Examples from Friday, July 14.

Partial Fractions — LONG DIVISION

Example: $\displaystyle\int \frac{x^3+2}{(x-1)(x+2)}\,dx = \int \frac{x^3+2}{x^2+x-2}\,dx$

degree numerator $\geq$ degree denominator

Before we can do a partial fraction decomposition, we first need to do LONG DIVISION:

$$
\begin{array}{r|l}
 & \boxed{x-1} \rightarrow \text{whole part} \\
x^2+x-2 & x^3+2 \\
 & x^3+x^2-2x \\
\hline
 & 0-x^2+2x+2 \\
 & -x^2-x+2 \\
\hline
 & 0+\boxed{3x}+0
\end{array}
$$

Remainder

$$\Rightarrow \frac{x^3+2}{x^2+x-2} = \underbrace{x-1}_{\text{whole part}} + \frac{3x}{x^2+x-2}$$

So $\displaystyle\int \frac{x^3+2}{(x-1)(x+2)}\,dx = \underbrace{\int (x-1)\,dx}_{\text{easy}} + \underbrace{\int \frac{3x}{(x-1)(x+2)}\,dx}_{\text{partial fractions as before.}}$

Now, $\displaystyle\frac{3x}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$

$$\Rightarrow \quad 3x = A(x+2) + B(x-1)$$

Method #1:

$$\underbrace{3x}_{=3x+0} = Ax + 2A + Bx - B = (A+B)x + (2A-B)$$

Therefore, we want

$$A+B=3 \quad \text{and} \quad 2A-B=0$$

$$\Rightarrow 2A = B$$

So $A + 2A = 3$

$$\Rightarrow 3A = 3$$

$$\Rightarrow \boxed{A=1}$$

$$\Rightarrow \boxed{B = 2A = 2}$$

Method #2:

$$3x = A(x+2) + B(x-1).$$

when $x=1$:

$$3(1) = A(1+2) + B(0-0)$$

$$\Rightarrow 3 = 3A$$

$$\Rightarrow \boxed{A=1.}$$

when $x=-2$:

$$3(-2) = A(-2+2) + B(-2-1)$$

$$\Rightarrow -6 = -3B$$

$$\Rightarrow \boxed{B=2.}$$

So, $\dfrac{3x}{(x-1)(x+2)} = \dfrac{1}{x-1} + \dfrac{2}{x+2}$.

Therefore

$$\int \frac{x^3+2}{(x-1)(x+2)}\,dx = \int (x-1)\,dx + \int\left(\frac{1}{x-1} + \frac{2}{x+2}\right)dx$$

$$= \frac{1}{2}x^2 - x + \ln|x-1| + 2\ln|x+2| + C$$