Announcing High Prices to Deter Innovation*

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Abstract

Price announcements—similar to the ones made by tech firms at media events—are effective in deterring innovation. By announcing (and setting) a high price, a firm increases its rivals' short-run profits, reducing the rival firms' incentives to innovate by magnifying their Arrow's replacement effect. We show that the equilibrium prices are greater and R&D investments lower relative to when price announcements cannot be used strategically. We call this the R&D deterrence effect of price and show that it induces equilibrium prices that may exceed the multiproduct monopoly prices and even dissipate the consumer benefits of innovation.

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1 Introduction

For many years now, Apple has unveiled and announced prices for its new products at media events. For example, Tim Cook unveiled the iPhone 11 Pro Max and announced its $1,099 price on September 10, 2019, at Apple's keynote event. A number of other recent examples suggest that price announcements at media events have become common practice in innovative industries (e.g., Microsoft unveiling its Surface Pro or Samsung launching the Galaxy S10 phone).[1] Price announcements are meaningful in that firms often choose not to revise these prices until they introduce a new generation of products (e.g., see the price history of the iPhone in Figure 1).

Price announcements have an impact on rival firms' incentives to innovate. To see this, consider a firm's unveiling of a new product and its price-announcement decision. On the one hand, announcing a high price reduces the amount of business that the new product will steal from substitute products sold by rival firms. If the profits of rival firms remain high despite the new product, rival firms will have less incentive to innovate or upgrade their existing products. On the other hand, announcing a low price may cause the new product to steal more of the business from rival products, which would make it more pressing for rival firms to innovate. Announcing a price that exceeds the static best response therefore has the cost of lower short-run profits but the benefit of less R&D activity by rival firms. These effects introduce a new trade-off in the pricing decision.

The mechanism by which price announcements impact the incentives to innovate is known as Arrow's replacement effect (Arrow, 1962). Arrow's replacement effect captures the tension between the profitability of a firm's existing products and the firm's incentives to upgrade those products (or invent new ones). The replacement effect has often been cited as a factor that kills innovation and even threatens the existence of firms in innovative industries (Christensen, 1993, 1997, Igami, 2017).[2] In this paper, we go one step further and show that firms can use

[1] Recent examples include announcements by Apple (see "Apple Unveils Smart Speaker Called HomePod," *The Wall Street Journal*, June 5, 2017), Microsoft (see "Microsoft's New Surface Pro Borrows From the Family to Revive Sales," *The Wall Street Journal*, May 23, 2017), Nintendo (see "Nintendo to Launch New 2DS XL Handheld Game Device in July," *The Wall Street Journal*, April 28, 2017), and Samsung (see "Samsung Has a Lot Riding on the Galaxy S8 Launch," *The Wall Street Journal*, March 29, 2017).

[2] See, for instance, "Nokia's New Chief Faces Culture of Complacency," *The New York Times*, September 26, 2010, or "Why Innovation and Complacency Don't Mix," *Associations Now*, September 2, 2014.

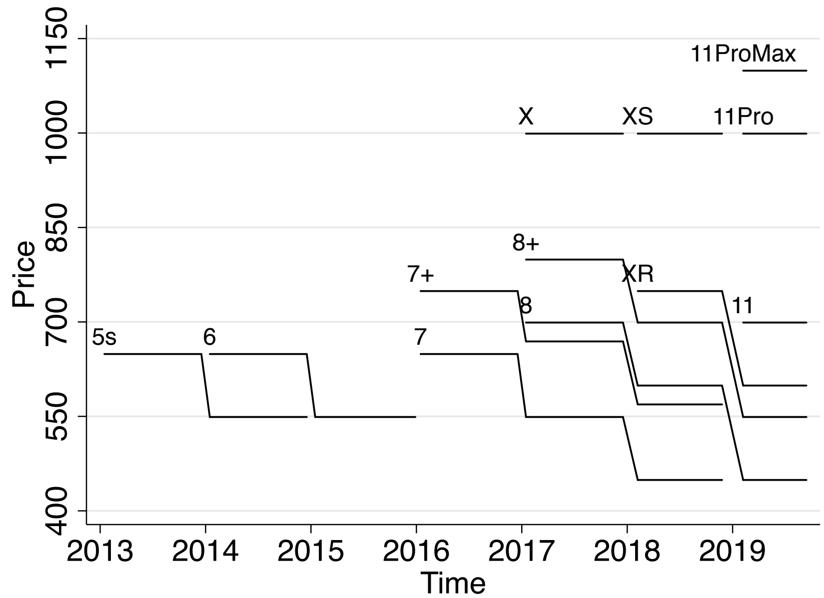


Figure 1: iPhone prices over time, by model

Source: Authors' calculations based on price announcements at Apple media events and past information posted on the Apple Store website. The prices are as of September of every year and correspond to the price of the cheapest version of each model when purchasing it unlocked (i.e., carrier free).

price announcements to affect the replacement effect of rivals and, consequently, their incentives to innovate—i.e., the *R&D deterrence effect* of price announcements. Specifically, we study the impact of price announcements on innovation outcomes and welfare in equilibrium via their R&D deterrence effect. Are price announcements moving industries into a state of complacency, increasing the level of prices, and eroding the consumer benefits of innovation?

We study price announcements in the context of a dynamic oligopoly model where firms compete both in prices and in developing a new product that improves upon the existing products. Motivated by the examples of price announcements at media events, we assume that firms make public price announcements when they start selling their products and then they choose how much to invest in R&D after observing the full profile of price announcements. We interpret these R&D investments as the resources that a firm can scale up (down) over time to accelerate (decelerate) its effort to make a preconceived product or technology ready for the market (e.g., the size of the research team devoted to the project).[3] That is, a

[3]As we discuss later in the paper, we do not model the one-time R&D investments that firms may incur to develop the prototype or concept of the new product/technology. "Direction

greater R&D investment on average increases the speed at which a firm brings a *new* product to market. To isolate the role played by price announcements in deterring R&D, we compare the equilibrium outcomes under price announcements with the equilibrium outcomes when firms cannot use price announcements to strategically influence R&D investments.

We show that price announcements can be used by firms to manipulate the (Arrow's) replacement effect of their rivals, thereby decreasing their incentives to innovate and moving the industry into a state of complacency. Price announcements cause equilibrium prices to be higher and innovation rates to be lower relative to the equilibrium without announcements. We show the existence of equilibria where no R&D activity takes place (*full deterrence*) and equilibria where some R&D activity takes place (*partial deterrence*). In some of these equilibria, the equilibrium prices can even surpass the multiproduct monopoly prices. In all equilibria, the higher prices are at the expense of static profits, but they increase the discounted value of the firm because of the lower innovation rates by rivals. Less innovation by rival firms benefits the firm because it decreases the probability that a rival will develop a competitive advantage.

We also examine the impact of competition on the R&D deterrence effect, where we use the degree of differentiation between products as our measure of competition. We show that the R&D deterrence effect vanishes (i.e., the equilibrium prices with and without price announcements are equal) in the two extreme cases of independent goods and homogeneous goods. This is either because the goods do not compete with each other (independent goods) or because the incentives to undercut the rival's price overwhelm the R&D deterrence motive of setting a higher price (homogeneous goods). The R&D deterrence effect peaks for intermediate levels of differentiation, which implies that the use of price announcements can reshape of the relationship between competition and innovation.

With respect to consumer welfare, we show that the higher prices and lower innovation rates in the equilibrium with price announcements make consumers worse off. We quantify the magnitude of the consumer-welfare loss in two ways. First, we show that the decrease in consumer welfare can lead to a decrease in total surplus relative to the case without price announcements, despite the increase in firms' profits. Second, the higher prices caused by the R&D deterrence effect of price announcements may completely dissipate the consumers' benefits from new

of research" decisions are likelier longer-run decisions and hence less likely to be affected by short-run price announcements.

products. That is, we find instances where consumers would be better off if firms did not engage in R&D activity (i.e., cases with products that stay the same forever) when price announcements are used. Both results suggest that the loss in consumer welfare caused by the R&D deterrence effect of prices is of first order.

A number of facts makes our theory plausible. First, Wu *et al.* (2004) show that the risk of cannibalizing existing products is a significant factor explaining why firms choose to delay the introduction of new products. This finding is significant for our work, as it links the mechanism that we study—i.e., manipulating rival profits via price announcements—to slowdowns in innovation. Second, data patterns in the smartphone industry align with our results on the impact of price announcements on market outcomes. In this industry, prices have been increasing at a rate faster than inflation (see Figure 1).[4] These price increases have been coupled with a worldwide slowdown in the rate at which consumers replace their smartphones. Tim Cook even acknowledged this in a letter to Apple investors on January 2, 2019: "iPhone upgrades also were not as strong as we thought they would be." Industry observers argue that among the factors explaining these smartphone-replacement numbers is that consumers are less impressed with the recent innovations found in smartphones.[5]

Our findings have several implications. First, they broaden our understanding of how prices can be used to soften competition along non-price dimensions. Second, they suggest that the R&D deterrence effect of price announcements has a first-order effect on consumer welfare, implying that the measurement of the welfare gains of innovation will be biased unless the strategic role of price announcements is accounted for. Third, they provide an economic argument for why firms in innovative industries make use of price announcements (although we acknowledge that marketing and other factors may also play a role).

Our paper contributes to several strands of the literature. First, it contributes to the literature on dynamic R&D competition. Our model resembles Loury (1979), Lee and Wilde (1980), and Reinganum (1982) in that innovation is uncertain and the arrival of the innovation follows a Poisson process with parameters that de-

[4]Price announcements have also been cited as unexpectedly high in other industries. For example, Nintendo announced a price for its Nintendo Switch that was higher than the market's expectation (see "Nintendo shares dive on pricing of Switch console," *Financial Times*, January 13, 2017).

[5]See, for example, "Samsung's new phone shows how hardware innovation has slowed," *Financial Post*, August 9, 2018, or "Upgrade rate slows by 33 percent as we hold onto our iPhones ever longer," *Cult of Mac*, February 10, 2019.

pend on the intensity of the firms' R&D investments. We extend these models by explicitly modeling the product-market game to study the interplay between price announcements and R&D investments (see Marshall and Parra 2019 for a similar model). In a related paper, Besanko *et al.* (2014) study pricing in a dynamic model of oligopoly with learning by doing. As in our work, the authors show that firms manipulate prices away from the static best response to impact learning/innovation, i.e., the authors show that learning by doing induces firms to decrease their prices to expand quantity and speed up their learning process (and slow down the learning process of their rivals).

Second, we contribute to the literature on the strategic use of prices to deter entry. Research has shown that firms may benefit from manipulating prices to signal cost efficiency or low market profitability (Milgrom and Roberts, 1982, Harrington, 1986), or to establish a reputation of being a "tough" competitor (Goolsbee and Syverson, 2008, Kreps and Wilson, 1982). More recently, Byford and Gans (2014, 2019) show that in markets where there is an upper bound on the firms that can operate profitably (e.g., natural duopoly), an efficient incumbent may have incentives to increase its price beyond its static best response to deter the exit of an inefficient rival (e.g., by acquisition) and ultimately prevent the entry of an efficient firm as its replacement. In all of these cases, firms sacrifice short-run profits to deter entry and increase the value of the firm in the long run.

Our paper is related more generally to the literature on strategic investments. A number of authors have studied sequential entry games in which a first mover can strategically invest in capacity to affect the entry incentives of a rival. These authors show conditions for when the equilibrium of the game features the first mover overinvesting in capacity—meaning that some capacity is left idle in the production stage—to deter entry (Spence, 1977, Dixit, 1980, Spulber, 1981, Bulow *et al.*, 1985).[6]

Strategic investments have also been studied in the context of R&D competitions. Fudenberg *et al.* (1984) show that in an R&D competition game where an incumbent has a first-mover advantage, the incumbent may choose to underinvest in improving its cost-efficiency level in the first period to limit its profitability in the second period, which is when entry occurs and the R&D competition begins. The firm underinvests because it wishes to manipulate its own R&D incentives

[6]Other studies have analyzed the strategic use of advertising, patenting, and production-sharing agreements to deter entry (e.g., Ellison and Ellison 2011, Gilbert and Newbery 1982, Chen and Ross 2000).

in future periods so as to avoid the "fat-cat effect"—i.e., a greater cost efficiency leads to a greater profit, which increases the firm's Arrow's replacement effect and makes it a weaker R&D competitor. This is in contrast to our work, where firms use price announcements to magnify the Arrow's replacement effect of rival firms so as to make them weaker competitors. Closest to our work is Gallini (1984), who shows that incumbents may use licensing agreements to share profits with potential entrants to decrease their incentives to innovate.

Lastly, our work relates to the economics literature on facilitating practices, which argues that advance price announcements can facilitate supracompetitive pricing in some settings (e.g., Rotemberg and Saloner 1990 and Blair and Romano 2002 when firms face asymmetric information).[7] In line with prior work, we show that the impact of price announcements on R&D competition creates unilateral incentives to set prices above the static best response, which causes prices and equilibrium profits to increase relative to the competitive benchmark. Unlike previous work, however, we show that advance price announcements can also have a negative impact on R&D investments, which has an additional effect on consumer welfare.

The rest of the paper is organized as follows. Section 2 introduces the model, and the equilibria with and without price announcements are discussed in Section 3 and Section 4, respectively. Section 5 discusses how the R&D deterrence effect of price announcements impacts the relationship between competition and innovation and Section 6 explores the impact of price announcements on welfare. Lastly, Section 7 discusses managerial implications and concludes.

2 Model Setup

Consider a continuous-time infinitely lived oligopoly, where firms sell differentiated goods and compete in prices. At every instant of time, and for a given vector of market prices $\mathbf{p}$, firm i earns a profit flow $\pi_i(\mathbf{p}) = (p_i - c_i)q_i(\mathbf{p})$, where q_i is the demand for firm i's product, and c_i is firm i's marginal cost of production. We assume $\partial q_i/\partial p_i < 0$ and $\partial q_i/\partial p_j > 0$ as well as some additional regularity conditions that guarantee a unique equilibrium in the static-price game and a unique solution to the problem of a multiproduct monopolist controlling the prices of all the goods (see Appendix B for details). For ease of exposition, we present

[7] See Buccirossi (2008) for an extensive literature review on facilitating practices.

our analysis for the case of a symmetric duopoly (i.e., $c_1 = c_2 = c$), and we later argue that our results generalize to the case of n firms and cost asymmetries.

Aside from competing in prices, firms compete in developing an innovation. We consider the case of a cost-saving innovation in our baseline model, but we later argue that our analysis also applies to quality-enhancing innovations. The firm that successfully innovates, which we call the *leader*, obtains a patented innovation that decreases its marginal cost to βc with $\beta \in (0, 1)$.

Firms invest in R&D by choosing a Poisson innovation rate x_i at a flow cost of $\kappa(x_i) = x_i^2/2$.[8] The Poisson processes are independent among firms, generating a memoryless stochastic process. Our preferred interpretation of the R&D competition is that firms are competing in bringing to market a clear and pre-determined product or idea. That is, the R&D investments do not affect the direction of innovation (i.e., the product is predetermined); they only affect the speed at which the firms can bring the product to market.[9] For example, the innovation rate x_i could represent the size of the team devoted to the research project, which can be scaled up or down over time.

We assume that after one firm successfully innovates, the industry reaches maturity.[10] Once the industry reaches maturity, firms no longer invest in R&D, and they play a static-asymmetric-price competition game at every instant of time. Define π_l and π_f to be the equilibrium profit flows earned by the leader and *follower* (i.e., the firm that loses the patent race), respectively, after the new technology is invented. Define the equilibrium prices of the leader and follower to be p_l and p_f, respectively.

As reference points, let π_s be the profit flow in the unique symmetric equilibrium of the static oligopoly price game, and let p_s be the price of each firm in this equilibrium. Similarly, let π_m be the per-product profit flow earned by a multiproduct monopolist controlling the prices of both goods, and let p_m be the multiproduct monopoly price for each good.

Lemma 1 (Profits and Prices). *In equilibrium,* $\pi_f < \pi_s < \pi_l$ *and* $p_l < p_f < p_s$.

After a firm has successfully innovated, the market reaches maturity and the

[8]A previous version of this article characterized the equilibrium of the game when allowing for a general strictly convex cost function. See the Online Appendix for the general characterization.

[9]We do not model the R&D investments that set the "direction of research" (i.e., the investments that establish the pre-determined product that the firm will invest to bring to market; c.f. Bryan and Lemus (2017)). We acknowledge that "direction of research" decisions are likelier longer-run decisions and thus less likely to be affected by short-run price announcements.

[10]We discuss the case of a sequence of innovations in the Online Appendix.

firms no longer invest in R&D. As established in Lemma 1, the innovation gives the leader a competitive advantage, increasing its profit flow relative to the static oligopoly payoff and decreasing that of the follower: $\pi_f < \pi_s < \pi_l$. The values of the innovator (or leader) and follower are given by $L = \pi_l/r$ and $F = \pi_f/r$, respectively, where r is the discount rate.

Definition 1 (Post-Innovation Values)**.** *When an innovation arrives, the market reaches maturity and the values of the technology leader and follower are given by* $L = \pi_l/r$ *and* $F = \pi_f/r$*, respectively.*

Depending on the initial level of the marginal costs c and the magnitude of the process innovation β, the profit flow of the leader may be higher or lower than the per-product monopoly profit flow, π_m. Because the distinction will be important in the analysis that follows, we call an innovation *incremental* when $\pi_l \leq \pi_m$ and *radical* when $\pi_l > \pi_m$.

Definition 2 (Incremental and Radical Innovations)**.** *An innovation is incremental when* $\pi_l \leq \pi_m$*, whereas it is radical when* $\pi_l > \pi_m$*.*

In what follows, we analyze the model under two different assumptions about the timing of play. First, in Section 3, we follow the dynamic oligopoly literature and study equilibrium outcomes when firms simultaneously decide on both prices and R&D investments (see, for instance, Ericson and Pakes 1995). In this case, prices cannot be used to strategically influence R&D investment decisions. The model with simultaneous decisions will serve as a benchmark and help us to isolate the strategic role played by price announcements.

In Section 4, we proceed to study the case where firms make simultaneous public price announcements at the beginning of the game, and credibly commit to those prices. Firms choose how much to invest in R&D in every period, conditioning their choices on the full profile of price announcements. As previously discussed, we interpret these investments as choosing the speed at which a new product is brought to market.

We will focus on studying the Markov perfect equilibria of the game. At each instant of time, firms' strategies will be a function of the state variables of the game only. In the case without price announcements, the only state variable is whether the innovation has arrived; strategies in the case with price announcements also depend on the prices announced by each firm.

3 No Price Announcements

We first analyze the game where firms choose both prices and R&D investments simultaneously at every instant of time.[11] Because of the timing of play, firms cannot use prices to affect their rivals' investment choices, making it a natural comparison for the model with price announcements.

Let r represent the discount rate. After a firm has successfully innovated, the values of the leader (or innovator) and follower are given by $L = \pi_l/r$ and $F = \pi_f/r$, respectively. Let V_i represent the value of firm i at time t before any firm has successfully innovated. Under Markov strategies and using the principle of optimality, we can write this value as

$$rV_i = \max_{p_i \geq 0, x_i \geq 0} \{\pi_i(p_i, p_j) + x_i(L - V_i) + x_j(F - V_i) - \kappa(x_i)\}. \tag{1}$$

Equation (1) shows that the flow value of participating in the market, rV_i, is equal to the sum of firm i's profit flow $\pi_i(p_i, p_j)$ net of the flow cost of its R&D, $\kappa(x_i)$, and the expected value gains and losses in the event of an innovation. Innovation makes firm i realize a value gain of $L - V_i$ at a rate x_i (firm i wins the competition) and a value loss of $F - V_i$ at a rate of x_j (firm i loses the competition).

Given the rival's strategy (x_j, p_j), the best-response functions of firm i are implicitly defined by the first-order conditions

$$x_i = L - V_i, \qquad \frac{\partial \pi_i(p_i, p_j)}{\partial p_i} = 0. \tag{2}$$

The first condition in equation (2) shows that firms choose their R&D investments by equating the incremental value of the innovation with the marginal cost of increasing the innovation rate, x_i. The incremental value of the innovation, $L - V_i$, depends on firm i's value under its *current* technology, V_i. The greater is V_i, the lower are firm i's incentives to invest in R&D (i.e., Arrow's replacement effect). The second condition in equation (2) shows that firms choose prices by maximizing their static product-market profits. Let V_{na}, p_{na}, and x_{na} represent the equilibrium values, prices, and investments, respectively, when there are no announcements. Equilibrium existence and uniqueness is established in the following proposition.

Proposition 1 (Equilibrium with No Announcements)**.** *There is a unique symmetric Markov perfect equilibrium, (p_{na}, x_{na}, V_{na}), that solves equations* (1) *and*

[11] A more general version of this model is discussed in Marshall and Parra (2019).

(2). *In equilibrium,* $p_{na} = p_s$*, where* p_s *is the unique static oligopoly price,* $x_{na} > 0$*, and* $V_{na} \in (F, L)$*.*

4 Price Announcements

We next consider the case with public price announcements. The timing of the game is as follows. At the beginning of the race, firms make simultaneous public price announcements, and they (credibly) commit to these prices until the next innovation arrives. Upon observing the announced prices, firms then choose how much to invest in R&D in every period.[12]

We solve the game by backward induction. We first analyze equilibrium investments and establish the existence of the firms' values for a given pair of prices $\mathbf{p} = (p_i, p_j)$. We then use the firms' values—which are a function of the price announcements—to analyze the equilibrium in the pricing stage. Multiple pricing equilibria may exist in this scenario. In every equilibrium, market prices are higher and R&D investments are lower than those when pricing and investment decisions are determined simultaneously (i.e., the case where price choices cannot affect the rivals' R&D decisions). When firms play symmetric strategies, we show the existence of an equilibrium where firms choose not to invest in R&D (*full deterrence*) and other equilibria in which the investments are positive but lower than those in the case without price announcements (*partial deterrence*). We also show that prices may exceed the multiproduct monopoly prices in equilibria with partial deterrence, which shows that deterrence incentives significantly distort equilibrium prices.

4.1 R&D Investments

Let $V_i(\mathbf{p})$ represent the value of firm i at time t before any firm has successfully innovated as a function of the announced prices $\mathbf{p} = (p_i, p_j)$. Using the principle of optimality, and given beliefs about the strategy of firm j, x_j, we can write firm i's R&D decision problem as

$$rV_i(\mathbf{p}) = \max_{x_i \geq 0} \{\pi_i(\mathbf{p}) + x_i(L - V_i(\mathbf{p})) + x_j(F - V_i(\mathbf{p})) - \kappa(x_i)\}, \tag{3}$$

[12]To simplify exposition, we assume that both firms invest in R&D. We emphasize, however, that a firm's incentive to announce a high price and deter a rival from introducing a new product exists even if the firm that makes the price announcement does not have R&D capabilities.

which depends on the vector of price announcements $\mathbf{p}$. This value function has an interpretation that is similar to that of equation (1), with the difference being that prices are now fixed at $\mathbf{p}$. We obtain firm i's R&D investment strategy by taking the first-order condition with respect to x_i,

$$x_i^*(\mathbf{p}) = L - V_i(\mathbf{p}). \tag{4}$$

As before, the investment rule equates the marginal cost of increasing the innovation rate, x_i, with the incremental value of an innovation, $L - V_i(\mathbf{p})$. Lemma 2 shows that for any price vector $\mathbf{p}$, there exists a value function $V_i(\mathbf{p}) \in (L, F)$ that simultaneously solves equations (3) and (4).

Lemma 2 (Value-Function Existence)**.** *Fix any R&D strategy for the opponent, $x_j \geq 0$. For any price vector $\mathbf{p}$ such that $\pi_i(\mathbf{p}) \in [\pi_f, \pi_l + x_j(L-F))$, there exists a unique function $V_i(\mathbf{p}) \in (L, F)$ that simultaneously solves equations (3) and (4).*[13]

4.2 Market Prices

Equipped with the value functions and R&D investment strategies, we can now proceed to study equilibria in the price-announcement stage. At this stage, firms take their future strategies, $x_i^*(\mathbf{p})$, and the strategies of their opponent, $x_j^*(\mathbf{p})$, as given. Given the beliefs about p_j, each firm i chooses its price announcement by solving $\max_{p_i} V_i(\mathbf{p})$.[14]

Firm i's first-order condition with respect to p_i is given by

$$\frac{d\pi_i(\mathbf{p})}{dp_i} + \frac{dx_i^*}{dp_i}\underbrace{(F - V_i(\mathbf{p}) - \kappa'(x_i))}_{= 0 \text{ by Envelope theorem}} + \underbrace{\frac{dx_j^*}{dp_i}(F - V_i(\mathbf{p}))}_{\text{R\&D deterrence effect}>0} = 0. \tag{5}$$

That is, when firm i considers the effect of p_i on its value, $V_i(\mathbf{p})$, it considers both the standard effect of price on its static profit π_i and also the impact of price on the expected value loss when losing the patent race; i.e., $x_j^*(\mathbf{p})(F - V_i(\mathbf{p}))$. This second force arises because firms can manipulate the R&D investments of rival firms with their price announcements, and firms wish to prevent their rivals from winning the

[13]Given x_j and $\mathbf{p}$, there is a unique value of V_i that solves equations (3) and (4). However, multiple equilibria may exist, as firms might coordinate at different values of x_j for different price announcements $\mathbf{p}$. The values of $V_i(\mathbf{p})$ differ across equilibria.

[14]Since equation (3) does not depend on t, and as long as no firm has successfully innovated, the firms would make the same price announcement regardless of the timing of the announcement.

patent race. To see how the firm manipulates the R&D investments of its rival, note that when the firm announces its price p_i, the firm affects the rival's profit flow π_j and consequently the rival's value $V_j(\mathbf{p})$. Because the R&D investments are determined by the incremental value of the innovation, $x_j^*(\mathbf{p}) = L - V_j(\mathbf{p})$, the announced price p_i has a direct impact on the rival's R&D investment.

Lemma 3 (R&D Deterrence Effect of Price Announcements)**.** *First-order condition* (5) *is equivalent to*

$$\frac{d\pi_i(\mathbf{p})}{dp_i} + R_i(\mathbf{p})\frac{d\pi_j(\mathbf{p})}{dp_i} = 0, \tag{6}$$

where $R_i(\mathbf{p})$ represents how much firm i internalizes firm j's profit when setting its price, p_i. $R_i(\mathbf{p})$ is a function of $\mathbf{p}$ given by

$$R_i(\mathbf{p}) \equiv \frac{dV_i}{dV_j} = \frac{V_i(\mathbf{p}) - F}{r + 2L - V_i(\mathbf{p}) - V_j(\mathbf{p})} > 0. \tag{7}$$

That is, the R&D deterrence effect of price announcements is positive.

Lemma 3 shows that the R&D deterrence effect of price announcements is positive, which results in an upward pressure on prices solely driven by deterrence motives. The lemma also shows that we can rewrite the first-order condition (6) in a way that resembles the first-order condition of a multiproduct monopolist. Aside from the standard effect of p_i on π_i, the term $d\pi_j/dp_i > 0$ captures the price complementarities between the substitute goods (i.e., how much firm j benefits from an increase in p_i), and the term $R_i(\mathbf{p}) = dV_i/dV_j > 0$ measures the impact of an increase in firm j's value on the value of firm i. That is, for every dollar that j obtains from an increase in p_i, $d\pi_j/dp_i$, firm i earns $R_i(\mathbf{p})$. The firm internalizes these pricing externalities because of the benefits of deterrence rather than because of ownership claims.[15]

Proposition 2. *In any interior equilibrium with price announcements, $p_i^* > p_s$ for every firm i, where p_s is the static oligopoly price.*

Proposition 2 is our main result. It says that in any equilibrium with price announcements—either symmetric or asymmetric—firms will announce prices higher

[15] It is important to highlight that the R&D deterrence effect is solely driven by the firms' incentives to influence the investments of their rivals and it is not driven by collusive behavior. For results on firms' investments under collusive behavior, see Fershtman and Pakes (2000).

than the static oligopoly price. That is, firms are willing to give up static profits in order to increase the overall value of their firm $V_i(\mathbf{p})$ through a lower pace of innovation by rivals.

4.3 Full Deterrence

We next characterize the set of symmetric equilibria of the game. In these equilibria, firms make symmetric price announcements in the first stage of the game, which yield symmetric payoffs and values; i.e., $\pi_i(\mathbf{p}) = \pi_j(\mathbf{p}) = \pi(\mathbf{p})$ and $V_i(\mathbf{p}) = V_j(\mathbf{p}) = V(\mathbf{p})$.

We start by considering the possibility of an equilibrium with full deterrence, which happens when the price announcements cause firms to choose not to invest in R&D: $x_i^* = x_j^* = 0$. Equation (4) shows that firms choose to invest in R&D according to the incremental benefit of the innovation: $x_i^* = L - V(\mathbf{p})$. Hence, for full deterrence to be achieved, the price announcements of firms must be such that $V(\mathbf{p}) \geq L$, which is equivalent to setting prices such that $\pi(\mathbf{p}) \geq \pi_l$, where π_l is the profit flow of the leader in the post-innovation subgame.[16] Such prices only exist when π_l is less than the multiproduct monopoly profit, π_m (i.e., the maximum symmetric profit flow that firms can achieve in the pre-innovation subgame). When $\pi_l \leq \pi_m$, we say that the innovations are incremental (see Definition 2).

Let p_{full} be the lowest price such that $\pi(p, p) = \pi_l$. Because $\pi_s < \pi_l \leq \pi_m$ in the case of incremental innovations (see Lemma 1), the intermediate value theorem implies that p_{full} is greater and lower than the static oligopoly and multiproduct monopoly prices, respectively: $p_s < p_{full} \leq p_m$. Proposition 3 shows that when innovations are incremental, there exists a full-deterrence equilibrium where firms announce p_{full} and no innovation takes place, $x_i^* = x_j^* = 0$. The value of both firms is equal to the value of the leader in the post-innovation subgame, $V(\mathbf{p}) = L$, even though the new product is never invented.

Proposition 3 (Full-Deterrence Equilibrium)**.** *Assume $L - F \geq r$. When innovations are incremental (i.e., $\pi_l \leq \pi_m$), there exists a* full-deterrence *equilibrium, in which $p_{full} = \min\{p : \pi(p, p) = \pi_l\}$, $x_{full} = 0$, and $V_{full} = L$. In this equilibrium, firms completely deter rivals from investing in R&D by announcing prices that are lower than the multiproduct monopoly price ($p_{full} \leq p_m$) but higher than the static oligopoly price ($p_{full} > p_s$).*

[16] To see this, note that when $x_i = x_j = 0$, $V(\mathbf{p}) = \pi(\mathbf{p})/r$.

We note that the derivative of V_i with respect to p_i at $\mathbf{p}_{full} = (p_{full}, p_{full})$ is positive, suggesting that the benefits of R&D deterrence exceed the costs of setting a price higher than the static best response even at the equilibrium prices. To see this, note that $V(\mathbf{p}_{full}) = L$ implies that $R(\mathbf{p}_{full}) = (L - F)/r \geq 1$ (see equation (6) for the definition of $R(\mathbf{p})$).[17] Hence, the derivative of V_i with respect to p_i at $\mathbf{p}_{full}$ is given by

$$\frac{d\pi_i(\mathbf{p}_{full})}{dp_i} + R_i(\mathbf{p}_{full})\frac{d\pi_j(\mathbf{p}_{full})}{dp_i} \geq \frac{d\pi_i(\mathbf{p}_{full})}{dp_i} + \frac{d\pi_j(\mathbf{p}_{full})}{dp_i} > 0,$$

where the first inequality follows from $R(\mathbf{p}_{full}) \geq 1$ and the second one follows from both $p_{full} < p_m$ and the stability of the multiproduct monopoly solution. Why are firms not increasing their prices beyond p_{full} if the derivative is positive at that point? Because firms cannot be further deterred, as they have already reached the zero lower bound of R&D investments: $x^* = 0$. Therefore, the R&D deterrence effect is zero for any price p_i beyond p_{full}. Mathematically, the derivative of V_i with respect to p_i is discontinuous at $\mathbf{p}_{full}$. The derivate is given by equation (6) for any price $p_i \leq p_{full}$, and it is given by $\partial\pi_i(p_i, p_{full})/\partial p_i < 0$ for any price $p_i > p_{full}$. As in the rest of the literature on strategic deterrence, these results show that firms are willing to sacrifice static market profits to increase their discounted value via R&D deterrence ($V_{full} > V_{na}$).

The assumption $L - F \geq r$ (or, equivalently, $\pi_l - \pi_f \geq r^2$) in the statement of the proposition restricts the value gain of being the industry leader $(L - F)$ to be at least r. Although in equilibrium firms end up increasing their value, the assumption guarantees that the net benefits of deterrence are high enough that a firm is willing to unilaterally pay the cost of setting a high price in order to deter their rival's R&D. From the analysis above one can see that the condition $L-F \geq r$ guarantees the existence of a full-deterrence equilibrium, but it is not necessary for its existence. That is, it is possible to have a full-deterrence equilibrium even if innovations provide a very small competitive advantage (i.e., if $\pi_l - \pi_f < r^2$).

4.4 Partial Deterrence

We next characterize the set of symmetric equilibria of the game where firms are only partially deterred, that is, equilibria where firms choose R&D investments that are positive but lower than the investments in the case without price an-

[17] Here, we make use of the assumption $L - F - r \geq 0$ in the statement of Proposition 3.

nouncements. We show that two equilibria with partial deterrence can exist: the low- and high-deterrence equilibria.[18]

The most notable difference between these equilibria is that firms are more aggressive in a high-deterrence equilibrium. This aggressiveness leads to lower R&D levels and higher prices relative to those in the low-deterrence equilibrium. The aggressiveness of firms in deterring their rivals shows up in the value of $R(\mathbf{p})$ (see equation (7)), which captures the extent to which firms internalize rival profits when setting prices. In the low-deterrence equilibrium, a firm less than fully internalizes the profits of its rival, $R(\mathbf{p}) \in (0, 1)$, whereas in the high-deterrence equilibrium, a firm more than fully internalizes the profits of its rival, $R(\mathbf{p}) > 1$.

The most notable difference between these equilibria is that the R&D levels in the high-deterrence equilibrium are lower than those in the low-deterrence equilibrium. This happens because firms are more aggressive in deterring their rivals in the high-deterrence equilibrium, which leads to higher equilibrium prices (and thus less incentives to innovate). The aggressiveness of firms in deterring their rivals shows up in the value of $R(\mathbf{p})$ (see equation (7)), which captures the extent to which firms internalize rival profits when setting prices. In the low-deterrence equilibrium, a firm less than fully internalizes the profits of its rival, $R(\mathbf{p}) \in (0, 1)$, whereas in the high-deterrence equilibrium, a firm more than fully internalizes the profits of its rival, $R(\mathbf{p}) > 1$.

We show that the low-deterrence equilibrium always exists, whereas the high-deterrence equilibrium can only exist when innovations are incremental (i.e., $\pi_l \leq \pi_m$) for reasons that relate to the discussion in the previous paragraph. Because firms are aggressively deterring their rivals in the high-deterrence equilibrium, firms need to coordinate at prices that earn them profit flows that are higher than the profit flow earned by the successful innovator (π_l). This is only possible when innovations are incremental (i.e., $\pi_l \leq \pi_m$). We summarize this discussion in the following lemma.

Lemma 4 (Properties of Partial-Deterrence Equilibria)**.**

i) When the innovations are radical (i.e., $\pi_l > \pi_m$), the V_{low} solution is the only candidate for equilibrium. When innovations are incremental (i.e., $\pi_l \leq \pi_m$), both solutions are candidates.

[18]This follows from the fact that the equilibrium value functions (i.e., $V(\mathbf{p}) = V_i(\mathbf{p}) = V_j(\mathbf{p})$) solve a quadratic equation with two possible solutions: V_{low} and V_{high} (see equation (13) in Appendix A).

ii) In any solution V_{low}, a firm less than fully internalizes the profits of its rival, $R(\mathbf{p}) \in (0,1)$, whereas in any solution V_{high}, a firm more than fully internalizes the profits of its rival, $R(\mathbf{p}) > 1$.

Low-Deterrence Equilibrium Because the V_{low} solution is defined for all types of innovation, we start by characterizing the low-deterrence equilibrium and showing that only mild conditions are needed for its existence.

In Proposition 2 we showed that any equilibria with price announcements feature prices that are higher than the static oligopoly prices. We can now also say that the equilibrium prices in any low-deterrence equilibria, p_{low}, are lower than the multiproduct monopoly price, p_m. To see this, we note that the derivative of the value function $V_i(\mathbf{p})$ with respect to p_i at the multiproduct monopoly prices $\mathbf{p}_m$ is negative,[19]

$$\frac{d\pi_i(\mathbf{p}_m)}{dp_i} + R(\mathbf{p}_m)\frac{d\pi_j(\mathbf{p}_m)}{dp_i} < \frac{d\pi_i(\mathbf{p}_m)}{dp_i} + \frac{d\pi_j(\mathbf{p}_m)}{dp_i} = 0,$$

where the inequality follows from $d\pi_j(\mathbf{p})/dp_i > 0$ and $R(\mathbf{p}) \in (0,1)$ in any low-deterrence equilibrium (see Lemma 4). Similarly, the derivative above evaluated at the static oligopoly prices $\mathbf{p}_s$ is positive both because the R&D deterrence effect is positive and $d\pi_i(\mathbf{p}_s)/dp_i = 0$ at p_s. Therefore, from the intermediate value theorem, it follows that there exists a symmetric price vector (p_{low}, p_{low}) with $p_{low} \in (p_s, p_m)$ that solves the first-order condition (6).

The existence and uniqueness of a low-deterrence equilibrium is discussed in the following proposition. In the proposition, we provide a sufficient (although not necessary) condition for equilibrium uniqueness based on a function of the primitives of the model,

$$\Psi(p) = \frac{\partial^2\pi_i(\mathbf{p})}{\partial p_i^2} + \max\left\{0, \frac{\partial^2\pi_j(\mathbf{p})}{\partial p_i^2}\right\} - \Lambda\frac{\partial\pi_i(\mathbf{p})}{\partial p_i}\frac{\partial\pi_j(\mathbf{p})}{\partial p_i}, \tag{8}$$

where $\mathbf{p} = (p,p)$ and $\Lambda = (3/(r + 2(L-F)))^2) > 0$. Assuming that $\Psi(p) < 0$ for all $p \in (p_s, p_m)$ is sufficient to guarantee that there exists a unique low-deterrence equilibrium. This condition guarantees that the pricing problem is concave (given the optimal R&D strategies in equation (4)). Below we provide examples with linear and logit demand functions where this condition is satisfied (see Table 1).

[19]For notational ease, we use $\mathbf{p}_s$ and $\mathbf{p}_m$ to denote the vectors (p_s, p_s) and (p_m, p_m), respectively.

Proposition 4 (Low-Deterrence Equilibrium)**.** *Assume* $\Psi(p) < 0$ *for all* $p \in (p_s, p_m)$*. There exists a unique* low-deterrence *symmetric Markov perfect equilibrium,* $(p_{low}, x_{low}, V_{low})$*. In this equilibrium, firms deter their rivals' R&D* ($x_{low} < x_{na}$) *by announcing higher prices* ($p_{low} \in (p_{na}, p_m)$) *and they earn greater profits* ($V_{low} > V_{na}$) *relative to the case without price announcements.*

High-Deterrence Equilibrium We next focus on the high-deterrence equilibrium. As noted in (see Lemma 4), in this equilibrium a firm aggressively deters R&D investments by more than fully internalizing the profits of its rival when setting its price: $R(\mathbf{p}) > 1$. This implies that the derivative of $V_i(\mathbf{p})$ with respect p_i (see equation (6)) is nonzero for any symmetric-price vector with prices between p_s and p_m (i.e., the static oligopoly and multiproduct monopoly prices, respectively). Hence, if an equilibrium exists, it must feature an equilibrium price, p_{high}, that exceeds the multiproduct monopoly price: $p_{high} > p_m$.

As noted in Lemma 4, the high-deterrence equilibrium can only exist when innovations are incremental (i.e., $\pi_l \leq \pi_m$) for reasons that relate to our discussion of the full-deterrence equilibrium. That is, firms will only choose to make a low R&D investment if their Arrow's replacement is high: $x_i^*(\mathbf{p}) = L - V_i(\mathbf{p})$. In particular, the degree of deterrence in the high-deterrence equilibrium can only be sustained if firms coordinate at prices that earn them profits that exceed those earned by the successful innovator, π_l, which is only feasible in the case of incremental innovations (see the proof of Proposition 1 for details).[20] We note that no such restriction on the level of profits exists in the low-deterrence equilibrium, which is why it always exists.

Although equilibrium profit levels exceed those earned by the successful innovator, $\pi > \pi_l$, the value of firms in a high-deterrence equilibrium is less than the value of the successful innovator: $V_{high} < L$. This happens because firms still perform some R&D in this equilibrium, which is costly both because the R&D has to be paid for and positive R&D levels create the possibility that a rival firm will win the patent race (and develop a competitive advantage). We summarize this discussion of the properties of a high-deterrence equilibrium in the following proposition.

[20]Depending on the parameters of the model, the vector $\mathbf{p}$ that solves equation (6) may or may not satisfy the restriction that the profit flow is in the interval $[\pi_l, \pi_m]$. For this reason, a high-deterrence equilibrium may not always exist.

Proposition 5 (High-Deterrence Equilibrium)**.** *Assume $L - F \geq r$. When innovations are incremental (i.e., $\pi_l \leq \pi_m$), there may exist a* high-deterrence *equilibrium, $(p_{high}, x_{high}, V_{high})$. In this equilibrium, firms announce prices that are higher than the multiproduct monopoly price ($p_{high} > p_m$), deter their rivals' R&D ($x_{high} < x_{low}$) by more, and earn greater profits ($V_{high} > V_{low}$) relative to the low-deterrence equilibrium.*

The statement of the proposition assumes that the innovation provides the leader with a sufficiently large competitive advantage: $L - F \geq r$. This condition is necessary because if it fails to hold, the R&D investments become negative, which is both infeasible and inconsistent with the assumption of an interior solution that was used in the construction of the equilibrium. In economic terms, the assumption guarantees that the benefits of R&D deterrence are high enough that a firm is willing to set a high price to deter its rival.[21]

4.5 Examples

To illustrate our results and the existence of multiple equilibria with different levels of R&D deterrence, we present a series of examples in Table 1. In Panels A and B of Table 1, we present examples with linear and logit demand functions, respectively. In both panels, we keep the demand function, the marginal cost (c), and the discount rate (r) fixed throughout the examples. The magnitude of the innovation (β) is the only parameter that varies across examples. Each panel has the same taxonomy. Columns I and II present examples with incremental innovations ($\pi_l \leq \pi_m$). From Lemma 4 we know that a high-deterrence equilibrium may exist whenever the innovation is incremental; however, we find that a high-deterrence equilibrium only exists in Column I of each panel (i.e., a high-deterrence equilibrium is not guaranteed to exist when the innovation is incremental). In contrast, the full-deterrence equilibrium always exists when innovations are incremental. Column III presents an example with a radical innovation ($\pi_l > \pi_m$), where we know from Lemma 4 that only a low-deterrence equilibrium may exist. In all of the examples we have that $\Psi(p) < 0$ for all $p \in (p_s, p_m)$ (see Proposition 4), implying a unique low-deterrence equilibrium.

[21] See the last paragraph of our discussion of the full-deterrence equilibrium for a related discussion.

	Panel A: Linear Demand			Panel B: Logit Demand		
	I	II	III	I	II	III
Demand	$q_i = \frac{2-4p_i+2p_j}{3}$			$q_i = \frac{\exp\{-p_i\}}{1+\exp\{-p_i\}+\exp\{-p_j\}}$		
β	0.9	0.75	0.4	0.88	0.8	0.5
Other parameters	c=0.2, $r = 0.05$			c=0.13, $r = 0.03$		
Innovation type	Increm.	Increm.	Rad.	Increm.	Increm.	Rad.
Existence/Uniqueness						
Existence high eq.	Yes	No	-	Yes	No	-
Uniqueness low eq.	Yes	Yes	Yes	Yes	Yes	Yes
Prices						
p_s	0.4667	0.4667	0.4667	1.3379	1.3379	1.3379
p_m	0.6	0.6	0.6	1.5532	1.5532	1.5532
p_{low}	0.5015	0.4894	0.4897	1.4119	1.3896	1.3883
p_{high}	0.6641	-	-	1.5961	-	-
p_{full}	0.5125	0.5999	-	1.4376	1.5530	-
R&D						
$pace_{na}$	0.2391	0.5973	1.4803	0.1481	0.2430	0.6022
$pace_{low}$	0.1836	0.5824	1.4739	0.1123	0.2276	0.5957
$pace_{high}$	0.0613	-	-	0.0403	-	-
$pace_{full}$	0	0	-	0	0	-
Consumer Surplus						
CS_{na}	3.8720	4.0184	4.3858	14.1265	14.1772	14.3726
CS_{low}	3.7654	3.9930	4.3752	13.9484	14.1084	14.3451
CS_{high}	2.8178	-	-	12.9471	-	-
CS_{full}	3.1686	2.1342	-	12.9549	11.7641	-
Total Surplus						
TS_{na}	7.6957	7.9064	8.4582	28.0147	28.0906	28.3855
TS_{low}	7.6446	7.8959	8.4540	27.8724	28.0374	28.3646
TS_{high}	6.8192	-	-	26.9430	-	-
TS_{full}	7.2313	6.6195	-	26.9912	25.9206	-

Table 1: R&D Deterrence Effect: Numerical Examples

Note: An innovation is incremental (Increm.) or radical (Rad.) when $\pi_l \leq \pi_m$ and $\pi_l > \pi_m$, respectively. Existence high eq. indicates whether a high-deterrence equilibrium exists. Uniqueness low eq. indicates whether the condition for low-deterrence equilibrium uniqueness in Proposition 4 is satisfied. $p_s, pace_{na}$ are the equilibrium outcomes under no price announcements, where $pace$ is defined as $2x$. $p_{low}, pace_{low}$ and $p_{high}, pace_{high}$ are the equilibrium outcomes with price announcements in a low- and high-deterrence equilibrium, respectively. Consumer surplus (CS) is defined in equation (11), and total surplus is given by $CS + 2V$, where V is the value of a firm at the beginning of the game. The condition $L - F - r > 0$ holds in all of these examples.

4.6 Discussion: The Role of the Assumptions

We finish the section by discussing the key assumptions needed for the R&D deterrence effect of price announcements to hold, and those that are immaterial to the result.

Price commitments In our baseline model, firms announce and commit to a price until the next innovation arrives. Two observations are in order. First, rather than being an assumption, the commitment until the next innovation arrives is a result of a lack of change in the state variables during this period (i.e., no demand or cost shocks in the pre-innovation phase). That is, because the environment remains unchanged in the pre-innovation phase of the game, firms face no incentives to revise their price announcements. Second, we emphasize that committing to a price until the next innovation arrives is not central to our results. As long as price announcements are a commitment lasting any positive measure of time, they will affect the continuation value of their opponents, rendering price announcements effective in deterring the R&D of rivals. To address both of these points, the Online Appendix presents an extension of the model where we allow for demand shocks that lead to price-announcement revisions in the pre-innovation phase of the game.[22] All of our results carry through.

Innovation uncertainty Innovation uncertainty plays an important role in the firms' ability to deter R&D. To illustrate this point, consider the firms' incremental value of the innovation when firms are certain that they will achieve a breakthrough at a known date $\bar{t}$. In this case, the incremental value of the innovation is the difference between the value of inventing the innovation and the value of choosing not to invent the product, both measured at time $\bar{t}$. The pre-innovation prices (or values) are irrelevant for computing the incremental value of the innovation, which implies that they do not impact R&D decisions.

Things are different when there is innovation uncertainty—i.e., when the firms do not know *when* they will achieve a breakthrough. With uncertainty, the incremental value of the innovation is the value of inventing the product minus the value of remaining in the innovation race (i.e., the pre-innovation value). Because

[22] Demand shocks have incentivized firms to revise their price announcements. See, for instance, "Amazon just drastically dropped the price of its Fire phone," *Business Insider*, November 26, 2014.

the pre-innovation price announcements directly affect the pre-innovation values, these are effective tools in deterring R&D investments.

Continuous versus lumpy R&D investments In our baseline model, firms invest in R&D at every instant of time. An alternative modeling assumption would be that the R&D investments are a one-time investment rather than an ongoing effort. We note that the R&D deterrence effect of price announcements would fail to hold in an environment with lumpy R&D investments because firms would be unable to credibly commit to a price announcement. This can be seen by noting that before the firms make their R&D investment decisions, every firm has incentives to deter their rivals' R&D investments. However, as soon as the firms make their R&D decisions, all firms have incentives to revise their price announcements and set their prices according to their static best-response functions. This is because firms are no longer conducting R&D so there are no reasons to increase the price beyond the static best-response price. Firms foresee that price announcements are meaningless and are thus undeterred by price announcements in this case.

While we acknowledge that designing a product has fixed R&D costs, we also believe that bringing the product to market requires an ongoing effort that can be scaled up or down (e.g., by allocating more researchers to the task). Our analysis suggests that price announcements are only effective in deterring the latter.

Timing versus magnitude of the innovation In our baseline model, the R&D investments impact the probability of inventing the innovation but do not affect the innovation itself (e.g., the magnitude of the cost reduction). We note that the R&D deterrence effect of price announcements does not arise in a model where R&D investments only impact the magnitude of the innovation (i.e., the pace of innovation is exogenous). In this setting, the incremental value of increasing the magnitude of the innovation in dx_i is given by how it changes the post-innovation value of the leader ($dL(x_i)$). Because the incremental value of the innovation does not depend on pre-innovation prices (or values) in this case, these prices cannot deter R&D investments.

Other assumptions In the Online Appendix, we extend the model to show that the R&D deterrence effect of price announcements also exists in more general environments. With respect to market structure, we consider the cases with n symmetric players and the duopoly case with asymmetric firms. With respect to the nature of innovations, we consider the case of sequential innovations (i.e., a sequence of patent races) and the case of quality-enhancing innovations. Lastly, we allow for more general cost functions. All of our results hold in these extensions.

5 Price Announcements and the Relationship between Innovation and Competition

The impact of competition on innovation outcomes is a long-standing question that stems from the work of Schumpeter (1942) and Arrow (1962). This question continues to attract new work both because of a lack of consensus in the literature and its relevance for competition policy (e.g., see Loury 1979, Lee and Wilde 1980, Aghion *et al.* 2001, 2005, Vives 2008, Marshall and Parra 2019). We contribute to this debate by studying whether price announcements have an impact on the relationship between competition and innovation.

To answer this question, we parameterize the model using the demand system in Singh and Vives (1984), where the demand for good i is given by

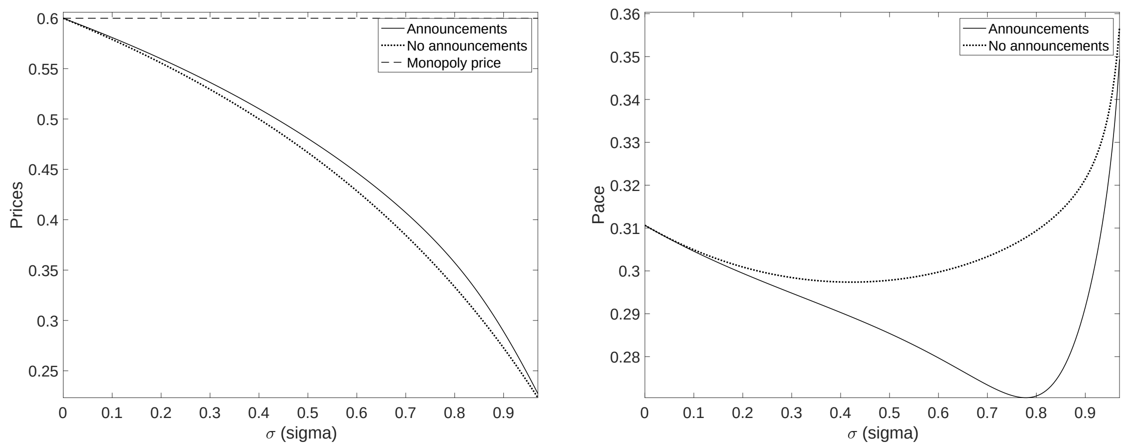
$$q_i = \frac{1}{1+\sigma} - \frac{1}{1-\sigma^2}p_i + \frac{\sigma}{1-\sigma^2}p_j, \tag{9}$$

and σ is restricted to be in the unit interval to capture the case of substitute goods. This demand system can capture various degrees of product differentiation. The two extreme cases of this system are the case of independent goods when $\sigma = 0$ (full differentiation) and the case of homogeneous goods when $\sigma \to 1$ (no differentiation). Henceforth, we use σ as our measure of competition between products.

We start by analyzing the impact of competition on the magnitude of the R&D deterrence effect: $p_a - p_s$ (i.e., the price difference between the equilibria with and without price announcements). We focus on the low-deterrence equilibrium, as it is the only equilibrium that exists for all types of innovations (i.e., incremental or radical). Using equations (6) and (9), we can write the equilibrium markup as

$$p_a - c = \frac{(1-\sigma^2)q(\mathbf{p}_a)}{1-\sigma R(\mathbf{p}_a)}. \tag{10}$$

Similary, we can write the equilibrium markup without price announcements as $p_s - c = (1-\sigma^2)q(\mathbf{p}_s)$.



a) Prices

b) Pace of innovation

Figure 2: R&D Deterrence Effect and Product Differentiation

Note: The figure shows the equilibrium prices with price announcements (p_a), without price announcements (p_s), and the multiproduct monopoly price for the following parameters: $r = 0.15$, $\beta = 0.65$, and $c = 0.2$, $\sigma \in [0, 0.97]$.

Equation (10) captures how competition affects the R&D deterrence effect. In the case of independent products ($\sigma = 0$), firms have no incentives to deter their rivals' R&D investments, as rival prices do not impact a firm's profits. When instead σ approaches one, the goods become homogeneous. When choosing prices, the firms face a tension between the incentives to undercut the rival in the product market and the incentives to increase prices to deter the R&D investments of the rival. Equation (10) shows that the incentives to undercut the rival are stronger, as the price markup go to zero when $\sigma \to 1$. In both of these extreme cases, equation (10) equals the equilibrium markup under no price announcements, suggesting that the use of price announcements does not impact the competition intensity between firms (i.e., prices and R&D investments are undistorted).

From equation (10), one can also see that the R&D deterrence effect is positive ($p_a > p_s$) for all intermediate values of competition (i.e., $\sigma \in (0, 1)$). This follows from the fact that $R(\mathbf{p}) < 1$ in any low-deterrence equilibrium, which causes the denominator in equation (10) to be strictly less than one. The greater equilibrium markups when firms use price announcements show the benefits of keeping innovation capabilities in industries where price announcements are used, as the threat of developing a competitive advantage prompts rivals to engage in "profit sharing" to decrease the probability of that competitive advantage materializing (i.e., the

R&D deterrence effect of price announcements).

The following proposition summarizes these results, and Figure 2 illustrates them with a numerical example.

Proposition 6 (R&D Deterrence Effect and Competition)**.** *The R&D deterrence effect is non-monotonic in the degree of competition (σ). The effect is positive for $\sigma \in (0, 1)$ (i.e., $p_a > p_s$ and $x_a < x_{na}$), and it vanishes both when products are independent ($\sigma = 0$) and when products are homogeneous ($\sigma \to 1$) (i.e., $p_a = p_s$ and $x_a = x_{na}$).*

Figure 2 also shows that because the R&D deterrence effect of price announcements increases in magnitude for intermediate values of competition, the R&D deterrence effect has an impact on the relationship between competition and the pace of innovation. Without price announcements, the pace of innovation decreases for values of σ in the range between 0 and 0.45 and then increases in σ for values larger than 0.45. The R&D deterrence effect of price announcements changes the relationship between innovation and competition in that the pace of innovation increases in σ for a smaller set of parameters ($\sigma \in (0.79, 1)$). This implies that there are values of σ for which the relationship between competition and innovation has different signs in the case with and without price announcements (e.g., $\sigma = 0.5$). These results suggest that if price announcements are not modelled, a researcher may incorrectly measure the relationship between competition and innovation in structural empirical studies of industries where price announcements are used.

6 Price Announcements and Consumer Welfare

The analysis in Section 4 reveals that price announcements unambiguously benefit firms in all equilibria with price announcements when compared to the equilibrium with no price announcements (see Propositions 3-5). We next turn to studying how price announcements impact consumer welfare in two different ways. We first compare consumer welfare in the equilibria with and without price announcements, and we then study the impact of innovation on consumer welfare in a world where firms always use price announcements.

6.1 Consumer Welfare

Propositions 3, 4, and 5 establish that consumers face both higher prices in the pre-innovation phase and a lower pace of innovation when firms make price announcements (i.e., $p_a > p_s$ and $x_a < x_{na}$). These effects lead to two types of price increases. The first is a price increase that is immediate and lasts until one firm succeeds in the innovation race (i.e., $p_a > p_s$). The second is a price increase that arises from the fact that the innovation will on average be invented later in time ($x_a < x_{na}$). That is, a slower pace of innovation implies that consumers will pay the higher prices of the pre-innovation phase of the game for a longer period of time. These price increases combined imply that consumers will face weakly greater prices throughout the industry lifetime in the equilibrium with price announcements, which leads us to conclude that price announcements negatively impact consumer welfare.

To see this more formally, define $cs(\mathbf{p})$ to be the consumer surplus flow at prices $\mathbf{p}$. We assume that consumers purchase both products and they have a downward sloping demand function for each good. This assumption implies that $cs(\mathbf{p})$ is strictly decreasing in each dimension of $\mathbf{p}$. Define the expected discounted consumer surplus in the market as a function of pre-innovation prices $\mathbf{p}$ and innovation pace λ by

$$CS(\mathbf{p}, \lambda) = \frac{1}{r}\left(\frac{rcs(\mathbf{p}) + \lambda cs(p_l, p_f)}{r + \lambda}\right), \tag{11}$$

where $cs(\mathbf{p})$ is the consumer surplus flow at the vector of pre-innovation prices $\mathbf{p}$, $cs(p_l, p_f)$ is the consumer surplus flow after the innovation reaches the market, and $\lambda = 2x$ is the pace of innovation in the pre-innovation period (see Appendix C for details).

$CS(\mathbf{p}, \lambda)$ equals the value of a perpetuity that pays consumers a convex combination of $cs(\mathbf{p})$ and $cs(p_l, p_f)$, where the weight on $cs(p_l, p_f)$ increases as a function of the pace of innovation λ. We can easily establish that $CS(\mathbf{p}, \lambda)$ is decreasing in the pre-innovation prices $\mathbf{p}$. $CS(\mathbf{p}, \lambda)$ is also increasing in λ for any vector $\mathbf{p}$ such that $cs(\mathbf{p}) < cs(p_l, p_f)$. These two properties imply that consumer welfare in any equilibrium with price announcements is less than consumer welfare in the equilibrium without price announcements: $CS(\mathbf{p}_s, \lambda_{na}) > CS(\mathbf{p}_a, \lambda_a)$.[23]

[23]To establish this inequality, we use Lemma 1, which establishes that the pre-innovation prices in the equilibria with and without price announcements are higher than the post-innovation prices: $p_l < p_f < p_s < p_a$.

Proposition 7 (R&D Deterrence Harms Consumers)**.** *Consumer welfare with price announcements is less than consumer welfare without price announcements.*

Table 1 shows examples of the consumer-welfare loss caused by price announcements. In all of the examples in the table, the loss in consumer welfare dominates the profit gains of price announcements, which implies that price announcements can also cause a decrease in total surplus.

6.2 Consumer Benefits of Innovation

Proposition 7 establishes that consumers are worse off in any equilibrium with price announcements relative to the equilibrium without price announcements. In this subsection, we pursue a different question. In a world where all firms use price announcements, would consumers be better off if firms did not conduct R&D? Although the answer is always affirmative in the equilibrium with full deterrence (i.e., consumers pay prices higher than the static oligopoly prices and the innovation never arrives), it is less obvious in the equilibria with partial deterrence.

If all firms use price announcements, why would consumers be better off if firms did not conduct R&D? From Propositions 3, 4, and 5 we know that the prices faced by consumers in the period before a firm successfully innovates (p_a) are higher than the static oligopoly prices (p_s), but Lemma 1 also establishes that consumers directly benefit when the innovation reaches the market, as prices fall: $p_l < p_f < p_s < p_a$. These opposing effects on prices could lead to cases where the R&D deterrence effect of price announcements ($p_a > p_s$) outweighs the positive impact of innovation on consumer welfare ($p_l, p_f < p_s$) even in equilibria with partial deterrence. That is, price announcements may cause innovation to be welfare decreasing despite the consumer benefits of a new technology.

To answer the question of whether consumers benefit from innovation despite the R&D deterrence effect of price announcements, we compare the expected discounted consumer surplus under price announcements $CS(\mathbf{p}_a, \lambda_a)$ (see equation (11)) with the consumer surplus that exists with no R&D competition whatsoever; i.e., $CS^{\text{No Innov}} = cs(\mathbf{p}_s)/r$. The difference between $CS(\mathbf{p}_a, \lambda_a)$ and $CS^{\text{No Innov}}$ can be written as

$$\Delta CS = \frac{1}{r(r+\lambda)}\left(r\underbrace{(cs(\mathbf{p}_a) - cs(\mathbf{p}_s))}_{<0} + \lambda\underbrace{(cs(p_l, p_f) - cs(\mathbf{p}_s))}_{>0} \right).$$

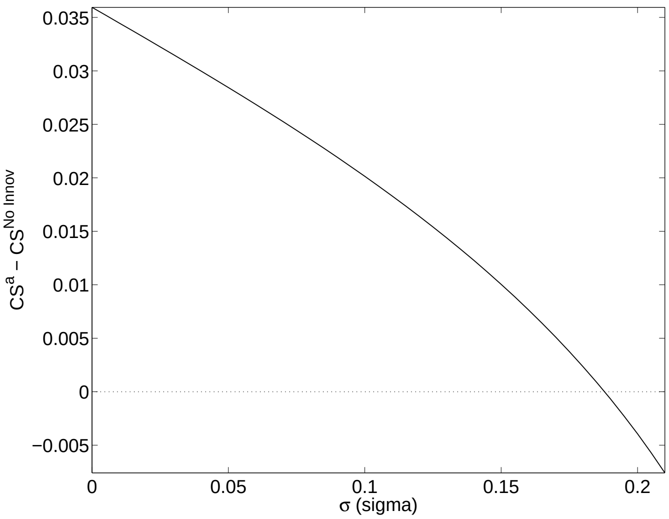


Figure 3: R&D Deterrence Effect and Consumer Benefits of Innovation

Note: The figure shows $\Delta CS = CS_a - CS^{\text{No Innov}}$ for the following parameters: $r = 0.04$, $\beta = 0.9$, $c = 0.08$, $q_i = \frac{1}{1+\sigma} - \frac{1}{1-\sigma^2}p_i + \frac{\sigma}{1-\sigma^2}p_j$ with $\sigma \in [0, 0.24]$ capturing the degree of differentiation between products (see Section 5 for a detailed discussion of this demand system). The sufficiency condition for equilibrium uniqueness, $\Psi(p) < 0$ for $p \in [p_s, p_m]$, is satisfied for every value of sigma. The equilibrium outcomes reported in the figure correspond to those in the low-deterrence equilibrium.

The first term in the parentheses is negative because of the R&D deterrence effect ($p_a > p_{na} = p_s$) and captures the loss in welfare due to the higher prices in the pre-innovation phase of the game. The second term in the parentheses is positive because consumers face lower prices once the innovation reaches the market ($p_l, p_f < p_a$). Depending on the relative magnitude of each of these terms, consumers may be worse off when firms perform R&D.

Figure 3 shows that depending on the parameters of the model, consumers may benefit or lose with R&D competition. For some parameters, the R&D deterrence effect fully dissipates the utility gains of innovation (e.g., $\sigma = 0.2$). While for other parameters, consumers are still better off with innovation competition despite the R&D deterrence effect (e.g., $\sigma = 0.1$). Regardless, we conclude that the R&D deterrence effect is of first order in understanding the consumer benefits of innovation.

Proposition 8 (Welfare Dissipation)**.** *The R&D deterrence effect of price dissipates the consumer benefits of innovation in all full-deterrence equilibria, and it may also do so in partial-deterrence equilibria.*

7 Concluding Remarks

We study how price announcements affect equilibrium market prices, firms' innovation rates, and welfare outcomes in a dynamic oligopoly model. We find that under price announcements, prices are always greater than the static oligopoly prices and may even exceed the multiproduct monopoly prices. Although price announcements are profitable for firms, price announcements decrease consumer surplus and may even decrease total surplus. The decrease in consumer surplus is caused by both higher prices and lower R&D investments. We show that the higher prices in the equilibrium with price announcements can even dissipate the consumer benefits of innovation. That is, we show instances where consumers would be better off if the industry exhibited no R&D activity whatsoever (i.e., when products stay the same forever). These results combined suggest that the effect of price announcements on consumer welfare is of first order and that the measurement of the welfare benefits of innovation will be biased unless the strategic price effects associated with R&D deterrence are considered.

These results provide an economic rationale for the use of price announcements in media events by tech firms. Our findings have strong managerial implications on the trade-offs that firms face when laying out their pricing strategies. Although pricing new products aggressively may attract new consumers—increasing the market share and short-run profits of the new products—competitors may react with an aggressive R&D plan. Depending on the firms' capabilities relative to competitors to come up with new products, the manager may prefer to avoid an intensification of R&D competition. This is particularly true in mature industries where product improvements are less clear and require large investments.

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A Omitted Proofs

Proof of Lemma 1. Using the envelope theorem and the demand regularity conditions (A), (B), and (D) (see section C of the Appendix) we find:

$$\frac{dp_i}{dc_i} = -\frac{-\frac{dq_i}{dp_i}}{2\frac{dq_i}{dp_i} + \frac{d^2q_i}{dp_i^2}(p_i - c)} \in (0,1), \qquad \frac{dp_j}{dp_i} = -\frac{\frac{dq_j}{dp_i} + \frac{d^2q_j}{dp_i dp_j}(p_j - c)}{2\frac{dq_j}{dp_j} + \frac{d^2q_j}{dp_j^2}(p_j - c)} \in (0,1).$$

Since $\frac{dp_j}{dc_i} = \frac{dp_j}{dp_i}\frac{dp_i}{dc_i} < \frac{dp_i}{dc_i}$, prices are lower when the cost-saving innovation arrives. In particular, $p_l < p_f < p_s$. Similarly for the profits we find:

$$\frac{d\pi_j}{dc_i} = \frac{dq_j}{dp_i}\frac{dp_i}{dc_i}(p_j - c) > 0, \qquad \frac{d\pi_i}{dc_i} = -q_i + \frac{dq_i}{dp_j}\frac{dp_j}{dp_i}\frac{dp_i}{dc_i}(p_i - c).$$

Given the results above, it is easy to see that $d\pi_j/dc_i$ is positive. For $d\pi_i/dc_i$, observe that at the prices satisfying the first order condition:

$$\frac{d\pi_i}{dc_i} = \left(\frac{dq_i}{dp_i} + \frac{dq_i}{dp_j}\frac{dp_j}{dp_i}\frac{dp_i}{dc_i}\right)(p_i - c) < \left(\frac{dq_i}{dp_i} + \frac{dq_i}{dp_j}\right)(p_i - c) < 0,$$

where we use dp_j/dp_i and $dp_i/dc_i \in (0,1)$ for the first inequality, and demand regularity condition (C) for the second. We conclude that $\pi_f < \pi_s < \pi_l$. ■

Proof of Proposition 1. The first-order condition for price in equation (2) and the demand regularity conditions imply that $\mathbf{p}_s = (p_s, p_s)$ is the unique equilibrium price vector and $\pi_s = \pi_i(\mathbf{p}_s)$ is the equilibrium profit earned by the firms.

Fix an arbitrary profit flow π. Replacing the R&D investment rule (2) in equation (3) and using symmetry, we obtain

$$rV = \pi + \frac{1}{2}(L - V)^2 + (L - V)(F - V) \tag{12}$$

which is a quadratic equation in V. Solving we obtain two candidate solutions

$$V_{low,high} = \left(2L + F + r \pm \sqrt{(L - F - r)^2 + 6(rL - \pi)}\right)\Big/\, 3. \tag{13}$$

This solution is well defined when the square root is non-negative. That is, whenever $\pi \le \overline{\pi} \equiv \pi_l + ((\pi_l - \pi_f)/r - r)^2/6$. Define $\rho = \sqrt{(L - F - r)^2 + 6(rL - \pi)}$. Replacing the solutions of V in the first order condition (2) yields the R&D investments, which are given by $x_{low} = (L - F - r + \rho)/3$ and $x_{high} = (L - F - r - \rho)/3$. It can be easily verified that x_{low} is non-negative whenever $\pi \le \pi_l$. Similarly, x_{high} is non-negative when $L - F \ge r$ and $\pi \in [\pi_l, \overline{\pi}]$. Since in the no-announcement scenario $\pi_s < \pi_l$, only V_{low} is a feasible solution and the equilibrium is given by $V_{na} = V_{low}(\pi_s)$ and $x_{na} = x_{low}$. ■

Proof of Lemma 2. Fix $\hat{x}_j$ and replace the R&D strategy (4) into equation (3) to define:

$$\phi(V) = \pi_i(\mathbf{p}) + \frac{1}{2}(L - V)^2 + \hat{x}_j(F - V) - rV. \tag{14}$$

Observe that $\phi(V) = 0$ defines a solution to problem (3). Note that $\phi(L) = \pi_i(\mathbf{p}) - \pi_l -$

$\hat{x}_j(L-F)$ is negative for all $\pi_i(\mathbf{p}) < \pi_l + \hat{x}_j(L-F)$. Also, $\phi(F) = \pi_i(\mathbf{p}) - \pi_f + \frac{1}{2}(L-F)^2 > 0$, which is positive for $\pi_i(\mathbf{p}) \in [\pi_f, \pi_l]$. Therefore, by the intermediate value theorem, there exists a value of $V \in (F, L)$ such that $\phi(V) = 0$. This value is unique as $\phi'(V) = -(L - V + \hat{x}_j + r) < 0$ for all $V \in [F, L]$ so that ϕ crosses zero only once. ∎

Proof of Lemma 3. We start by studying the expression for $dx_j(\mathbf{p})/dp_i$. Using the equilibrium R&D investments (4) we know that $dx_j(\mathbf{p})/dp_i = -dV_j(\mathbf{p})/dp_i$. Replacing the optimal investments into equation (3) for firm j and taking the total derivative with respect to p_i we obtain

$$r\frac{dV_j}{dp_i} = \frac{d\pi_j}{dp_i} + \frac{dx_j}{dp_i}(L - V_j) - x_j\frac{dV_j}{dp_i} + \frac{dx_i}{dp_i}(F - V_j) - x_i\frac{dV_j}{dp_i} - \kappa'(x_j)\frac{dx_j}{dp_i}.$$

Using equation (4) (i.e., $\kappa'(x_j) = x_j = (L - V_j)$) and the envelop theorem (i.e., $dx_i(\mathbf{p})/dp_i = -dV_i(\mathbf{p})/dp_i = 0$) we obtain

$$r\frac{dV_j}{dp_i} = \frac{d\pi_j}{dp_i} - x_j\frac{dV_j}{dp_i} - x_j\frac{dV_j}{dp_i} \qquad \Rightarrow \qquad \frac{dV_j}{dp_i} = \frac{d\pi_j/dp_i}{r + x_j + x_i}.$$

Replacing back into equation (5) and rearranging we obtain equation (6). Finally, to see that $R_i(\mathbf{p})$ can be interpreted as $R_i(\mathbf{p}) = dV_i/dV_j$, replace the optimal investments into equation (3) for firm i and take the total derivative with respect V_j to obtain

$$r\frac{dV_i}{dV_j} = \frac{dx_i}{dV_j}(L - V_i) - x_i\frac{dV_i}{dV_j} + \frac{dx_j}{dV_j}(F - V_i) - x_j\frac{dV_i}{dV_j} - \kappa'(x_i)\frac{dx_i}{dV_j}.$$

Using equation (4), $\kappa'(x_i) = (L - V_i)$, and $dx_j(\mathbf{p})/dV_j(\mathbf{p}) = -dV_j/dV_j = -1$, we obtain

$$r\frac{dV_i}{dV_j} = -x_i\frac{dV_i}{dV_j} + (V_i - F) - x_j\frac{dV_i}{dV_j} \qquad \Rightarrow \qquad \frac{dV_i}{dV_j} = \frac{V_i - F}{r + x_j + x_i} > 0$$

and the interpretation given in equation (7) follows. ∎

Proof of Proposition 2. Consider first the scenario without price announcements. Using the first order condition for price in (2) we can construct the best response function of firm i to any price p chosen by firm j, call it $b_i^s(p)$. Because of symmetry, both firms have the same best response function in the this scenario. We show that the best response function with price announcement, $b_i^a(p)$, satisfies $b_i^a(p) > b_i^s(p)$ for all p. Therefore, if the best responses with price announcement cross each other, they must do so at a higher price than p_s.

Start by observing that the first-order condition (6) evaluated at b^s satisfies

$$\frac{d\pi_i(b^s(p_j), p_j)}{dp_i} + R(b^s(p_j), p_j)\frac{d\pi_j(b^s(p_j), p_j)}{dp_i} = R(b^s(p_j), p_j)\frac{d\pi_j(b^s(p_j), p_j)}{dp_i} > 0,$$

where the equality holds because $d\pi_i(b^s(p_j), p_j)/dp_i = 0$ by definition of a best response function. This implies that equation (6) is greater than zero at every price that is a best response b_i^s to p_j. Given that, for every vector of market prices $\mathbf{p}$, $R_i(\mathbf{p})d\pi_j(\mathbf{p})/dp_i > 0$ and that $d^2\pi_i(\mathbf{p})/dp_i^2 < 0$ due to the strict concavity of the profit function, if equation (6) holds, it must be at a $p_i > b^s(p_j)$. That is, $b^a(p_j) > b^s(p_j)$. ∎

Proof of Proposition 3. We start by showing that p_{full} exists. Then we show that it is an equilibrium. Observe that the function $f(p) \equiv \pi(p,p)$ is continuous and concave. Since $f(p_s) = \pi_s$, $f(p_m) = \pi_m$, and innovations being incremental (i.e., $\pi_l < \pi_m$) by the intermediate value theorem there exists at most two prices satisfying $f(p) = \pi_l$; one above and one below p_m.[24] We show that the lowest price is part of an equilibrium. First, observe that if p_{full} is announced by both firms, then in the R&D stage we have that $V_{high} = L$ and $x_{high} = 0$ (see the *high* candidate for equilibrium in the proof of Proposition 1).

We now show that p_{full} is an equilibrium by showing that there is no profitable deviation. Firm i will never deviate to a price $p_i > p_{full}$, as this price cannot deter firm j further ($x_j(\mathbf{p}_{full}) = 0$) and it decreases firm i's profit flow.[25] To show that there is no profitable deviation to a price $p_i < p_{full}$ we start by observing that, in the price-announcing stage, if both firms announce p_{full}, the first-order condition is not satisfied. In particular, the derivative (6) is positive as[26]

$$\frac{d\pi_i(\mathbf{p}_{full})}{dp_i} + \frac{L-F}{r}\frac{d\pi_j(\mathbf{p}_{full})}{dp_i} > \frac{d\pi_i(\mathbf{p}_{full})}{dp_i} + \frac{d\pi_j(\mathbf{p}_{full})}{dp_i} > 0,$$

where in the first expression we used that $R_i(\mathbf{p}_{full}) = (L-F)/r$. The first inequality follows from replacing $(L-F)/r > 1$ by one. The second inequality follows from $p_{full} \in (p_s, p_m)$, the global concavity of the monopolist problem, and observing that the second expression is the derivative of the monopolist profit function with respect to p_i.

The positive derivative at the vector of prices $\mathbf{p}_{full}$ implies that a unilateral decrease in p_i decreases V_i locally. We show that for every price $p_i < p_{full}$ the derivative is positive—hence, there can never be a price $p^* < p_{full}$ such that $V_i(p^*, p_{full}) > L$. By contradiction, suppose that there exist a range of prices such that $V_i(p_i, p_{full})$ is decreasing in p_i. Because for prices lower than p_{full}, V_i is a continuous function of p_i, there must exist a local minimum. That is, there must exist a price $\underline{p}$ such that $dV(\underline{p}, p_{full})/dp_i = 0$. We show that $\underline{p}$ cannot exist. Start by observing that $x_i(\underline{p}, p_{full}) = L - V_i$ must be positive, as the function $V_i(p_i, p_{full})$ was increasing in p_i in the interval $(\underline{p}, p_{full})$. Similarly, $x_j(p_{full}, p_i)$ is positive as the value of V_j is lower than L, as firm j is facing more price competition (i.e., $p_i < p_{full}$) and greater R&D competition (i.e., $x_i > 0$) compared to a situation in which $p_i = p_{full}$. Consider $p_i = \underline{p}$ and observe that an infinitesimal decrease in p_i does not change x_i as, by construction of $\underline{p}$, $dV_i(\underline{p}, p_{full})/dp_i = 0$. Firm j, however, experiences a decrease in V_j as it faces more product market competition (it faces a lower price by a competitor). In Lemma 3 we showed that firms values move in the same direction, that is $R_i(\mathbf{p}) = dV_i/dV_j > 0$ for every $\mathbf{p}$. This is a contradiction, as we have an decrease in V_j that does not lead to a decrease in V_i and thus no such $\underline{p}$ can exist. ∎

[24] When $\pi_l = \pi_m$, there only is one price satisfying $f(p) = \pi_l$, which is given by $p = p_m$. The proof that this is an equilibrium price follows analogous steps as those in the $\pi_l < \pi_m$ scenario.

[25] Formally, there is a jump in $dV_i(p_i, p_{full})/dp_i$ at $p_i = p_{full}$. Because $x_j(p_i, p_{full}) = 0$ for $p_i \geq p_{full}$, the derivative (6) reduces to $d\pi_i(\mathbf{p}_{full})/dp_i$. This derivative is negative, as $p_{full} > p_s$ and the problem of the firm being strictly concave.

[26] Observe that (5), and consequently (6), holds even when the announced prices $\mathbf{p}$ lead R&D investments into a corner solution. In particular, when $L = V_i(\mathbf{p})$ the envelop theorem still holds. When $L < V_i(\mathbf{p})$, investments are zero in a neighborhood of p_i, which implies $dx_i^*/dp_i = 0$ guaranteeing that the second term in (5) is zero.

Proof of Lemma 4. *i)* See discussion about feasibility of *high* and *low* solutions in the proof of Proposition 1. *ii)* It follows from replacing the solutions V_{high} and V_{low} (see equation (13) in the proof of Proposition 1) into equation (7) and verifying the inequalities. ■

Proof of Proposition 4. The existence of the function $V_{low}(\mathbf{p})$ follows from the proof of Proposition 1. The existence of $p_{low} \in (p_s, p_m)$ follows from Lemma 4 and the application of the intermediate value theorem, as discussed in the text. We prove that p_{low} is indeed an equilibrium by showing that the second-order condition holds at every V_{low} solution in which the price vector satisfies the first-order condition (hence, the equilibrium is unique). Then, we show that $\pi(\mathbf{p}_{low}) > \pi_s$ which, by Lemma 4, immediately implies $V_{low} > V_{na}$ and $x_{low} < x_{na}$.

Let $\mathbf{p}^*$ be any symmetric price vector satisfying first-order condition (6). By Proposition 2, $p^* > p_s$. Observe that, by construction, $dV_i(\mathbf{p}^*)/dp_i = 0$ and $-d\pi_i(\mathbf{p}^*)/dp_i = R_i(\mathbf{p}^*)d\pi_j(\mathbf{p}^*)/dp_i$. Differentiating (6) we obtain:

$$\frac{d^2V_i(\mathbf{p}^*)}{dp_i^2} = \frac{d^2\pi_i(\mathbf{p}^*)}{dp_i^2} + R_i(\mathbf{p}^*)\frac{d^2\pi_j(\mathbf{p}^*)}{dp_i^2} + \frac{1}{(r+2x^*(\mathbf{p}^*))^2}\left(-\frac{d\pi_i(\mathbf{p}^*)}{dp_i}\right)\frac{d\pi_j(\mathbf{p}^*)}{dp_i}. \quad (15)$$

We show that $\Psi(p) < 0$ for every $p \in (p_s, p_m)$ implies that at every price where equation (6) is satisfied, an upper-bound of equation (15) is negative.

We know that $d^2\pi_i(\mathbf{p}^*)/dp_i^2 < 0$ because the profit function is concave. Bound the second term in equation (15) by $\max\{0, d^2\pi_j(\mathbf{p}^*)/dp_i^2\}$. This works because if $d^2\pi_j(\mathbf{p}^*)/dp_i^2 > 0$ take $R_i(\mathbf{p}^*) = 1$, if $d^2\pi_j/dp_i^2 < 0$ take $R_i(\mathbf{p}^*) = 0$. Before bounding the third (and last) term, we show that it is positive. First, by definition, $r + 2x^* > 0$. $d\pi_i(\mathbf{p}^*)/dp_i < 0$ because $p^* > p_s$ and demand regularity condition (D). Finally, $d\pi_j(\mathbf{p}^*)/dp_i > 0$ because $dq_j/dp_i > 0$.

The third term in equation (15) is still a function of an endogenous object, V (as $x^* = L - V$). It is, however, increasing in V. To find an upper bound notice that V_{low} in equation (13) is increasing in π. Also, $\overline{\pi}$ is the maximum feasible value of π. Then, at that value, $V_{low}(\overline{\pi}) = (2L + F + r)/3$. Using this value of V to bound the third term we obtain that $\Psi(\mathbf{p}^*)$ is an upper-bound of (15). Thus, $d^2V_i(\mathbf{p}^*)/dp_i^2 < \Psi(\mathbf{p}^*) < 0$, where the second inequality follows from the assumption that $\Psi(p) < 0$ for all $p \in (p_s, p_m)$. $d^2V_i(\mathbf{p}^*)/dp_i^2 < 0$ implies that p^* is a local maximum, and because this is true for every p satisfying (6), there is no other local minimum satisfying the first-order condition. Hence, the equilibrium is unique; i.e., $p^* = p_{low}$ is the only equilibrium.

Proof of $\pi(\mathbf{p}_{low}) > \pi_s$: The solution to the problem of a firm that controls the price of all products in the case of symmetric demand functions (i.e., $\max_p \pi(p, p)$) is given by the multiproduct monopoly price, p_m. This implies that, given concavity of the monopoly problem, $\pi(p, p)$ is increasing in p until the price reaches the monopoly price, p_m. Because $p_s < p_{low} < p_m$, the result follows. ■

Proof of Proposition 5. The necessity of $L - F > r$ follows from the proof of Proposition 1. High-deterrence equilibria are shown in Table 1. For $p_{high} > p_m$ (and therefore $p_{high} > p_{low}$) see the discussion in the text. By Lemma 1, we know V_{high} (if it exists) is larger than V_{low}. This, in conjunction with equation (4), implies $x_{high} < x_{low}$. Finally, to check that a price satisfying equation (6) is indeed a high-deterrence equilibrium, we need to check the second-order condition (15). We check this condition numerically in the examples in the table. ■

Proof of Proposition 6. See argument following equation (10) in the main text. ■

Proof of Proposition 7. See argument in the text. ■

Proof of Proposition 8. See argument in the text. ■

B Demand Regularity Conditions

We assume that firm i's maximization problem is strictly concave; that is, $d^2\pi_i(\mathbf{p})/dp_i^2 < 0$ for any price vector $\mathbf{p}$. In addition, for every price vector such that the first-order condition $q_i + dq_i/dp_i\ (p_i - c) = 0$ is satisfied, we assume

$$-\frac{dq_i}{dp_i} > \frac{d^2q_i}{dp_i^2}(p_i - c), \quad \text{(A)} \qquad \frac{dq_i}{dp_j} > -\frac{d^2q_i}{dp_j dp_i}(p_i - c), \quad \text{(B)} \qquad -\frac{dq_i}{dp_i} > \frac{dq_i}{dp_j}, \quad \text{(C)}$$

$$-\frac{dq_i}{dp_i} > \frac{dq_i}{dp_j} + \left(\frac{d^2q_i}{dp_i^2} + \frac{d^2q_i}{dp_j dp_i}\right)(p_i - c). \qquad \text{(D)}$$

(A) guarantees that $dp_i/dc_i < 1$. (B) and (D) are required for j to not over react to i's price change; i.e., $dp_j/dp_i \in (0, 1)$. Finally, (C) requires that own price effects are larger than those of the opponent.

To illustrate the conditions for when the multiproduct monopolist's problem has a unique solution, define the function: $F(p_1, p_2) \equiv \pi_1(p_1, p_2) + \pi_2(p_1, p_2)$. Let F_i represent the derivative of F with respect to p_i. We assume that F is globally concave, that is guaranteed $F_{i,i} < 0$ and $F_{1,1}F_{2,2} - (F_1F_2)^2 > 0$ at any price vector. This guarantees a unique global maximum and a unique maximum when the price of a good is exogenously fixed.

C Discounted Expected Consumer Surplus

The discounted expected consumer surplus at time t, before the innovation has arrived, and given a vector of price announcements, $\mathbf{p}$, is given by

$$CS(\mathbf{p}) = \int_t^{\infty} e^{-(r+\lambda)(s-t)} \left(cs(\mathbf{p}) + \lambda cs(p_l, p_f)/r\right) ds = \frac{rcs(\mathbf{p}) + \lambda cs(p_l, p_f)}{r(r + \lambda)},$$

where $cs(\mathbf{p})$ is the consumer surplus flow at prices $\mathbf{p}$, $cs(p_l, p_f)/r$ is the discounted consumer surplus after the innovation arrives, λ is the pace of innovation, and r is the discount rate. The interpretation of $CS(\mathbf{p})$ is similar to that of equation (1).

Online Appendix

Announcing High Prices to Deter Innovation

by Guillermo Marshall and Álvaro Parra

Supplemental Material – Not for Publication

In this appendix, we extend the model to show that the R&D deterrence effect of price announcements exists more generally. We start by generalizing our quadratic cost assumption. In particular, we assume that the flow cost $\kappa(x_i)$ is strictly increasing, strictly convex (i.e., $\kappa''(x) > 0$ for all $x \geq 0$), and satisfies $\kappa(0) = \kappa'(0) = 0$, $\kappa''(0) < 1$, and $\kappa'''(x) \geq 0$ for all $x \geq 0$. We, then, show that our results extend to scenarios with n symmetric players and the duopoly case with asymmetric firms. We also consider the case of sequential innovations and the case of quality-enhancing innovations (as opposed to the process innovations in our baseline model). Lastly, we discuss the case where firms face demand shocks that incentivize firms to revise their price announcements.

D Duopoly with General R&D costs

D.1 Preliminaries

We will analyze the symmetric equilibrium of the game. Start by assuming that the profit flow is symmetric across firms and, for the moment, exogenously given by $\pi \in [\pi_f, \pi_l]$. Using the principle of optimality, and conditioning on the opponents' strategy x_j, we can write the R&D problem above as

$$rV_i = \max_{x_i \geq 0} \{\pi + x_i(L - V_i) + x_j(F - V_i) - \kappa(x_i)\}. \tag{16}$$

Differentiating the right hand side of equation (16) gives us an implicit expression for firm i's R&D investment,

$$\kappa'(x_i) = L - V_i. \tag{17}$$

As before, the investment rule equates the marginal cost of increasing the innovation rate, x_i, with the incremental value of an innovation, $L - V_i$. Due to the strict convexity of κ, equation (17) has a unique solution and can be inverted and written as $x_i = f(L - V_i)$, where f is a strictly increasing and concave function.

Lemma 5. *The function $f(z)$ implicitly defined by $\kappa'(f(z)) = z$ satisfies $f(0) = 0$ and is increasing ($f'(z) > 0$ for all $z \geq 0$). When $\kappa'''(x) > 0$, $f(z)$ is concave $f''(z) < 0$ for all $z > 0$, and when $\kappa'''(x) = 0$, $f''(z) = 0$ for all $z > 0$.*

Proof. The first statement follows from $\kappa'(0) = 0$. The second follows from the derivative of $f(z)$ being equal to $f'(z) = 1/\kappa''(f(z))$ and the fact that $\kappa(x)$ is strictly convex (i.e., $\kappa''(x) \geq 0$ for all $x \geq 0$). Similarly, we have that $f''(z) = -\kappa'''(f(z))/(\kappa''(f(z))^3)$ and the results follows from the value of $\kappa'''(x)$. ■

Replacing the equilibrium investment decisions of both firms back into equation (16), and restricting attention to symmetric values among firms ($V_i = V_j = V$), we define the continuous function

$$\phi(V, \pi) \equiv \pi + f(L - V)(L + F - 2V) - \kappa(f(L - V)) - rV. \tag{18}$$

The function $\phi(V, \pi)$ will help us characterize how prices relate to the equilibrium value of V. Start by observing that, given a vector of (symmetric) prices $\mathbf{p}$, both firms earn the profit flow $\pi = \pi_i(\mathbf{p})$. For each π we can search for a candidate solution to the value function by looking at values of V satisfying $\phi(V, \pi) = 0$. Because deterrence is costly and firms cannot be deterred beyond the point where they choose not to invest in R&D (i.e., $x_j \geq 0$), no equilibrium with $V > L$ exists. Also, it can be easily shown that values of $V < F$ are never candidate solutions for economically relevant values of π. For these reasons we restrict the analysis of $\phi(V, \pi)$ to values of $V \in [F, L]$. Also, for ease in exposition, we assume that $L - F \geq r$. As in the main text, $L - F \geq r$ is necessary for the existence of a high deterrence equilibrium and sufficient for the existence of a full deterrence equilibrium.

Lemma 6 (Shape of ϕ)**.** *(i) $\phi(F, \pi) > 0$ for any $\pi \geq \pi_f$. (ii) $\phi(L, \pi_l) = 0$ and $\phi(L, \pi) < 0$ for any $\pi < \pi_l$. (iii) For any given value of π, $\phi'(F, \pi) < 0$, $\phi'(L, \pi) > 0$, and $\phi''(V, \pi) > 0$ for all $V \in (F, L)$.*[27] *(iv) There exists a unique value $\underline{V} \in (F, L)$, independent of π, where $\phi(V, \pi)$ is minimized.*

Proof. (i) and (ii): Observe that $\phi(F, \pi) = \pi - \pi_f + f(L - F)(L - F) - \kappa(f(L - F))$. Due to the convexity of κ and $\kappa(0) = 0$, we have that $x\kappa'(x) > \kappa(x)$ for all $x > 0$. Equation (17) implies $\kappa'(f(L - F)) = (L - F)$. Replacing back, we obtain $\phi(F, \pi) = \pi - \pi_f + x\kappa'(x) - \kappa(x)$ with $x = f(L - F)$. Thus, $\pi \geq \pi_f$ is sufficient for

[27] The prime denotes derivatives with respect to the V dimension.

$\phi(F,\pi) > 0$. Similarly, $\phi(L,\pi) = \pi - \pi_l$. Thus $\phi(L,\pi) < 0$ whenever $\pi < \pi_l$ and $\phi(L,\pi) = 0$ when $\pi = \pi_l$.

(iii) Differentiating ϕ with respect to V we obtain: $\phi'(V,\pi) = -r - 2f(L-V) + f'(L-V)(V-F)$ and $\phi''(V,\pi) = 3f'(L-V) - f''(L-V)(V-F)$. Start by observing that this derivatives are independent of π. Also, $\phi''(V,\pi) > 0$ for $V \in (F,L)$ as $f'(x) > 0$ and $f''(x) \leq 0$ (see Lemma 5). Using $\phi'(V,\pi)$ we have that $\phi'(F,\pi) = -r - 2f(L-F) < 0$. Similarly, $\lim_{V\to L}\phi'(V,\pi) = -r + (L-F)\lim_{V\to L} f'(L-V)$. Here we have two cases (see Lemma 5), if $\kappa''(0) = 0$ then $\lim_{V\to L} f'(L-V) = \infty$ and $\phi'(L,\pi) > 0$. If $\kappa''(0) \in (0,1]$, then $\phi'(L,\pi) = -r + \alpha(L-F)$ where $\alpha > 1$ and $\phi'(L,\pi) > 0$ by the assumption that $L - F \geq r$.

(iv) Since $\phi'(F,\pi) < 0$ and $\phi'(L,\pi) > 0$, the intermediate value theorem implies that there exists a $\underline{V} \in (F,L)$ such that $\phi'(\underline{V},\pi) = 0$. Because $\phi'' > 0$, $\underline{V}$ is unique and corresponds to a minimum, which is independent of π, as $\phi'(V,\pi)$ is independent of π. ■

The previous lemma characterizes ϕ as a function of V. It tells us that for any given value of π, $\phi(V,\pi)$ is U-shaped with $\phi'(V,\pi)$ being monotonically increasing (see Figure 4). Due to the linearity of ϕ in π, a change in π only results in a vertical shift of ϕ. These two facts imply that ϕ is uniquely minimized at $\underline{V}$, which is independent of π. Define $\bar{\pi}$ to be the unique value of π satisfying $\phi(\underline{V},\bar{\pi}) = 0$. Observe that, by construction, $\bar{\pi} > \pi_l$.

Lemma 7 (Equilibrium Candidates)**.** *For $\pi \in [\pi_f,\pi_l) \cup \{\bar{\pi}\}$ the equation $\phi(V,\pi) = 0$ has a unique solution. For $\pi \in [\pi_l,\bar{\pi})$, $\phi(V,\pi) = 0$ has two solutions. For $\pi > \bar{\pi}$, $\phi(V,\pi) = 0$ has no solution.*

Proof. Since $\phi(\underline{V},\bar{\pi}) = 0$ and $\phi''(V,\pi) > 0$, there is a unique solution when $\pi = \bar{\pi}$. Observe that $\phi(\underline{V},\pi) < 0$ for $\pi < \bar{\pi}$ and, by Lemma 6, that $\phi(F,\pi) > 0$ for $\pi \geq \pi_f$. Consequently, for any $\pi \in [\pi_f,\bar{\pi})$, the intermediate value theorem guarantees that there exists a solution to $\phi(V,\pi) = 0$, call it V_{low}, such that $V_{low} \in (F,\underline{V})$. Because ϕ is monotone (decreasing) in V for $V \in [F,\underline{V})$, the V_{low} solution is unique. Similarly, because ϕ is monotonically increasing in V for $V \in (\underline{V},L]$, there is no solution to $\phi(V,\pi) = 0$ larger than $\underline{V}$ when $\phi(L,\pi) < 0$ (i.e., $\pi < \pi_l$) and a unique solution when $\phi(L,\pi) \geq 0$ (i.e., $\pi \geq \pi_l$), call it V_{high}. ■

The intuition of Lemma 7 can be seen graphically in Figure 4. Start by analyzing $\phi(V,\pi_l)$. Since $\phi(L,\pi_l) = 0$ and $\phi'(L,\pi_l) > 0$ we know that ϕ approaches

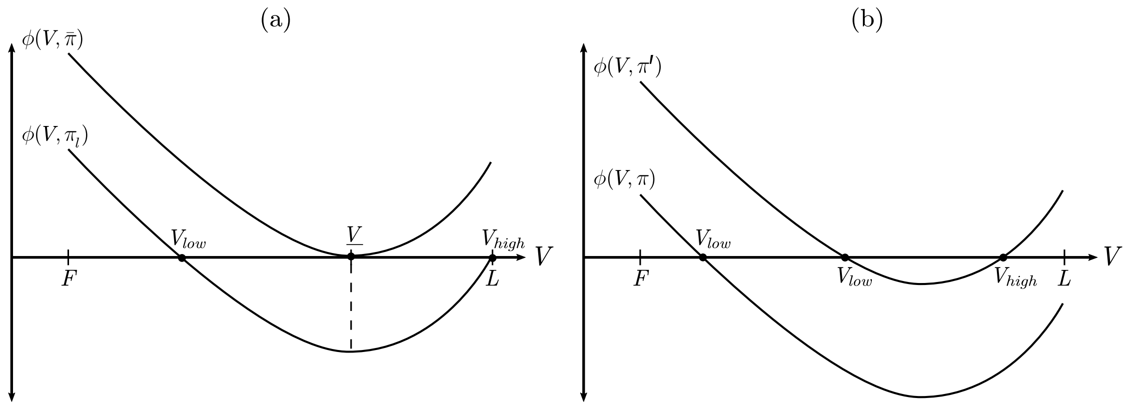

Figure 4: Equilibrium Multiplicity.

Note: (i) In panel (b) $\pi \in (\pi_f, \pi_l)$ and $\pi' \in (\pi_l, \bar{\pi})$. (ii) Both panels depicted under the $L_F \geq r$ assumption. If $L_F < r$, ϕ might be u-shaped (as depicted above) or decreasing in V.

zero at L from below. Also, since $\phi(F, \pi_l) > 0$ we know that $\phi(V, \pi_l)$ must cross zero at a value of $V \in (F, L)$. This value is represented by V_{low} in Figure 4 (Panel A). Moreover, since a lower π implies a downward parallel shift of ϕ, and because $\phi(F, \pi) > 0$ for any $\pi \geq \pi_f$, a solution to the left of $\underline{V}$ always exists for this profit range but no solution to the right of $\underline{V}$ exists (see V_{low} for π and π' in Figure 4 (Panel B)). Similarly, when $\pi \in [\pi_l, \bar{\pi})$ the solution V_{low} still exists, but now a new solution to the right of $\underline{V}$, denoted by V_{high}, also exists (see $\phi(V, \pi')$ in Figure 4 (Panel B)). When $\pi = \bar{\pi}$, $\underline{V}$ is the only solution. Finally, when $\pi > \bar{\pi}$, no solution exists.

Since ϕ may never cross the horizontal axis for a sufficiently high profit level, we henceforth assume that $\pi_m \leq \bar{\pi}$ in order to guarantee the existence of $V_i(\mathbf{p})$. To see why this is sufficient, observe that π_m is the maximum profit flow that can be attained in a symmetric equilibrium. Therefore $\pi_m \leq \bar{\pi}$ guarantees that $V_i(\mathbf{p})$ is well defined for any relevant vector of symmetric prices.[28]

The two solutions of ϕ have different qualitative features. For instance, while V_{low} is increasing in π, the solution V_{high} is decreasing. By implicitly differentiating $\phi(V, \pi) = 0$ with respect to V_j, we can define the ratio

$$R(\mathbf{p}) \equiv \frac{dV_i}{dV_j} = \frac{f'(L - V(\mathbf{p}))(V(\mathbf{p}) - F)}{r + 2f(L - V(\mathbf{p})))} > 0. \tag{19}$$

$R(\mathbf{p})$ captures how a solution $V_i(\mathbf{p})$ changes with an increase in the opponents'

[28]Note, however, that $\pi_m \leq \bar{\pi}$ is a sufficient condition. It is possible to find model parameters for which the unique equilibrium price vector under price announcements $\mathbf{p}^*$ satisfies $\pi(\mathbf{p}^*) < \bar{\pi}$—so that $V(\mathbf{p}^*)$ is well defined—but $\bar{\pi} < \pi_m$.

value. As we can observe, firms' values are complements, since an increase in firm j's value increases firm i's value by $R(\mathbf{p})$. $R(\mathbf{p})$ will play an important role when we characterize the equilibrium of the pricing game, as it measures how increasing firm j's value benefits firm i (i.e., the payoff pass-through of increasing the rival's value).

Lemma 8 (Properties of High and Low Equilibria)**.** *The equilibrium value in every solution V_{low} is lower than the value in any solution V_{high}. In any solution V_{low}, $R(\mathbf{p}) \in (0,1)$. In any solution V_{high}, $R(\mathbf{p}) > 1$. $V_{low}(\pi)$ is increasing in π, while $V_{high}(\pi)$ is decreasing.*

Proof. From the proof of Lemma 7 we know that $V_{low} < \underline{V} < V_{high}$, which proves the first claim. For the second claim simply observe that $\phi'(V,\pi) < 0$ (or $\phi'(V,\pi) = 0$ or $\phi'(V,\pi) > 0$) is equivalent to $R(\mathbf{p}) < 1$ (or $R(\mathbf{p}) = 1$ or $R(\mathbf{p}) > 1$, respectively) after manipulating the inequality and using the definition of $R(\mathbf{p})$. Since $R(\mathbf{p}) > 0$, $\phi'(\underline{V},\pi) = 0$ and $\phi''(V,\pi) > 0$, it follows that $R(\mathbf{p}) \in (0,1)$ at V_{low} and $R(\mathbf{p}) > 1$ at V_{high}. For the last statement, at any solution $V^a \in \{V_{low}, V_{high}\}$ of $\phi(V,\pi)$, observe that $dV^a/d\pi = -\phi'(V^a,\pi)^{-1}$. The result follows. ■

Because the equilibrium value in every solution V_{low} is lower than the value in any solution V_{high}, and because larger V's are associated with lower R&D investments levels (see equation (4)), we refer to a V_{low} solution as part of a *low deterrence* equilibrium. Similarly, we refer to V_{high} solutions as part of a *high deterrence* equilibrium. Recall from Lemma 7 that the solution V_{high} only exists when $\pi \geq \pi_l$. Since in equilibrium $\pi(\mathbf{p}) \leq \pi_m$—i.e., only profit flows less than the monopoly profits are attainable—we know that V_{high} solutions are not feasible when the innovation is radical ($\pi_l > \pi_m$). That is, a high deterrence equilibrium may only exist for incremental innovations ($\pi_l \leq \pi_m$).

Lemma 9 (Feasibility of Equilibria)**.** *Under radical innovations (i.e., $\pi_l > \pi_m$) only the V_{low} solution is a candidate for equilibrium. Under incremental innovations (i.e., $\pi_m \geq \pi_l$), both solutions are candidates.*

D.2 No Price Announcements

In the absence of price announcements, it is not hard to see that the value of participating in the patent race is still given by (1). The first-order conditions are now given by

$$\kappa'(x_i) = L - V_i, \qquad \frac{\partial \pi_i(p_i, p_j)}{\partial p_i} = 0. \tag{20}$$

As it can be seen, the only change with respect to the main text occurs in the first-order condition for R&D investments, which now incorporates the general cost structure. The statement about existence and uniqueness of an equilibrium with no announcements extends without modification.

Proposition 9 (Equilibrium with No Announcements)**.** *There is a unique symmetric Markov perfect equilibrium, (p_{na}, x_{na}, V_{na}), that solves equation (1) and conditions (20). In equilibrium, $p_{na} = p_s$, $x_{na} > 0$, and $V_{na} \in (F, L)$.*

Proof. The first-order condition for price in (20) and the demand regularity conditions imply that $\mathbf{p}_s = (p_s, p_s)$ is the unique equilibrium price vector and $\pi_s = \pi_i(\mathbf{p}_s)$ is the equilibrium profit earned by the firms. Using the condition (20) for R&D investments, we can construct $\phi(V, \pi_s)$ which, by Lemma 7 single crosses zero at a value $V^{na} \in (F, L)$ since $\pi_s \in (\pi_f, \pi_l)$ (see Lemma 1). Lastly, strict convexity of $\kappa(x)$ guarantees that x_{na} is the unique solution to equation (20) given V_{na} and L. ■

D.3 Price Announcements

We now consider the case of symmetric equilibria with public price announcements. The timing of the game remains unchanged. At the beginning of the race, firms make simultaneous public price announcements, and they (credibly) commit to these prices until the next innovation arrives. Upon observing the announced prices, firms then choose how much to invest in R&D in every period.

Solving by backward induction, we know that for a given pair of prices $\mathbf{p} = (p_i, p_j)$ firms choose investments strategies given by $x_i^*(\mathbf{p}) = L - V_i(\mathbf{p})$. Taking these strategies as given, in the price announcement stage firms solve $\max_{p_i} V_i(\mathbf{p})$, which deliver the first-order condition

$$\frac{d\pi_i(\mathbf{p})}{dp_i} + \underbrace{\frac{dx_j}{dp_i}(F - V_i(\mathbf{p}))}_{\text{R\&D Deterrence Effect}>0} = \frac{d\pi_i(\mathbf{p})}{dp_i} + R(\mathbf{p})\frac{d\pi_j(\mathbf{p})}{dp_i} = 0 \tag{21}$$

Where the equality can be shown using the analogous version to Lemma 3. These equations have the same interpretation as the ones given in the main text.

Proposition 10 (Full Deterrence Equilibrium)**.** *Assume $L - F \geq r$, when innovations are incremental (i.e., $\pi_l \leq \pi_m$), there exists a* full deterrence *equilibrium, in which $p_{full} = \min\{p : \pi(p,p) = \pi_l\}$, $x_{full} = 0$, and $V_{full} = L$. In this equilibrium, firms completely deter rivals from investing in R&D by announcing prices that are lower than the multiproduct monopoly price ($p_{full} \leq p_m$) but higher than the static oligopoly price ($p_{full} > p_s$).*

The intuition and proof of the result is identical to that of Proposition 3. The only difference in the proofs is that now $R(\mathbf{p}_{full}) = \alpha(L - F)/r$ with $\alpha = f'(0) \equiv \kappa(0)^{-1} \geq 1$ (see Lemma 5). This implies that $R(\mathbf{p}_{full}) > 1$, and consequently the existence of a full deterrence equilibrium, can be guaranteed by a weaker assumption than $(L - F)/r > 1$.

To establish the existence and uniqueness of a low deterrence equilibrium define the following function,

$$\Psi(p) = \frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i^2} + \max\left\{0, \frac{\partial^2 \pi_j(\mathbf{p})}{\partial p_i^2}\right\} - \Lambda \frac{\partial \pi_i(\mathbf{p})}{\partial p_i} \frac{\partial \pi_j(\mathbf{p})}{\partial p_i}, \tag{22}$$

where $\mathbf{p} = (p, p)$,

$$\Lambda = \frac{1}{K^2} \left(\frac{f'(L - \underline{\mathrm{V}})^2 - f''(L - \underline{\mathrm{V}})K}{f'(L - \underline{\mathrm{V}})} \right) > 0,$$

$K = r + 2f(L - \underline{\mathrm{V}}) > 0$ and $\underline{\mathrm{V}}$ is defined in Lemma 6. For the sufficiency proof we also assume that $\kappa(x) = x^\gamma/\gamma$ with $\gamma \geq 2$ (that is $f(z) = z^{\frac{1}{\gamma-1}}$). Observe that $\Psi(p)$ is a function of only fundamentals of the game. Assuming that $\Psi(p) < 0$ for all $p \in (p_s, p_m)$ is sufficient to guarantee that there exists a unique low deterrence equilibrium.

Proposition 11 (Low Deterrence Equilibrium)**.** *Assume $\Psi(p) < 0$ for all $p \in (p_s, p_m)$. There exists a unique* low deterrence *symmetric Markov perfect equilibrium, $(p_{low}, x_{low}, V_{low})$. In this equilibrium, firms deter their rivals' R&D ($x_{low} < x_{na}$) by announcing higher prices ($p_{low} \in (p_{na}, p_m)$) and they earn greater profits ($V_{low} > V_{na}$) relative to the case without price announcements.*

Proof. The existence of the function $V_{low}(\mathbf{p})$ follows from Lemmas 6 and 7. The existence of $p_{low} \in (p^s, p^m)$ follows from Lemma 4 and the application of the intermediate value theorem, by an analogous argument as the one given in the main text. For completeness and to better show the differences between a *high* and *low* deterrence equilibrium, we present a complete derivation of the first order conditions. Then, we prove that p_{low} is indeed an equilibrium by showing that the second-order condition holds at every vector of prices which the first-order condition holds (hence, the equilibrium is unique). Then, we show that $\pi(\mathbf{p}_{low}) > \pi^s$ which, by Lemma 4, immediately implies $V_{low} > V_{na}$ and $x_{low} < x_{na}$.

First-order condition: Using implicit differentiation, the derivative of V_i with respect to p_i is

$$\frac{dV_i}{dp_i} = \frac{\frac{d\pi_i}{dp_i} + f'\left(L - V_j\right)\frac{dV_j}{dp_i}\left(V_i - F\right)}{r + f\left(L - V_j\right) + f\left(L - V_i\right)},$$

where

$$\frac{dV_j}{dp_i} = \frac{\frac{d\pi_j}{dp_i} + f'\left(L - V_i\right)\frac{dV_i}{dp_i}\left(V_j - F\right)}{r + f\left(L - V_j\right) + f\left(L - V_i\right)}.$$

Observe that at the optimum $dV_i/dp_i = 0$. Using this condition in dV_j/dp_i and replacing back into dV_i/dp_i delivers equation (21). More generally (i.e., not necessarily at the optimum), we obtain

$$\frac{dV_i}{dp_i} = \frac{1}{K}\frac{\frac{d\pi_i}{dp_i} + R_i\frac{d\pi_j}{dp_i}}{\left(1 - R_iR_j\right)} \tag{23}$$

where $K = r + f(L - V_i) + f(L - V_j) > 0$ and $R_i = f'(L - V_j)(V_i - F)/K > 0$. The expression above captures how in a *high* deterrence equilibrium the derivative of V_i with respect to p_i has the opposite sign of the same derivative in a *low* deterrence equilibrium, since $R_i > 1$ for $i \in \{1, 2\}$ in any high deterrence equilibrium.

Second-order condition: Differentiating (23) with respect to p_i and using that in equilibrium $-d\pi_i/dp_i = R_i d\pi_j/dp_i$, we obtain:

$$\frac{d^2V_i}{dp_i^2} = \frac{\frac{d^2\pi_i}{dp_i^2} + R_i\frac{d^2\pi_j}{dp_i^2} - \frac{1}{K^2}\left(\frac{f'(L-V_j)^2 - f''(L-V_j)K}{f'(L-V_j)}\right)\frac{d\pi_i}{dp_i}\frac{d\pi_j}{dp_i}}{K\left(1 - R_iR_j\right)}. \tag{24}$$

The denominator is positive in a low deterrence equilibrium whereas it is negative in a high deterrence equilibrium (Lemma 4). We show that $\Psi(p) < 0$ for every $p \in (p_s, p_m)$ implies that at every price where (21) is satisfied, an upper-bound of (24) is negative.

Because the denominator is positive, we ignore it to determine the sign of d^2V_i/dp_i^2. We know that $d^2\pi_i/dp_i^2 < 0$ because of demand regularity condition (A), which guarantees uniqueness of the static oligopoly game. To bound the second term from above, we use $\max\{0, d^2\pi_j/dp_i^2\}$ (if $d^2\pi_j/dp_i^2 > 0$ take $R_i = 1$, if $d^2\pi_j/dp_i^2 < 0$ take $R_i = 0$). Before bounding the third (and last) term, we show that it is positive (i.e., the negative 1 multiplies a negative term). $d\pi_i/dp_i < 0$ because every price that satisfies (21) is greater than p_s, and we have that $d\pi_i(\mathbf{p}_s)/dp_i = 0$ and $d^2\pi_i/dp_i^2 < 0$. $d\pi_j/dp_i > 0$ because $dq_j/dp_i > 0$. Finally the term in parenthesis is positive by Lemma 5. We bound the term in parenthesis using the parametric specification of the cost function, $\kappa(x) = x^\gamma/\gamma$, which implies $f(z) = z^{1/(\gamma-1)}$, and

$$\frac{1}{K^2}\left(\frac{f'\left(L-V\right)^2 - f''\left(L-V\right)K}{f'\left(L-V\right)}\right) = \frac{(\gamma-1)^{-1}}{(r+2f(L-V))^2}\left(\frac{1}{(L-V)^{\frac{\gamma-2}{\gamma-1}}} + \frac{\gamma-2}{L-V}\right).$$

The expression above is increasing in V. Because every low deterrence equilibrium satisfies $V_{low} \leq \underline{\mathrm{V}}$, we use $\underline{\mathrm{V}}$ to bound the expression above. Thus, $d^2V_i/dp_i^2 < \Psi(p_{low})/(K(1-R_iR_j)) < 0$, where the second inequality follows from the assumption that $\Psi(p) < 0$ for all $p \in (p_s, p_m)$. $d^2V_i/dp_i^2 < 0$ implies that p_{low} is a local maximum, and because this is true for every p satisfying (6), there is no local minimum satisfying the first order condition. Hence, the equilibrium is unique.

Proof of $\pi(\mathbf{p}_{low}) > \pi_s$*:* The solution to the problem of a firm that controls the price of all products in the case of symmetric demand functions (i.e., $\max_p \pi(p,p)$) is given by the multiproduct monopoly price, p_m. This implies that $\pi(p,p)$ is increasing in p until the price reaches the monopoly price, p_m. Because $p_s < p_{low} < p_m$, the result follows. ■

As in the main text, we can construct examples of high-deterrence equilibria satisfying the assumptions of the model, but its existence cannot be guaranteed within the assumptions of the model.

E Other Extensions

In this section we present various extensions of the model to show the robustness of the results.

E.1 n Symmetric Competitors

Consider a industry served by n symmetric firms. As before, let π_l and π_f represent the profit flow obtained by the innovating firm and the non-successful followers after the innovation occurred. Likewise, let $L = \pi_l/r$ and $F = \pi_f/r$ represent the discounted values of being the technology leader and follower after the innovation arrives.

$V_i(\mathbf{p})$ represents the value of firm i at time t before any firm has successfully innovated as a function of the vector of Markov strategies $\mathbf{p}$,

$$rV_i(\mathbf{p}) = \max_{x_i} \; \pi_i(\mathbf{p}) + x_i(L - V_i(\mathbf{p})) + \sum_{j \neq i} x_j(F - V_i(\mathbf{p}))) - \kappa(x_i),$$

where $V_i(\mathbf{p})$ now captures that an invention by any of the $n-1$ rivals makes firm i become a follower. Firm i's first-order condition with respect to the R&D investment is given by equation (4).

Firm i chooses its price announcements by maximizing $V_i(\mathbf{p})$ given beliefs about its rivals' strategies. The first-order condition with respect to price is equivalent to

$$\frac{dV_i}{dp_i} = 0 \Leftrightarrow \frac{d\pi_i(\mathbf{p})}{dp_i} \underbrace{- \sum_{j \neq i} \frac{dx_j}{dp_i}(V_i(\mathbf{p}) - F)}_{\text{R\&D Deterrence Effect}} = \frac{d\pi_i(\mathbf{p})}{dp_i} + \underbrace{R(\mathbf{p}) \sum_{j \neq i} \frac{d\pi_j}{dp_i}}_{>0} = 0,$$

where

$$R(\mathbf{p}) = \frac{f'(L - V(\mathbf{p}))(V(\mathbf{p}) - F)}{r + nf(L - V(\mathbf{p}))) - (n-2)f'(L - V(\mathbf{p}))(V(\mathbf{p}) - F)} > 0.$$

As before, firm i takes into consideration how its price will impact its product market profit as well as the R&D investment of each of its rivals. It is possible to show that $R(\mathbf{p}) \in (0, 1)$ in any low deterrence equilibrium. Thus, the existence of a low deterrence equilibrium with prices satisfying $p_{low} \in (p_s, p_m)$ follows from analogous arguments to those presented in Proposition 4.

E.2 Asymmetric Firms

Consider an asymmetric duopoly where firm 1 is investing in R&D to increase its cost advantage (i.e., lower its marginal cost from βc to $\beta^2 c$), and firm 2 is investing to match firm 1's marginal cost (i.e., lower its marginal cost from c to βc). Once one of the two firms succeeds, the industry reaches maturity and the firms no longer invest in R&D. The firms' post-innovation values differ depending on which firm successfully innovates. If firm i innovates, the values are given by L_i and F_i, with $L_1 > L_2 = F_1 > F_2$.[29]

Let $V_i(\mathbf{p})$ represent the value of firm i at time t before any firm has successfully innovated as a function of the firms' Markov strategies,

$$rV_i(\mathbf{p}) = \max_{x_i} \ \pi_i(\mathbf{p}) + x_i(L_i - V_i(\mathbf{p})) + x_j(F_j - V_i(\mathbf{p})) - \kappa(x_i),$$

where $\pi_1(\mathbf{p}) = (p_1 - \beta c)q_1(\mathbf{p})$ and $\pi_2(\mathbf{p}) = (p_2 - c)q_2(\mathbf{p})$. Firm i's first-order condition with respect to the R&D investment is given by $\kappa'(x_i) = L_i - V_i(\mathbf{p})$.

The price announcements are chosen by maximizing $V_i(\mathbf{p})$ given beliefs about p_j, and must satisfy

$$\frac{dV_i}{dp_i} = 0 \Leftrightarrow \frac{d\pi_i(\mathbf{p})}{dp_i} \underbrace{- \frac{dx_j}{dp_i}(V_i(\mathbf{p}) - F_i)}_{\text{R\&D Deterrence Effect}} = \frac{d\pi_i(\mathbf{p})}{dp_i} + \underbrace{R_i(\mathbf{p})\frac{d\pi_j}{dp_i}}_{>0} = 0,$$

with

$$R_i(\mathbf{p}) = \frac{f'(L - V_j(\mathbf{p}))(V_i(\mathbf{p}) - F)}{r + f(L - V_i(\mathbf{p}))) + f(L - V_j(\mathbf{p})))} > 0.$$

Arguments analogous to those in Proposition 4 establish existence of a low deterrence equilibrium with the properties discussed in Section 4.

E.3 Sequential Innovations

We next use the analysis above to argue that the existence of the R&D deterrence effect extends to scenarios where firms compete to develop a sequence of new technologies. Consider the case where two symmetric firms compete to develop a sequence of two technologies.[30] Each new technology reduces a firm's marginal cost by a factor of β. Once the two technologies have been invented, the industry reaches maturity, and the firms no longer invest in R&D.

The analysis of the game after the first technology has been invented is equivalent to the analysis in Section E.2, where we studied the asymmetric case where one of the firms has a cost advantage of magnitude β. The analysis of the game before the first technology has been invented is equivalent to the symmetric analysis in Section 4 when replacing L and F for V_1 and V_2, respectively, and where V_1 and V_2 are the pre-innovation equilibrium values in Section E.2. Because the R&D deter-

[29] These inequalities follow from arguments analogous to those in Lemma 1.

[30] The argument provided here extends to the case of k innovations. We consider the case of two innovations for ease of exposition.

rence effect exists in both of these versions of the model, it follows that the R&D deterrence effect and its properties extend to the case of sequential innovations.

E.4 Quality-Enhancing Innovations

In the previous sections we focused on cost-saving innovations. Our results, however, also apply to markets in which firms compete in developing quality-enhancing innovations.

To illustrate this, let θ be a vector describing the (single-dimensional) quality of each product, and let $q_i(\mathbf{p}, \theta)$ be the demand of firm i given the price vector $\mathbf{p}$ and the vector of product qualities θ. In addition to the assumptions discussed in Section 2, we assume that products are *substitutes* in quality, that is, $q_i(\mathbf{p}, \theta)$ strictly increases in θ_i, but strictly decreases in θ_j. This assumption is quite general and includes a wide class of discrete-choice demand models (e.g., logit model).[31]

In this environment with substitutes goods, it is not hard to verify that an increase in the quality of product i benefits firm i and makes every rival of firm i worse off. Consequently, pre and post-innovation profits satisfy Lemma 1 ($\pi_f < \pi_s < \pi_l$). This post-innovation asymmetry between market leader and market follower generates the same incentives to deter R&D as in the case of cost-saving innovations. Because the analysis in Section 4 only relied on the profit ordering in Lemma 1, the innovation deterrence results extend to this environment.

E.5 Demand Shifters and Revised Price Announcements

In the baseline model, we assume that firms make simultaneous public price announcements at the beginning of the game, and they (credibly) commit to these prices until the next innovation arrives. In our baseline model, price announcements are not revised prior to the discovery of the next innovation because this period presents no changes to state variables that that would incentivize a firm to revise its price. In this extension, we allow for changes to state variables prior to the arrival of the next innovation (e.g., demand shocks), which lead to revised price announcements. This extension is empirically relevant—e.g., Amazon revised its price for the Amazon Fire smartphone because of disappointing sales—and illustrates that our results do not depend on a price commitment that lasts until the

[31] This assumption, however, does exclude cases where an innovation increases the desirability of every product in the market (e.g., improved industry standard).

next innovation arrives.[32] That is, the same results obtain if firms make a sequence of short-term price announcements over time.

To make this argument, we consider a version of our baseline model where the industry may face a permanent demand shock. We assume that the demand shock arrives at Poisson rate μ, and it may arrive before or after the arrival of an innovation. Because the demand shock impacts profits, firms have incentives to revise their price announcements when faced with the demand shock.

Let L and F represent the discounted values of being the technology leader and follower after the innovation arrives, and L^S and F^S the discounted values once the demand shock impacts the industry,

$$L = \frac{\pi_l + \mu L^S}{r+\mu}, \quad F = \frac{\pi_f + \mu F^S}{r+\mu}, \quad L^S = \frac{\pi_l^S}{r}, \quad F^S = \frac{\pi_f^S}{r},$$

where π_j and π_j^S ($j \in \{f, l\}$) are the profit flows with and without the demand shock.

Let $V_i(\mathbf{p})$ and $V_i^S(\mathbf{p})$ represent the value of firm i at time t before any firm has successfully innovated as a function of both the firms' Markov strategies and whether the demand shock has arrived,

$$\begin{aligned} V_i(\mathbf{p}) &= \max_{x_i} \frac{\pi_i(\mathbf{p}) + x_i L_i + x_j F_j + \mu V^S - \kappa(x_i)}{r + x_i + x_j + \mu}, \\ V_i^S(\mathbf{p}) &= \max_{x_i^S} \frac{\pi_i^S(\mathbf{p}) + x_i^S L_i^S + x_j^S F_j^S - \kappa(x_i^S)}{r + x_i^S + x_j^S}, \end{aligned}$$

where $V^S = \max_{p_i} V_i^S(\mathbf{p})$.

The price announcements are chosen by maximizing $V_i(\mathbf{p})$ and $V_i^S(\mathbf{p})$ given beliefs about p_j, and must satisfy

$$\frac{dV_i}{dp_i} = 0 \Leftrightarrow \frac{d\pi_i(\mathbf{p})}{dp_i} \underbrace{- \frac{dx_j}{dp_i}(V_i(\mathbf{p}) - F)}_{\text{R\&D Deterrence Effect}} = \frac{d\pi_i(\mathbf{p})}{dp_i} + \underbrace{R_i(\mathbf{p})\frac{d\pi_j}{dp_i}}_{>0} = 0,$$

[32] See, for instance, https://www.businessinsider.com/amazon-fire-phone-price-drop-2014-11.

and $dV_i^S/dp_i = 0 \Leftrightarrow$

$$\frac{d\pi_i^S(\mathbf{p})}{dp_i} \underbrace{- \frac{dx_j^S}{dp_i}(V_i^S(\mathbf{p}) - F^S)}_{\text{R\&D Deterrence Effect}} = \frac{d\pi_i^S(\mathbf{p})}{dp_i} + \underbrace{R_i^S(\mathbf{p})\frac{d\pi_j^S}{dp_i}}_{>0} = 0,$$

with

$$R(\mathbf{p}) = \frac{f'(L - V(\mathbf{p}))(V(\mathbf{p}) - F)}{r + 2f(L - V(\mathbf{p})) + \mu} > 0$$

and

$$R^S(\mathbf{p}) = \frac{f'(L^S - V^S(\mathbf{p}))(V^S(\mathbf{p}) - F^S)}{r + 2f(L^S - V^S(\mathbf{p}))} > 0.$$

The existence of a low deterrence equilibrium, with the properties discussed in Section 4, follows from arguments analogous to those in Proposition 4.