(T1) General Thin-Airfoil Theory

- Airfoil is represented by its mean Camber line
- $\frac{\text{Maximum thincKness}}{\text{Chord}} \simeq 12\%$
- $\frac{\text{Maximum mean Camber}}{\text{Chord}} \simeq 2\%$

- Assumptions
  1) The lifting characteristics of an airfoil below stall are negligibly affected by the presence of the boundary layer.

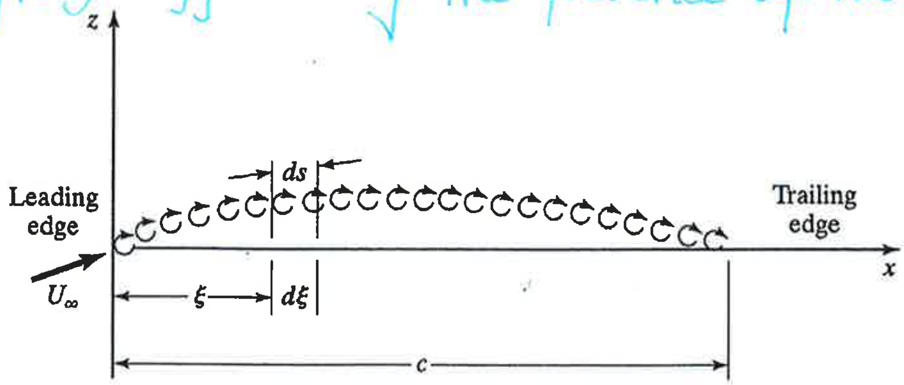


Figure 6.4 Representation of the mean camber line by a vortex sheet whose filaments are of variable strength $\gamma(s)$. [2]

  2) The airfoil is operating at a Small angle of attack.
  3) The resultant of the pressure forces is only slightly influenced by the airfoil thicKness.

- A vortex sheet coincident with the mean camber line produces a velocity distribution that exhibits the required velocity jump, i.e. lift-generating pressure difference.

- The total circulation is the sum of the circulations of the vortex filaments

(1) $$\Gamma = \int_0^c \gamma(s)\, ds ,$$ $\gamma(s)$: distribution of vorticity for the line vortices

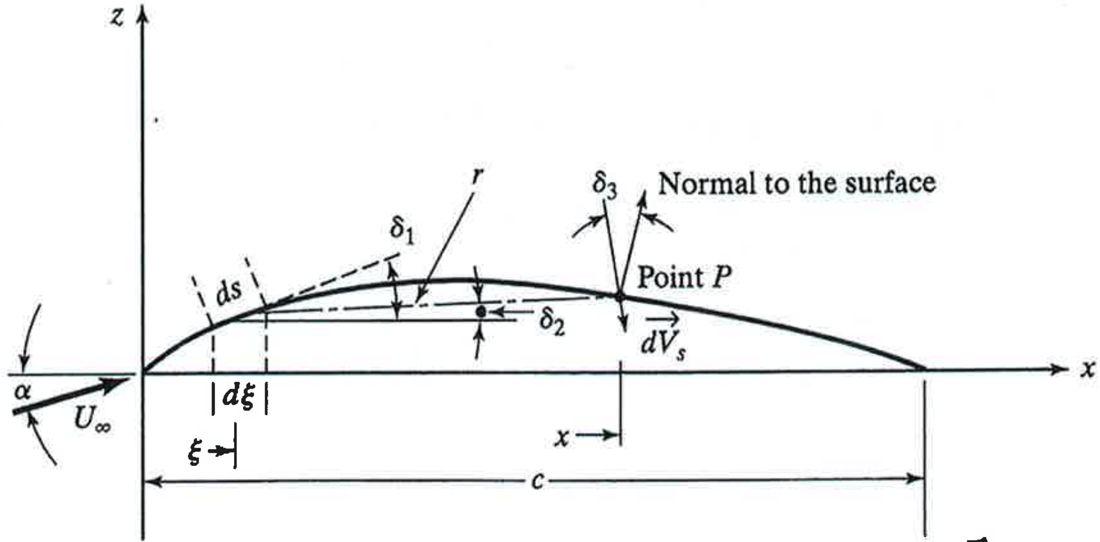


Figure 6.5 Thin-airfoil geometry parameters. [2]

- The velocity field around the sheet is the sum of the free-stream velocity and the velocity induced by all the vortex filaments that make up the vortex sheet.

- For the vortex sheet to be a streamline of the flow, it is necessary that the resultant velocity to be tangent to the mean camber line at each point.
  - The sum of the components normal to the surface for these two velocities is zero.

- The condition that the flows from the upper surface and the lower surface join smoothly at the trailing edge (the Kutta-Condition) requires that $\gamma = 0$ at the trailing edge.

(T2) The portion of the vortex sheet designated ds produces a velocity at point P which is perpendicular to the

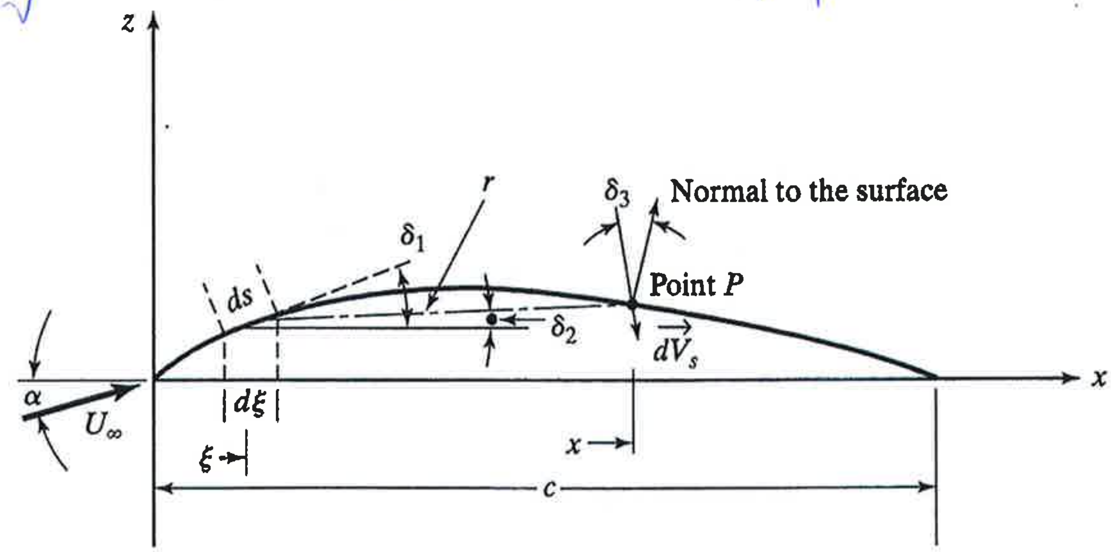


Figure 6.5 Thin-airfoil geometry parameters. [2]

line whose length is r and which joins the element ds and the point P. The induced velocity component normal to the camber line at P due to the vortex element ds is

(2) $$dV_{s,n} = -\frac{\gamma ds}{2\pi r} \cos\delta_3$$

- The chordwise location of the point of interest P will be designated in terms of x coordinate.
- The chordwise location of a given element of vortex sheet ds will be given in terms of ξ coordinate.

Thus, to calculate the cumulative effect of all the vortex elements, it is necessary to integrate over the

ξ- coordinate from the leading edge ($\xi = 0$) to the trailing edge ($\xi = c$). Noting that

(3) $$\cos\delta_2 = \frac{x-\xi}{r}, \qquad ds = \frac{d\xi}{\cos\delta_1}$$

the resultant vortex-induced velocity at any point P (which has the chordwise location x) is given by

(4) $$V_{s,n} = -\frac{1}{2\pi}\int_{\xi=0}^{\xi=c} \frac{\gamma(\xi)\cos\delta_2\cos\delta_3}{(x-\xi)\cos\delta_1}\, d\xi$$

The component of the free-stream velocity normal to the mean camber line at P is given by

(5) $$U_{\infty,n}(x) = U_\infty \sin(\alpha - \delta_p)$$

where α is the angle of attack and δ_p is the slope of the camber line at the point of interest. Thus,

(6) $$\delta_p = \tan^{-1}\frac{dz}{dx}$$

Where $z(x)$ describes the mean camber line. As a result

(7) $$U_{\infty,n}(x) = U_\infty \sin\left(\alpha - \tan^{-1}\frac{dz}{dx}\right).$$

(T3)

Since the Sum of the velocity components normal to the Surface must be Zero at all points along the Vortex Sheet,

(8) $$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\cos\delta_2 \cos\delta_3}{(x-\xi)\cos\delta_1}\,d\xi = U_\infty \sin\left(\alpha - \tan^{-1}\frac{dz}{dx}\right)$$

The vorticity distribution $\gamma(\xi)$ that satisfies this integral equation makes the vortex sheet (the mean camber) a streamline of the flow. The desired vorticity distribution must also satisfy the Kutta Condition that $\gamma(c)=0$.

Note Within the assumptions of thin-airfoil theory, the angles δ_1, δ_2, δ_3, and α are Small. Hence, Eq.(8) becomes

(9) $$\frac{1}{2\pi}\int_{\xi=0}^{\xi=c} \frac{\gamma(\xi)}{x-\xi}\,d\xi = U_\infty\left(\alpha - \frac{dz}{dx}\right)$$

This is the fundamental equation of thin airfoil theory; it is simply a statement that the camber line is a streamline of the flow.

Remark For a given airfoil and a given angle of attack, both α and $\frac{dz}{dx}$ are known values in eqn.(9). Indeed, the only unknown is Eqn.(9) is the vortex strength $\gamma(\xi)$. Hence, Eqn.(9) is an integral equation, the solution of which yields the variation of $\gamma(\xi)$ such that the camber line is a streamline of the flow.

Remark 2. The central problem of thin airfoil theory is to solve Eqn.(9) for $\gamma(\xi)$, subject to the Kutta condition, i.e., $\gamma(c) = 0$

Symmetric Airfoil.

A symmetric airfoil has no camber; the camber line is coincident with the chord line. Hence, $\frac{dz}{dx} = 0$. Thus, eqn.(9) becomes

$$(10) \qquad \frac{1}{2\pi} \int_{\xi=0}^{\xi=c} \frac{\gamma(\xi)}{x-\xi} \, d\xi = U_\infty \alpha.$$

T4

Remark. Equation (10) is an exact expression for the <u>inviscid</u>, <u>incompressible</u> flow over a flat plate at angle of attack.

It is convenient to introduce the coordinate transformation

(11) $$\xi = \frac{c}{2}\left(1 - \cos\theta\right)$$

Since x is a fixed point in Eqs. (9) and (10), it corresponds to a particular value of θ, namely θ_0, such that

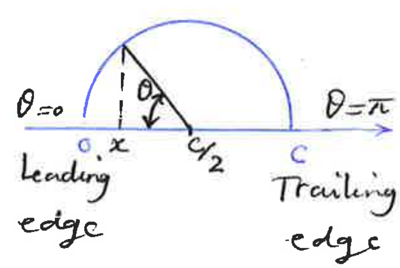
(12) $$x = \frac{c}{2}\left(1 - \cos\theta_0\right)$$

The corresponding limits of integration are

(13) $$\xi = 0, \ \theta = 0 \qquad \text{and} \qquad \xi = c, \ \theta = \pi$$

Also from Eqn. (11)

(14) $$d\xi = \frac{c}{2}\sin\theta \, d\theta$$



Eqn. (10) becomes

(15) $$\frac{1}{2\pi}\int_0^{\pi} \frac{\gamma(\theta)\sin\theta}{\cos\theta - \cos\theta_0}\, d\theta = U_\infty \alpha$$

The required vorticity distribution, $\gamma(\theta)$, must not only satisfy this integral, but must satisfy the Kutta Condition, $\gamma(\pi)=0$. The Solution is

$$\gamma(\theta) = 2\alpha U_\infty \frac{1+\cos\theta}{\sin\theta} \tag{16}$$

You can verify this solution by substituting Eqn (16) into (15) and using the following identity

$$\boxed{\int_0^\pi \frac{\cos n\theta}{\cos\theta - \cos\theta_0}\, d\theta = \pi \frac{\sin n\theta_0}{\sin\theta_0}} \tag{17}$$

Glauert's Integral

Note. At the trailing edge, where $\theta=\pi$, Eqn. (16) yields

$$\gamma(\pi) = 2\alpha U_\infty \frac{0}{0} \tag{18}$$

which is an indeterminant form. Using L'Hospital's rule

$$\gamma(\pi) = 2\alpha U_\infty \frac{-\sin\pi}{\cos\pi} = 0 \tag{19}$$

So the expression for $\gamma(\theta)$ also satisfies the Kutta condition.

(T5) # Lift Coefficient Calculation

The total circulation around the airfoil is

(20) $$\Gamma = \int_0^c \gamma(\xi)\, d\xi$$

Using the coordinate transformation, (11), we obtain

(21) $$\Gamma = \int_0^{\pi} \frac{c}{2}\, \gamma(\theta) \sin\theta \, d\theta$$

Substituting Eqn. (16) into Eqn. (21), we obtain

(22) $$\Gamma = \alpha c U_\infty \int_0^{\pi} (1+\cos\theta)\, d\theta = \pi \alpha c U_\infty .$$

Substituting Eqn. (22) into the Kutta-Joukowski theorem, we find that the lift per unit span is

(23) $$L' = \rho U_\infty \Gamma = \pi \alpha c \rho U_\infty^2$$

The lift coefficient is

(24) $$C_L = \frac{L'}{\frac{1}{2}\rho U_\infty^2 A} = \frac{\pi \alpha c \rho_\infty U_\infty^2}{\frac{1}{2}\rho U_\infty^2 \, c\,(1)} = 2\pi\alpha .$$

or

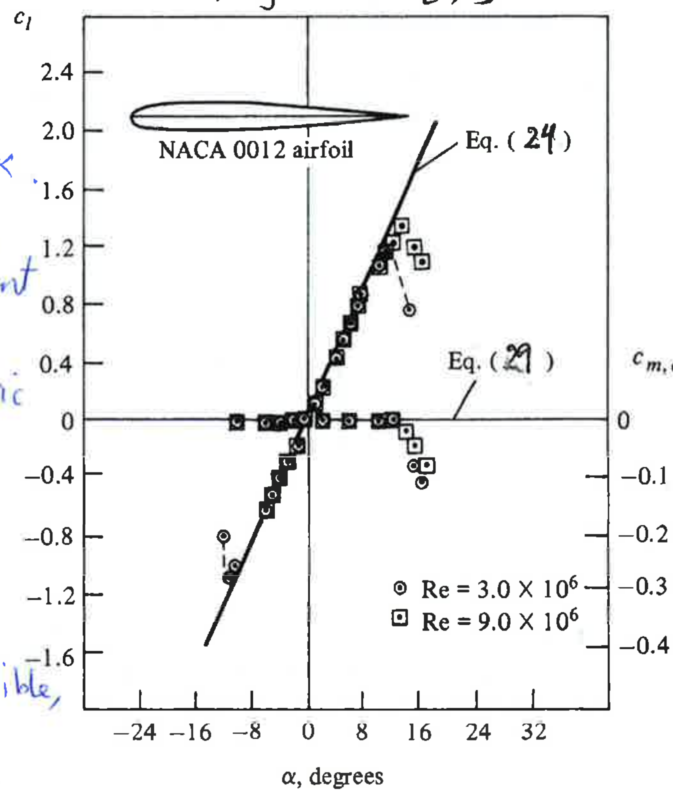
(25) $$\text{Lift Slope} = \frac{dC_L}{d\alpha} = 2\pi$$

Remark.

Eqn.(24) States that the left coefficient is linearly proportional to angle of attack.

The experimental lift coefficient data for an NACA 0012 symmetric airfoil are given in Figure.

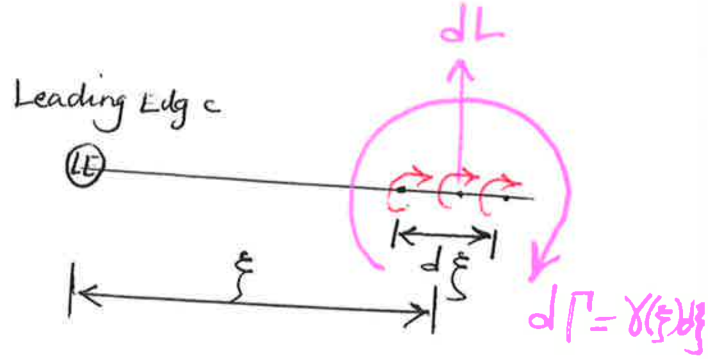
Reference [1]



Remark 2. Consider an incompressible, inviscid flow about a symmetric airfoil at a small angle of attack. There is a stagnation point on the lower surface of the airfoil just downstream of the leading edge. From this stagnation point, flow accelerates around the leading edge to the upper surface. The approximate theoretical solution for a thin, symmetric airfoil with two sharp edges yields an infinite velocity at the leading edge. The high velocities for flow over the leading edge result in low pressures in this region, producing a component of force along the leading edge, known as the leading-edge suction force, which exactly cancels the streamwise component of the pressure distribution acting on the rest of the airfoil, resulting in zero drag.

(T6) The pressure distribution also produces a pitching moment about the leading edge (per unit span), given by

$$M'_{LE} = \int_0^c -\xi \, dL$$



(26)
$$= \int_0^c -\rho U_\infty \xi \, \gamma(\xi) \, d\xi$$

$$= -\frac{1}{2}\rho U_\infty^2 \alpha c^2 \int_0^\pi (1-\cos^2\theta) \, d\theta$$

$$= -\frac{\pi}{4}\rho U_\infty^2 \alpha c^2$$

The section moment coefficient is

(27)
$$C_m = \frac{M'_{LE}}{\frac{1}{2}\rho U_\infty^2 S c}$$ where $S = c(1)$.

Hence

(28)
$$C_m = \frac{M'_{LE}}{\frac{1}{2}\rho U_\infty^2 c^2} = \frac{-\frac{\pi}{4}\rho U_\infty^2 \alpha c^2}{\frac{1}{2}\rho U_\infty^2 c^2} = -\frac{\pi}{2}\alpha = -\frac{C_L}{4}$$

Symmetric Airfoil

Note. The Centre of pressure x_{cp} is the x-coordinate, where the resultant lift force could be placed to produce the pitching moment M'_{LE}. Equating the moment about the leading edge (Eqn. 26) to the product of the lift (Eqn. 23) and the center of pressure yields

$$-\frac{\pi}{4}\rho U_\infty^2 \alpha c^2 = -\left(\pi \alpha c \rho U_\infty^2\right) x_{cp}$$

(29)
$$\Rightarrow \quad x_{cp} = \frac{c}{4}$$

The result is independent of angle of attack, and demonstrates the theoretical result that the center of pressure is at the quarter-chord point for a symmetric airfoil.

Remark The point on an airfoil where moments are independent of angle of attack is called the aerodynamic center. Hence, for a symmetric airfoil we have the theoretical result that the quarter-chord point is both the center of pressure and the aerodynamic center.

T7

b. *Discretization and Grid Generation* [3]

At this phase the thin-airfoil camberline (Fig. 11.2) is divided into N subpanels, which may be equal in length. The N vortex points (x_j, z_j) will be placed at the quarter-chord point of each planar panel (Fig. 11.2). The zero normal flow boundary condition can be fulfilled on the camberline at the three-quarter point of each panel. These N collocation points (x_i, z_i) and the corresponding N normal vectors $\mathbf{n}_i$ along with the vortex points can be computed numerically or supplied as an input file. Note that by discretizing the camberline as shown in Fig. 11.2, we end up with only the panel edges remaining on the original camberline. For convenience, the normal vector is evaluated at the actual camberline and the effect of this choice will be investigated at the end of this section. Consequently, the normal vectors $\mathbf{n}_i$, pointing outward at each of these points, are approximated by using the

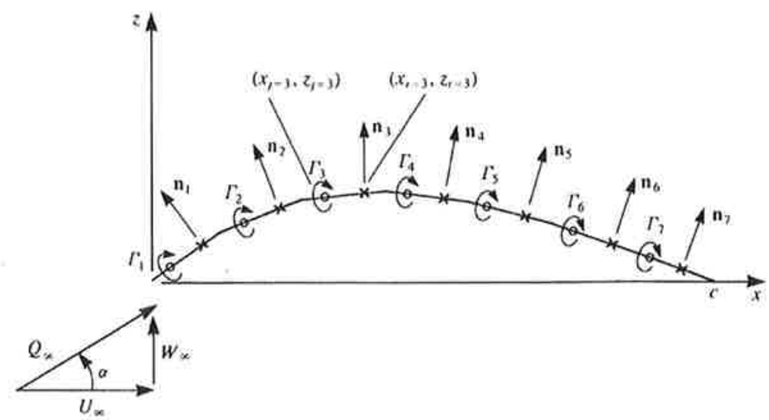


Figure 11.2 Discrete vortex representation of the thin, lifting airfoil model.

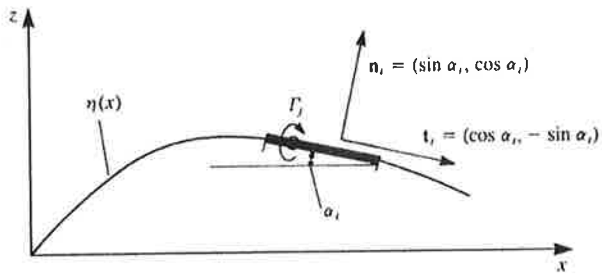


Figure 11.3 Nomenclature used in defining the geometry of a point singularity based surface panel.

surface shape $\eta(x)$, as shown in Fig. 11.3:

$$\mathbf{n}_i = \frac{(-d\eta/dx, 1)}{\sqrt{(d\eta/dx)^2 + 1}} = (\sin\alpha_i, \cos\alpha_i) \tag{11.3}$$

where the angle α_i is defined as shown in Fig. 11.3. Similarly the tangential vector $\mathbf{t}_i$ is

$$\mathbf{t}_i = (\cos\alpha_i, -\sin\alpha_i) \tag{11.3a}$$

Since the lumped-vortex element is based on the Kutta condition, the last panel will inherently fulfill this requirement, and no additional specification of this condition is needed.

c. *Influence Coefficients*

The normal velocity component at each point on the camberline is a combination of the self-induced velocity and the free-stream velocity. Therefore, the zero normal flow boundary condition can be presented as

$$\mathbf{q} \cdot \mathbf{n} = 0 \quad \text{on solid surface}$$

Division of the velocity vector into the self-induced and free-stream components yields

$$(u, w) \cdot \mathbf{n} + (U_\infty, W_\infty) \cdot \mathbf{n} = 0 \quad \text{on solid surface} \tag{11.4}$$

where the first term is the velocity induced by the singularity distribution on itself (hence "self-induced part") and the second term is the free-stream component $\mathbf{Q}_\infty = (U_\infty, W_\infty)$, as shown in Fig. 11.2.

Thin, Cambered Airfoil

A vorticity distribution is sought which satisfies both the condition that the mean camber line is a streamline and the Kutta condition. Again, the coordinate transformation

(30)
$$\xi = \frac{c}{2}(1 - \cos\theta)$$

is used, so the integral equation to be solved is

(31)
$$\frac{1}{2\pi}\int_0^{\pi} \frac{\gamma(\theta)\sin\theta}{\cos\theta - \cos\theta_0}\, d\theta = U_\infty\left(\alpha - \frac{dz}{dx}\right)$$

The vorticity distribution $\gamma(\theta)$ that satisfies the integral equation makes the vortex sheet (which is coincident with the mean camber line) a streamline of the flow.

Vorticity Distribution

The desired vorticity distribution may be represented by the series involving

1. A term of the form for the vorticity distribution for a symmetric airfoil,

(32)
$$2U_\infty A_0 \frac{1+\cos\theta}{\sin\theta}$$

2. A Fourier sine series whose terms automatically satisfy the Kutta condition

(33)
$$2U_\infty \sum_{n=1}^{\infty} A_n \sin n\theta$$

(T8)

The Coefficient A_n of the Fourier series depend on the shape of the Camber line. Thus

$$(34) \qquad \gamma(\theta) = 2U_\infty \left(A_0 \frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta \right)$$

Since each term is zero when $\theta = \pi$, the Kutta Condition is Satisfied

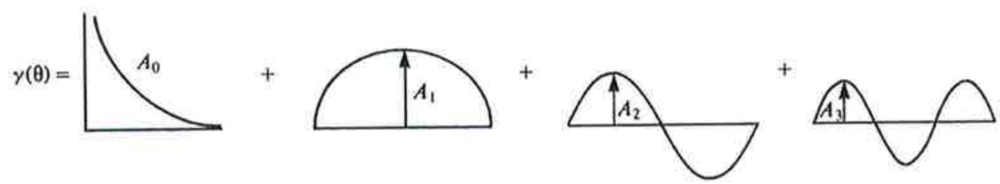


Figure 5.10 Schematic description of the first four terms in the series describing the circulation. [3]

Substituting the vorticity distribution (34) into (31) yields

$$(35) \qquad \frac{1}{2\pi} \int_0^{\pi} 2U_\infty \left[A_0 \frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta \right] \frac{\sin\theta}{\cos\theta - \cos\theta_0} \, d\theta = U_\infty \left[\alpha - \frac{dz}{dx} \right]$$

Using the Glauert's integral

$$(36) \qquad \int_0^{\pi} \frac{\cos n\theta_0}{\cos\theta_0 - \cos\theta} \, d\theta_0 = \frac{\pi \sin n\theta}{\sin\theta} \quad , \; n = 0, 1, 2, \ldots$$

and the following trigonometric relation

$$(37) \qquad \sin n\theta_0 \sin\theta_0 = \frac{1}{2} \left[\cos(n-1)\theta_0 - \cos(n+1)\theta_0 \right] , \quad n = 1, 2, 3, \ldots$$

Eqn. (35) reduces to

$$A_0 - \sum_{n=1}^{\infty} A_n \cos n\theta_0 = \alpha - \frac{dz}{dx} \tag{38}$$

or

$$\frac{dz}{dx} = (\alpha - A_0) + \sum_{n=1}^{\infty} A_n \cos n\theta_0$$

This is actually a Fourier expansion of the right-hand side of the equation that includes the information on the airfoil geometry. Multiplying both sides of the equation by $\cos m\theta$ and performing an integral from $0 \to \pi$, for each value of n, will result in the cancellation of all nonorthogonal multipliers (where $m \neq n$). Consequently, for each value of n the value of the corresponding coefficient A_n is obtained

$$\boxed{A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi} \frac{dz(\theta)}{dx} d\theta \quad , \; n = 0} \tag{39}$$

$$\boxed{A_n = \frac{2}{\pi}\int_0^{\pi} \frac{dz(\theta)}{dx} \cos n\theta \, d\theta, \quad n = 1, 2, 3, \ldots} \tag{40}$$

19

Aerodynamic Coefficients for a Cambered Airfoil

Useful Integral

(41)
$$\int_0^{\pi} \sin n\theta \sin\theta \, d\theta = \begin{cases} \frac{\pi}{2} & \text{when } n=1 \\ 0 & \text{when } n \neq 1 \end{cases}$$

The total circulation due to the entire vortex sheet from the leading edge to the trailing edge is

(42)
$$\Gamma = \int_0^{c} \gamma(\xi)\, d\xi = \int_0^{\pi} \gamma(\theta) \frac{c}{2} \sin\theta \, d\theta$$

$$= 2U_\infty \int_0^{\pi} \left[A_0 \frac{1+\cos\theta}{\sin\theta} + \sum A_n \sin n\theta \right] \frac{c}{2} \sin\theta \, d\theta$$

$$= c\,U_\infty \left[A_0 \int_0^{\pi} (1+\cos\theta)\, d\theta + A_1 \int_0^{\pi} \sin\theta \sin\theta \, d\theta \right]$$

$$= c\,U_\infty \left(\pi A_0 + \frac{\pi}{2} A_1 \right).$$

The lift coefficient is obtained as

(43)
$$C_L = \frac{L'}{\frac{1}{2}\rho U_\infty^2 c(1)} = \frac{\rho U_\infty \Gamma}{\frac{1}{2}\rho U_\infty^2 c} = \pi (2A_0 + A_1)$$

Note. This equation indicates that only the first two terms of the circulation will have an effect on the lift.

Recall that the coefficients A_0 and A_1 are given by Eqs. (39) and (40). Hence, Eqn. (43) becomes

(44) $$C_L = 2\pi\left[\alpha + \frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}(\cos\theta - 1)\,d\theta\right]$$

and

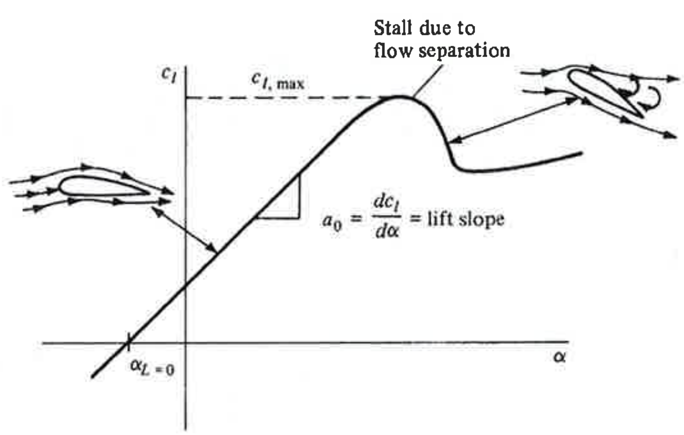
(45) $$\boxed{\text{Lift Slope} = \frac{dC_L}{d\alpha} = 2\pi}$$

Remark. The angle of zero lift is denoted by $\alpha_{L=0}$ and is a negative value. From the geometry shown in figure, clearly

$$C_L = \frac{dC_L}{d\alpha}(\alpha - \alpha_{L=0})$$

(46) $$= 2\pi(\alpha - \alpha_{L=0})$$

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}(\cos\theta - 1)\,d\theta$$



T10 Since the pitching moment about the y-axis is positive for a clockwise rotation, a minus sign needs to be included when calculating the moment M_0 relative to the airfoil leading edge:

$$(47) \quad M_0 = -\int_0^c \Delta p \, x \, dx = -\int_0^c \rho U_\infty \, \gamma(\xi) \, \xi \, d\xi$$

$$= -\rho U_\infty \int_0^c \gamma(\theta) \frac{c}{2}(1-\cos\theta)\frac{c}{2}\sin\theta \, d\theta$$

$$= \rho U_\infty \left[-\frac{c}{2} \underbrace{\int_0^\pi \gamma(\theta) \frac{c}{2} \sin\theta \, d\theta}_{\Gamma} + \frac{c^2}{4} \int_0^\pi \gamma(\theta) \sin\theta \cos\theta \, d\theta \right]$$

$$= -\frac{c}{2} \rho U_\infty \Gamma + \rho \frac{c^2}{4} U_\infty \left(A_0 \pi + A_2 \frac{\pi}{2} \right)$$

$$= -\rho U_\infty^2 \frac{\pi c^2}{4} \left(A_0 + A_1 - \frac{A_2}{2} \right)$$

The moment M along the x-axis can be described in terms of the lift and the moment at the leading edge as

$$(48) \quad M(x) = M_0 + x \cdot F \simeq M_0 + x \cdot L$$ Lift

The center of pressure x_{cp} is defined as the point where the moment is zero (this can be considered to be the point where the resultant lift force acts)

(49) $$M = M_o + x_{cp} \cdot L = 0$$

which yields

(50) $$x_{cp} = -\frac{M_o}{L} = \frac{c}{4}\,\frac{A_o + A_1 - \frac{A_2}{2}}{A_o + \frac{A_1}{2}}$$

In Summary the airfoil section aerodynamic coefficient are

(51) $$C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2 c} = 2\pi\left(A_o + \frac{A_1}{2}\right)$$

(52) $$C_D = \frac{D}{\frac{1}{2}\rho U_\infty^2 c} = 0$$

(53) $$C_{M_o} = \frac{M_o}{\frac{1}{2}\rho U_\infty^2 c^2} = -\frac{\pi}{2}\left[A_o + A_1 - \frac{A_2}{2}\right]$$

T11

Taken from Ref. [1]. PP 311-314.

Consider an NACA 23012 airfoil. The mean camber line for this airfoil is given by

$$\frac{z}{c} = 2.6595\left[\left(\frac{x}{c}\right)^3 - 0.6075\left(\frac{x}{c}\right)^2 + 0.1147\left(\frac{x}{c}\right)\right] \qquad \text{for } 0 \le \frac{x}{c} \le 0.2025$$

and

$$\frac{z}{c} = 0.02208\left(1 - \frac{x}{c}\right) \qquad \text{for } 0.2025 \le \frac{x}{c} \le 1.0$$

Calculate (a) the angle of attack at zero lift, (b) the lift coefficient when $\alpha = 4°$, (c) the moment coefficient about the quarter chord, and (d) the location of the center of pressure in terms of x_{cp}/c, when $\alpha = 4°$. Compare the results with experimental data.

Solution

We will need dz/dx. From the given shape of the mean camber line, this is

$$\frac{dz}{dx} = 2.6595\left[3\left(\frac{x}{c}\right)^2 - 1.215\left(\frac{x}{c}\right) + 0.1147\right] \qquad \text{for } 0 \le \frac{x}{c} \le 0.2025$$

and

$$\frac{dz}{dx} = -0.02208 \qquad \text{for } 0.2025 \le \frac{x}{c} \le 1.0$$

Transforming from x to θ, where $x = (c/2)(1 - \cos\theta)$, we have

$$\frac{dz}{dx} = 2.6595\left[\frac{3}{4}(1 - 2\cos\theta + \cos^2\theta) - 0.6075(1 - \cos\theta) + 0.1147\right]$$

or

$$= 0.6840 - 2.3736\cos\theta + 1.995\cos^2\theta \qquad \text{for } 0 \le \theta \le 0.9335 \text{ rad}$$

and

$$= -0.02208 \qquad \text{for } 0.9335 \le \theta \le \pi$$

(a) From Equation (4.61),

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}(\cos\theta - 1)\,d\theta$$

(*Note:* For simplicity, we have dropped the subscript zero from θ; in Equation (4.61), θ_0 is the variable of integration—it can just as well be symbolized as θ for the variable of integration.)

Substituting the equation for dz/dx into Equation (4.61), we have

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{0.9335} (-0.6840 + 3.0576\cos\theta - 4.3686\cos^2\theta + 1.995\cos^3\theta)\,d\theta \qquad \textbf{[E.1]}$$

$$-\frac{1}{\pi}\int_{0.9335}^{\pi} (0.02208 - 0.02208\cos\theta)\,d\theta$$

From a table of integrals, we see that

$$\int \cos\theta\,d\theta = \sin\theta$$

$$\int \cos^2\theta\,d\theta = \tfrac{1}{2}\sin\theta\cos\theta + \tfrac{1}{2}\theta$$

$$\int \cos^3\theta\,d\theta = \tfrac{1}{3}\sin\theta(\cos^2\theta + 2)$$

Hence, Equation (E.1) becomes

$$\alpha_{L=0} = -\frac{1}{\pi}[-2.8683\theta + 3.0576\sin\theta - 2.1843\sin\theta\cos\theta$$
$$+ 0.665\sin\theta(\cos^2\theta + 2)]_0^{0.9335}$$
$$-\frac{1}{\pi}[0.02208\theta - 0.02208\sin\theta]_{0.9335}^{\pi}$$

Hence, $$\alpha_{L=0} = -\frac{1}{\pi}(-0.0065 + 0.0665) = -0.0191 \text{ rad}$$

or $$\boxed{\alpha_{L=0} = -1.09^\circ}$$

(*b*) $\alpha = 4^\circ = 0.0698$ rad
From Equation (4.60),

$$c_l = 2\pi(\alpha - \alpha_{L=0}) = 2\pi(0.0698 + 0.0191) = \boxed{0.559}$$

(*c*) The value of $c_{m,c/4}$ is obtained from Equation (4.64). For this, we need the two Fourier coefficients A_1 and A_2. From Equation (4.51),

$$A_1 = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos\theta\,d\theta$$

$$A_1 = \frac{2}{\pi}\int_0^{0.9335} (0.6840\cos\theta - 2.3736\cos^2\theta + 1.995\cos^3\theta)\,d\theta$$
$$+ \frac{2}{\pi}\int_{0.9335}^{\pi} (-0.02208\cos\theta)\,d\theta$$
$$= \frac{2}{\pi}[0.6840\sin\theta - 1.1868\sin\theta\cos\theta - 1.1868\theta + 0.665\sin\theta(\cos^2\theta + 2)]_0^{0.9335}$$
$$+ \frac{2}{\pi}[-0.02208\sin\theta]_{0.09335}^{\pi}$$
$$= \frac{2}{\pi}(0.1322 + 0.0177) = 0.0954$$

T12

From Equation (4.51),

$$A_2 = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos 2\theta\, d\theta = \frac{2}{\pi}\int_0^{\pi}\frac{dz}{dx}(2\cos^2\theta - 1)\,d\theta$$

$$= \frac{2}{\pi}\int_0^{0.9335}(-0.6840 + 2.3736\cos\theta - 0.627\cos^2\theta$$

$$- 4.747\cos^3\theta + 3.99\cos^4\theta)\,d\theta$$

$$+ \frac{2}{\pi}\int_{0.9335}^{\pi}(0.02208 - 0.0446\cos^2\theta)\,d\theta$$

Note:

$$\int \cos^4\theta\, d\theta = \tfrac{1}{4}\cos^3\theta\sin\theta + \tfrac{3}{8}(\sin\theta\cos\theta + \theta)$$

Thus,

$$A_2 = \frac{2}{\pi}\left\{-0.6840\theta + 2.3736\sin\theta - 0.628\left(\frac{1}{2}\right)(\sin\theta\cos\theta + \theta)\right.$$

$$\left.- 4.747\left(\frac{1}{3}\right)\sin\theta(\cos^2\theta + 2) + 3.99\left[\frac{1}{4}\cos^3\sin\theta + \frac{3}{8}(\sin\theta\cos\theta + \theta)\right]\right\}_0^{0.9335}$$

$$+ \frac{2}{\pi}\left[0.02208\theta - 0.0446\left(\frac{1}{2}\right)(\sin\theta\cos\theta + \theta)\right]_{0.9335}^{\pi}$$

$$= \frac{2}{\pi}(0.11384 + 0.01056) = 0.0792$$

From Equation (4.64)

$$c_{m,c/4} = \frac{\pi}{4}(A_2 - A_1) = \frac{\pi}{4}(0.0792 - 0.0954)$$

$$\boxed{c_{m,c/4} = -0.0127}$$

(*d*) From Equation (4.66)

$$x_{cp} = \frac{c}{4}\left[1 + \frac{\pi}{c_l}(A_1 - A_2)\right]$$

Hence,

$$\frac{x_{cp}}{c} = \frac{1}{4}\left[1 + \frac{\pi}{0.559}(0.0954 - 0.0792)\right] = 0.273$$

Comparison with Experimental Data The data for the NACA 23012 airfoil are shown in Figure 4.22. From this, we make the following tabulation:

	Calculated	Experiment
$\alpha_{L=0}$	$-1.09°$	$-1.1°$
c_l (at $\alpha = 4°$)	0.559	0.55
$c_{m,c/4}$	-0.0127	-0.01

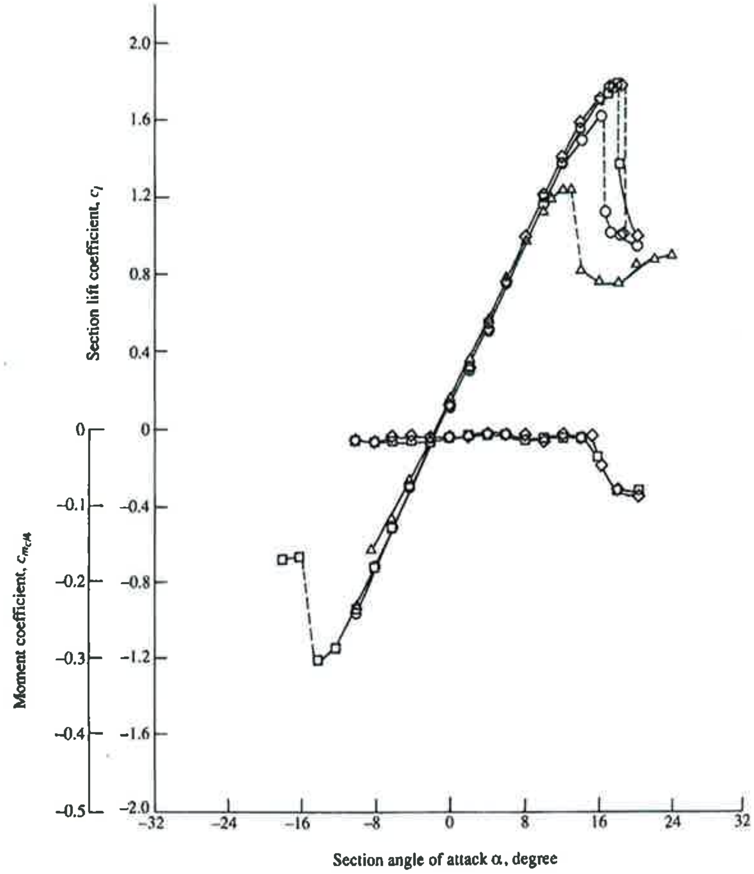


Figure 4.22 Lift- and moment-coefficient data for an NACA 23012 airfoil, for comparison with the theoretical results obtained in Example 4.2. [1]

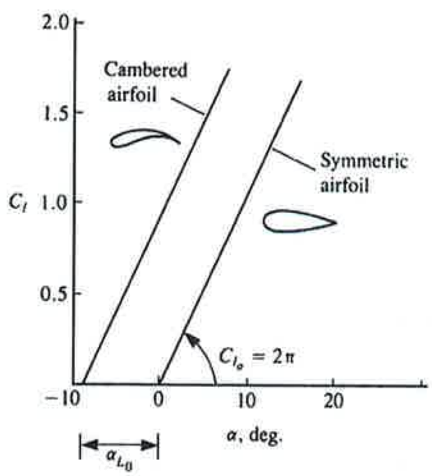


Figure 5.22 Schematic description of airfoil camber effect on the lift coefficient. [3]

References.

[1]. J.D. Anderson, Fundamental of Aerodynamics, Third Edition, MC-Graw Hill, 2001, Chapter 4.

[2]. J.J. Bertin, Aerodynamics for Engineers, 4th Edition, Prentice Hall, 2002, chapter 6.

[3]. J. Katz, A. Plotkin, Low-speed Aerodynamics, 2nd Edition, 2001, Chapter 5.