# Lecture Examples – July 14th

## Integration by Parts:

simpler when differentiated — Can integrate it

① $\int_0^1 x e^x \, dx$ Let $u = x$, $dv = e^x dx$.
Then $du = dx$, $v = e^x$.

So $\int_0^1 \underset{u}{\underline{x}}\, \underset{dv}{\underline{e^x dx}} \overset{\text{IBP}}{=} \underset{u}{\underline{x}}\,\underset{v}{\underline{e^x}} \Big|_0^1 - \int_0^1 \underset{v}{\underline{e^x}}\, \underset{du}{\underline{dx}}$

$= 1e^1 - 0 \cdot e^0 - \left( e^x \Big|_0^1 \right)$

$= e - (e^1 - 1)$

$= 1$ //

② $\int_0^1 x^2 e^x \, dx$ Let $u = x^2$, $dv = e^x dx$.
Then $du = 2x\,dx$, $v = e^x$.

So

$\int_0^1 \underset{u}{\underline{x^2}}\, \underset{dv}{\underline{e^x dx}} \overset{\text{IBP}}{=} \underset{u}{\underline{x^2}}\, \underset{v}{\underline{e^x}} \Big|_0^1 - \int_0^1 \underset{du}{\underline{2x}}\, \underset{v}{\underline{e^x}}\, \underline{dx}$

$= 1^2 e^1 - 0^2 e^0 - 2 \underbrace{\int_0^1 x e^x \, dx}_{= 1 \text{ by } ①}$

$= e - 2$ //

* Integration by parts was really performed twice here – once, as demonstrated, and then again on the resulting integral as in question ①.

we don't know how to integrate $\ln x$, so we're forced to choose $u$ this way.

3. $\int x \ln x \, dx$ Let $u = \ln x$, $dv = x\,dx$

$du = \frac{1}{x}\,dx$, $v = \frac{1}{2}x^2$

So $$\int x \ln x \, dx \overset{IBP}{=} \ln x \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^{\cancel{2}} \cdot \frac{1}{\cancel{x}}\,dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C.$$

4. $\int e^{-x} \cos x \, dx$ Let $u = \cos x$, $dv = e^{-x}\,dx$

$du = -\sin x\,dx$, $v = -e^{-x}$

So $$\int e^{-x}\cos x\,dx = -e^{-x}\cos x - \int (\cancel{-}e^{-x})(\cancel{-}\sin(x))\,dx + C$$

$$= -e^{-x}\cos x - \underbrace{\int e^{-x}\sin x\,dx}_{} + C$$

doesn't look any simpler — kind of looks the same.
→ Try IBP again...

(For $\int e^{-x}\sin x\,dx$, let

$u = \sin x$, $dv = e^{-x}\,dx$

$du = \cos x\,dx$, $v = -e^{-x}$).

$$\therefore \int e^{-x}\cos x = -e^{-x}\cos x - \left[-e^{-x}\sin x - \int -e^{-x}\cos x\,dx\right] + C$$

$$= -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x\,dx + C$$

same integral!
→ solve equation algebraically for $\int e^{-x}\cos x\,dx$

$$\Rightarrow \underbrace{\int e^{-x}\cos x\,dx + \int e^{-x}\cos x\,dx}_{=2\int e^{-x}\cos x\,dx} = e^{-x}\sin x - e^{-x}\cos x + C$$

$$\Rightarrow \int e^{-x}\cos x\,dx = \frac{e^{-x}\sin x - e^{-x}\cos x}{2} + \underbrace{\tilde{C}}_{= C/2}$$