}+\vec{v}$ and $\vec{u}-\vec{v}$.

$$\vec{u}+\vec{v} = \langle 1,1,4\rangle + \langle 2,0,-3\rangle = \underbrace{\langle 1+2, 1+0, 4-3\rangle}_{\text{add component-wise.}} = \langle 3,1,1\rangle.$$

$$\vec{u}-\vec{v} = \langle 1,1,4\rangle - \langle 2,0,-3\rangle = \langle 1-2, 1-0, 4--3\rangle = \langle -1,1,7\rangle$$

Example: If $\vec{u} = \langle 1,1,4 \rangle$, and $\vec{v} = \langle 2,0,-3 \rangle$, find $5\vec{u}$, $-\frac{1}{2}\vec{u}$ and $0\vec{v}$.

$$5\vec{u} = 5\langle 1,1,4\rangle = \underbrace{\langle 5\cdot 1, 5\cdot 1, 5\cdot 4\rangle}_{\text{multiply component-wise.}} = \langle 5,5,20\rangle$$

$$-\tfrac{1}{2}\vec{u} = -\tfrac{1}{2}\langle 1,1,4\rangle = \langle -\tfrac{1}{2}\cdot 1, -\tfrac{1}{2}\cdot 1, -\tfrac{1}{2}\cdot 4\rangle = \langle -\tfrac{1}{2}, -\tfrac{1}{2}, -2\rangle.$$

$$0\cdot\vec{v} = 0\langle 2,0,-3\rangle = \langle 0\cdot 2, 0\cdot 0, 0\cdot(-3)\rangle = \langle 0,0,0\rangle$$

Example: If $\vec{u} = \langle 1,1,2\rangle$, $\vec{v} = \langle 3,0,-1\rangle$, $\vec{w} = \langle 2,2,3\rangle$, calculate the following.

$$\vec{u}\cdot\vec{v} = \langle 1,1,2\rangle\cdot\langle 3,0,-1\rangle = 1\cdot 3 + 1\cdot 0 + 2\cdot(-1) = 3-2 = 1$$

$$\vec{u}\cdot\vec{w} = \langle 1,1,2\rangle\cdot\langle 2,2,3\rangle = 1\cdot 2 + 1\cdot 2 + 2\cdot 3 = 2+2+6 = 10$$

$$\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w} = 1 + 10 = 11.$$

$$5(\vec{u}\cdot\vec{v}) = (5\vec{u})\cdot\vec{v} = \vec{u}\cdot(5\vec{v}) = 5\cdot 1 = 5.$$

Functions of 2 Variables

Example: If $f(x,y) = x^2 y + e^{x+y}$, find $f(0,0)$, $f(1,0)$ and $f(1,2)$.

$$f(0,0) = 0^2 \cdot 0 + e^{0+0} = e^0 = 1.$$

$$f(1,0) = 1^2 \cdot 0 + e^{1+0} = e^1 = e.$$

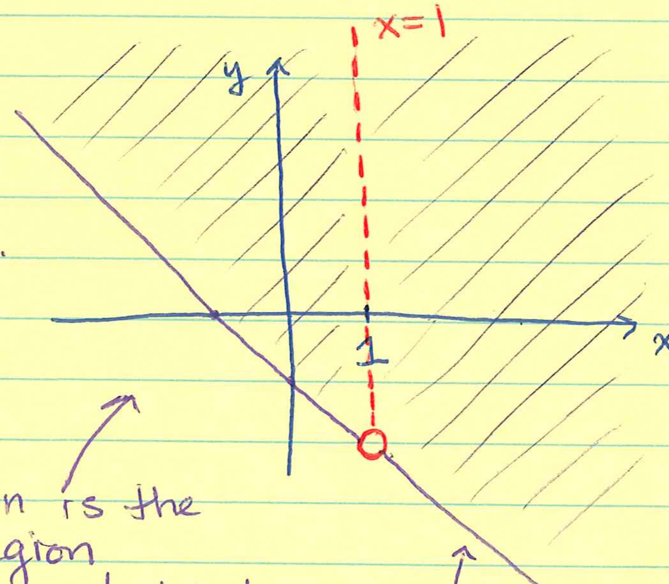
$$f(1,2) = 1^2 \cdot 2 + e^{1+2} = 2 + e^3$$

Example: Find the domains of the following functions.

① $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$

Need $x+y+1 \geq 0$ and $x-1 \neq 0$.

$\Leftrightarrow$ $\underbrace{y \geq -1-x}_{\text{focus on } y=-1-x}$ and $x \neq 1$.

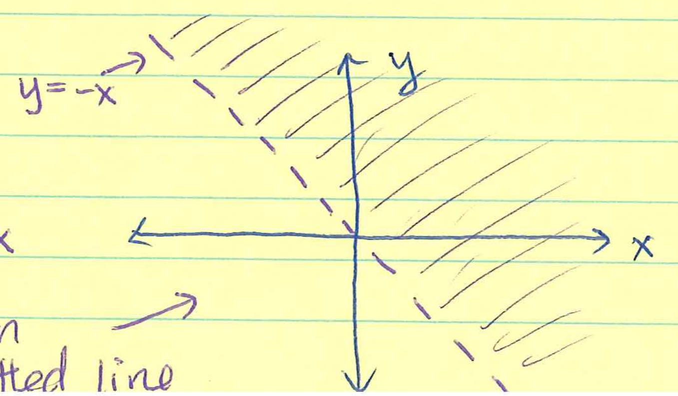


the domain is the shaded region (excluding the red dotted line)

$y=-1-x$

② $f(x,y) = x \cdot \ln(x+y)$

Need $x+y>0 \Leftrightarrow y>-x$



domain excludes dotted line

③ $f(x,y) = \sqrt{16 - x^2 - y^2}$

Need $16 - x^2 - y^2 \geq 0$

$\iff \underbrace{x^2 + y^2 \leq 16}$

focus on $x^2+y^2=16 \to$ circle of radius 4