Lecture Examples — August 9th

Tangent Planes

1) Example: Find the equation of the plane tangent to $z = \underbrace{x e^{x+y^2}}_{f(x,y)}$ at $(1,0) = (x_0, y_0)$

First, $z_0 = f(x_0, y_0) = f(1,0) = 1e^{1+0^2} = e.$

Now

$$\frac{\partial f}{\partial x} = 1 \cdot e^{x+y^2} + x \cdot e^{x+y^2} \Rightarrow \left.\frac{\partial f}{\partial x}\right|_{(1,0)} = e^{1+0} + 1e^{1+0} = 2e$$

and

$$\frac{\partial f}{\partial y} = x e^{x+y^2} \cdot 2y \Rightarrow \left.\frac{\partial f}{\partial y}\right|_{(1,0)} = 1e^{1+0} \cdot 2 \cdot 0 = 0$$

$\therefore$ The equation of the tangent plane is

$$z - e = 2e(x-1) + 0 \cdot (y-0)$$

$$\Leftrightarrow \quad z - e = 2e(x-1).$$

Critical Points

2) Example: Find the critical points of

$$f(x,y) = x^4 + y^4 - 4xy + 10.$$

We have that

$$\frac{\partial f}{\partial x} = 4x^3 - 4y = 0 \Rightarrow y = x^3$$

and $$\frac{\partial f}{\partial y} = 4y^3 - 4x = 0 \Rightarrow x = y^3$$

$$\therefore \quad x = y^3 = (x^3)^3 = x^9$$

Either $x=0$ or we can divide both sides by x to get

$$1 = x^8 \Leftrightarrow x = \pm 1.$$

If $x=0$, then $y = x^3 = 0^3 = 0 \Rightarrow (0,0)$ is a CP

If $x=1$, then $y = (1)^3 = 1 \Rightarrow (1,1)$ is a CP

If $x=-1$, then $y = (-1)^3 = -1 \Rightarrow (-1,-1)$ is a CP.

[Note: $(0,1)$, $(0,-1)$, $(1,0)$, $(1,-1)$, $(-1,0)$ and $(-1,1)$ are NOT critical points.]

So $f(x,y)$ has critical points $(0,0)$, $(1,1)$ and $(-1,-1)$.