On the Interaction between Patent Screening and its Enforcement*

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Abstract

This paper explores the interplay between patent screening and patent enforcement. Costly enforcement involves type I and type II errors. When the patent office takes the rates at which such errors occur as given, granting some invalid patents is socially optimal even in the absence of screening costs because it encourages innovation. When the influence on courts' enforcement effort is considered, these same forces imply that screening and enforcement are complementary. This means that, contrary to common wisdom, better screening induces better enforcement but also that an increase in enforcement costs could be optimally accommodated with less rather than more ex-ante screening.

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1 Introduction

In recent decades, we have witnessed a surge in patenting activity. The large number of applications has put a strain on patent offices everywhere. There is a concern that this process might have led to a proliferation of likely invalid patents that could hinder future technological progress, particularly in areas where innovation is cumulative. Between 75% and 97% of the applications reviewed by the US Patent and Trademark Office (USPTO) end up in a patent being granted (Lemley and Sampat, 2008). To address this issue, many authors advocate to increase the resources of patent offices (c.f., Farrell and Merges, 2004).[1] Others, however, argue that the high approval rates are the consequence of *rational ignorance* (Lemley, 2001): Since only a tiny fraction of patents is ever litigated,[2] performing in-depth ex-post screening through litigation, rather than carefully analyzing ex-ante whether each patent is valid and relevant, can be more cost effective.[3]

In this article, we study the interplay between ex-ante screening by the patent office and ex-post enforcement by courts, and their impact on innovation and welfare. We show that when courts are imperfect—i.e., there is a positive chance to make an incorrect ruling—and these mistakes are regarded independent from the patent office's behavior, some rational ignorance by the patent office is socially optimal even in the *absence* of screening costs. That is, even when ex-ante screening could be perfect at no cost, it could be optimal to allow a percentage of invalid patents. On the other hand, when courts' mistakes depend on judges' endogenous effort (as in Daughety and Reinganum (2000)) ex-ante screening and ex-post enforcement are complementary. That is, an increase in screening by the patent office facilitates court decisions, leading to better rulings.

To study the interaction between patent screening and enforcement, we propose a tractable industry-dynamics model with sequential innovation and endogenous entry. The

[1] This view has been adopted in the Leahy-Smith America Invents Act of 2011 that increases funding and provides new screening mechanisms to better discern the quality of patent applications.

[2] Lemley (2001) estimates that 2% of patents are litigated and only about 0.2% of patents reach a courtroom.

[3] Frakes and Wasserman (2019) challenge the results of this cost-benefit analysis.

industry is made up of a continuum of business niches, each of which can be thought of as the market for a distinct product. Successful developers of improved versions of each product contribute to welfare and appropriate temporary monopoly profits as in a standard quality ladder model with limit pricing (Grossman and Helpman, 1991; Aghion and Howitt, 1992). These temporary monopolies are based on intellectual property (IP) protection and are threatened by the endogenous arrival of two kinds of entrants: developers of better versions of the product (that we denote as genuine innovators) and entrants that contribute minor improvements with little social value (that we denote as obvious innovators).[4]

In every period, entrants observe market conditions and invest in R&D until the quasi-rents from entry are dissipated. Entrants face uncertainty on whether their product will infringe existing IP rights and, as a result, they suffer from the "tragedy of anticommons" (Heller and Eisenberg, 1998). This assumption is consistent with Lemley (2008) who argues that, due to the large number of overlapping rights, firms decide to innovate first, ignoring related patents, and deal later with the lawsuits that ensue from existent patent holders. This strategy is also supported by the large proportion of patents brought to court that end up invalidated (Allison and Lemley, 1998).

Upon entry, firms may randomly produce a genuine or an obvious innovation. Entrants apply first for a patent and only learn the quality of their innovation through the commercialization of their products. Patent applications are presumed valid, as patent examiners have to find prior art and articulate an appropriate basis for rejection. This means that genuine innovators always receive a patent but the success of an obvious innovator depends negatively on the amount of resources that the patent office devotes to screening—the screening rate. This assumption is consistent with the findings in Frakes and Wasserman (2017), which shows that lack of resources by patent examiners (e.g.,

[4]We assume that obvious innovations do not fulfill the novelty requirements for patentability nor represent a sufficient innovative step to place them outside the breadth of existing patents. Genuine innovations satisfy both.

tighter time constraints) results in a bias towards an increase in the approval rate.[5]

After a patent has been obtained, an entrant may randomly reach a competitive niche or one monopolized by an incumbent. A genuine innovator monopolizes a competitive niche, while an obvious innovator keeps the niche competitive, as it introduces a product of similar quality to those in the market. If the entrant lands in a monopolized niche, the incumbent goes to court to preserve its rents by claiming that the entrant infringes on its patent. If the court rules in favor of the entrant both firms compete in the same niche, driving the incumbent's profits to zero, whereas the entrant makes profits according to its quality. If the court determines that the entrant has infringed the incumbent's patent, the innovation goes to waste.

The strength of patent protection is endogenous. Each case that arrives to court is decided by a different judge. Judges can err in their rulings. Consequently, patent protection is probabilistic, reflecting the uncertainty on the enforcement of existing patents (c.f., Lemley and Shapiro, 2005; Farrell and Shapiro, 2008).[6] For each case, a judge decides how much costly effort to devote to analyzing the evidence. Although both the patent office and judges choose their effort with the objective to maximize social welfare, their decisions are taken at different stages of the entry process. Whereas the patent office oversees every patent application, courts only evaluate the validity of a patent conditional on an entrant having reached a monopolized niche. As we explain below, this asymmetry creates a dynamic (in)consistency problem within the patent system.

A judge's objective function can be written as a weighted average of the welfare costs of committing type I and type II errors. A type I error arises when a judge rules against an entrant with a genuine innovation, depriving society from the benefit of that improvement. A type II error arises when an obvious innovator is allowed to compete with the existing

[5] The authors also show that the bias towards patenting created by shortening the allocated time for the reviewing process is more prominent in industries where technologies are complex and innovation is sequential, as in the framework discussed in this paper. See also Lei and Wright (2017).

[6] Other work in which courts make probabilistic rulings include Spier (1994), Daughety and Reinganum (1995), and Landeo et al. (2007). By adopting this probabilistic approach, we abstract away from the traditional patent length and patent breadth discussion (Scotchmer, 2004).

incumbent. Type II errors have non-trivial effects on entry and welfare. On the one hand, they shorten the expected duration of the incumbency of genuine innovators, discouraging entry. On the other hand, type II errors modify market structure: they turn previously monopolized niches competitive producing two kinds of benefits. First, the social costs of the existing monopoly are dissipated. Second, the entrants' prospect of facing opposition in a niche improve, encouraging entry. That is, allowing obvious innovators to compete mitigate the distortions associated to the tragedy of the anticommons. On the net, in our model type II errors always have a strictly positive effect on entry and welfare. This finding is consistent with Galasso and Schankerman (2015), who show that patent invalidation is positively correlated with future entry.

Despite the benefits of incurring in a type II error, the joint social costs of both type of errors are always positive. Better screening by the patent office increases the importance of the type I error relative to the type II error and fosters judge's effort. This complementarity between ex-ante screening and ex-post enforcement is further reinforced by the complementarity between the decisions of the current and future judges in a given niche. When the rulings of the judges that will oversee the same niche in the future become more accurate, genuine innovators are more likely to succeed regardless of the competitive state of the niche. As a result, the gains from altering market structure that result from the type II error are reduced, increasing the current cost from an inaccurate court ruling.

To conclude, we explore the optimal level of ex-ante patent screening taking into account the endogenous response of ex-post enforcement. The optimal level of patent screening balances off two important forces. On the one hand, there is the traditional substitution effect consistent with the idea of rational ignorance. A decrease in enforcement costs, which leads to more enforcement, should be accommodated with less ex-ante screening to decrease the overall costs of the patent system. On the other hand, and due to the complementarities described above, less ex-ante screening also induces worst enforcement. We show that both effects manifest in the optimal policy, but the complementarity effect prevails: relative to a situation in which ex-post enforcement is exogenous, optimal ex-ante screening is higher when the courts response is taken into account.

The article is organized as follows. Section 2 introduces the model which is stylized along several dimensions to preserve tractability. The implications of relaxing some of its simplifying assumptions are discussed in Section 5. Section 3 shows that, for a given time-invariant combination of screening intensity decided by the patent office and enforcement intensity decided by judges, the model displays a unique steady-state equilibrium. We provide the comparative statics of such equilibrium and characterize the socially optimal intensities of screening and enforcement in a frictionless world in which both could be costlessly set by a social planner.

In Section 4 we study the case where both screening and enforcement are costly and separately undertaken by the patent office and the corresponding judges, respectively. The model shows that both the screening rate of the patent office and the effort of future judges are complementary to a single judge's effort. These complementarities lead to more ex-ante screening in equilibrium. After discussing the robustness of the main results to relaxing some of the simplifying assumptions of the model in Section 5, Section 6 concludes the article. All proofs are in the Appendix.

Related Literature To our knowledge, this is the first article providing a formal model to understand how patent screening and enforcement interact, and the corresponding impact on innovation and market dynamics. We do, however, build upon several strands of literature.

Our model belongs to the category of sequential (or cumulative) innovation models. In that literature various dimensions of patent policy have been studied, such as: patentability requirements (Scotchmer and Green, 1990; O'Donoghue, 1998), patent breadth and length (O'Donoghue et al., 1998), forward protection (Denicolò, 2000; Denicolò and Zanchettin, 2002), or lack of protection (Bessen and Maskin, 2009). Other works within the sequential innovation framework study issues such as: antitrust (Segal and Whinston, 2007), optimal buyouts schemes (Hopenhayn et al., 2006), growth and industry dynamics (Denicolò and Zanchettin, 2014), and product-market competition (Marshall and Parra, 2019).

This article contributes to the literature on IP rights and entry. Gilbert and Newbery (1982) emphasizes that patents may be used preemptively to deter entry. In our model, the fraction of niches occupied by incumbents protected by patents affects the probability of success of subsequent entrants, acting as an entry barrier. Parra (2019) studies optimal patent design when market-structure is endogenously determined by (an exogenous) patent strength. We add to the analysis of IP protection on market structure by considering a setup in which patent strength is itself an endogenous object determined by subsequent entry as well as by the screening efforts of the patent office and the enforcement effort of the courts.

The literature on law and economics has long recognized that courts might be imperfect in their rulings (see Spier, 2007, for an extensive survey on litigation). When endogenizing the decisions of courts, most of the literature assumes that prosecutors are driven by social welfare maximization (Grossman and Katz, 1983; Reinganum, 1988), career concerns (Daughety and Reinganum, 2020) or a mixture of both (Daughety and Reinganum, 2016). Abstracting from other agents involved in the functioning of courts, we consider welfare-maximizing judges that can improve the quality of their rulings (reduce their errors) by exerting costly effort.

Finally, our paper contributes to the literature on the optimal screening behavior by the patent office. Schankerman and Schuett (2020) study patent screening and fees in a single-innovation framework where court rulings are perfect. By adding imperfect and endogenous enforcement, we are able to unveil the complementarity and substitution effects arising from the interaction between patent screening and court behavior.

2 The Baseline Model

2.1 The Market

We characterize the evolution of an industry in an infinite-horizon discrete-time model with discount factor $\beta < 1$. This industry is comprised of a continuum of business niches of measure one. Each niche can be interpreted as the market for a different product.[7] Each niche can be in one of the following two situations. A firm might be the sole producer of the good, with the monopolist earning a per period profit flow $\pi > 0$. Alternatively, the niche might operate under competition and, in that case, all firms earn 0 profits. We denote the proportion of monopolized niches as $x_t \in [0, 1]$.[8]

In every period t there is an endogenous measure $e_t \in [0, 1]$ of entrants extracted from a large population of potential firms that face an entry cost normalized to 1. Each entrant develops a new product (innovation) and applies for a patent. Each innovation can be of two types, genuine and obvious, with exogenous probabilities α and $1 - \alpha$, respectively. A *patent office* screens all applications and determines whether to grant a patent or not. Applications based on genuine innovations are always successful, whereas those based on obvious innovations succeed with probability λ. We interpret λ as a measure of the quality of the screening made by the patent office.[9] Only innovators that receive a patent can enter a niche. Potential entrants decide on entry without knowing their innovations' type, which they learn by competing in their niche. Entry is untargeted and uniformly distributed over the existing niches so that in every niche and period there will be an independent-across-niches probability e_t of having just one entrant and a probability $1 - e_t$ of having no entrant. The probability that an entrant lands in a monopolized niche is x_t, which is

[7] This simplification allows us to abstract from cross-product competition and to focus on competition related to concomitant and future entry into each niche.

[8] We interpret each niche as a quality ladder under price competition for a single unit of the good (see section 3.2). Under zero marginal cost of production, profits (and prices) are equal to the quality improvement brought to the market by the innovator.

[9] Because applications are presumed valid, our modeling can be interpreted as the result of a search for prior art; λ, thus, represents the probability that the patent office fails to find similar existing products when they exist and they deem the innovation obvious.

independent of the entrants' type.[10]

2.2 Litigation and Judges's Decisions

Incumbent firms in monopolized niches might lose their status due either to competition with an obvious innovator (i.e., a firm with a similar-quality product) or to the replacement by a genuine innovator (a firm with a superior substitute product). An incumbent can respond to entry by filing a patent-infringement lawsuit. For simplicity, we assume that litigation is costless but, whenever indifferent, the incumbent avoids it. This means that incumbents in already competitive niches (i.e., with no profits at stake) will not engage in litigation.

Each infringement claim is reviewed by a different judge that makes a probabilistic decision. If the judge rules in favor of the incumbent, entry is blocked, allowing the incumbent to preserve the monopoly status, and the entrant's innovation goes to waste.[11] If the judge rules in favor of the entrant, this firm replaces the existing incumbent and receives a profit flow according to the quality of its innovation.[12] A judge rules in favor of entrant with an obvious and genuine innovation with endogenous probabilities denoted by μ_0 and μ_1, respectively.

We assume that judges make evidence-based rulings. That is, they rule in favor of the incumbent if and only if they possess evidence that the entrant's innovation was obvious, so that it infringes the incumbent's patent. When a case reaches the court, the judge, who does not directly observe the quality of the entrant's patent, can exert effort

[10]This formulation simplifies exposition by focusing on the "anticommons" problem, captured by x_t, and abstracting away from competition between simultaneous innovators, which would introduce niche congestion—as in the literature on random search—and patent races (e.g., Loury, 1979; Lee and Wilde, 1980).

[11]For ease in exposition, we do not consider the possibility of licensing. If we were to allow licensing, however, blocked entry would emerge as part of the equilibrium. Blocking entry preserves the monopoly rents of π per period, dominating any payoff that the incumbent would obtain from licensing to the entrant. We rule out the scenario where an entrant sells the patent to the incumbent, as it would be problematic from an antitrust point of view (Green and Scotchmer, 1995).

[12]Because entry drives the incumbent profits to zero and firms only litigate to defend their profits, our modeling is equivalent to a *de facto* invalidation of the incumbent's patent if the entrant succeeds in court.

(that is, invest resources) to receive a costly signal σ about the merit of the infringement case. The outcome of the signal is binary, taking a value 0 when the judge finds no evidence of infringement (indicating that the innovation is likely to be genuine) and a value of 1 when the judge concludes otherwise. The precision of this signal depends on the unobservable evidence-gathering effort of each judge, $s \in [0, 1]$, according to the following simple specification:

$$\mu_0(s) = \Pr[\sigma = 0|\text{obvious}] = \frac{1-s}{2} \quad \text{and} \quad \mu_1(s) = \Pr[\sigma = 0|\text{genuine}] = \frac{1+s}{2}. \qquad (1)$$

Thus, if no effort is exerted, $s = 0$, the signal classifies genuine and obvious innovators as infringers with equal probability, $\mu_0(0) = \mu_1(0) = 1/2$. Under maximum effort, $s = 1$, the signal solely and perfectly classifies obvious innovators as infringers, $\mu_0(1) = 0$ and $\mu_1(1) = 1$. Judges face a cost of effort $c(s)$ increasing in s. This costs captures the effort of gathering evidence, analyzing and deliberating on the case.[13]

Importantly, we assume that each infringement claim is overseen by a different and independent judge. This judge decides how much effort to carry out in order to maximize the social surplus (welfare) associated with determining the right of the entrant to produce in the niche under dispute. As we explain in detail later, this welfare maximization is akin to minimizing the weighted cost of Type I errors (not allowing a genuine innovation to be implemented) and Type II errors (allowing an entrant with an obvious innovation to compete with the incumbent). In doing so, each judge takes the effort of the other judges as given, as well as the screening rate of the patent office, λ.

3 Exogenous Courts

To ease the exposition, and to distill the direct impact of the patent office on the innovation outcomes, we first solve the model under exogenously given values of the probabilities μ_0 and μ_1, which we will endogenize in the next section as the result of the judges' decisions.

[13]We have explored the implications of the model when judges' rulings are not entirely based on evidence and get influenced by either some utilitarian pro-competitive bias or some pro-incumbent anti-competitive bias. In all these extensions the main features of the baseline model remain essentially unchanged. Details can be provided upon request.

For given μ_0 and μ_1, we denote as v_t the value of being the incumbent in a monopolized niche at date t (that is, holding a patent that has not been infringed or whose infringement has been fended off in court) and we can characterize it recursively as

$$v_t = \pi + \beta[1 - e_{t+1}((1-\alpha)\lambda\mu_0 + \alpha\mu_1)]v_{t+1}. \tag{2}$$

This value is composed of the current flow of monopoly profits π and the discounted future value of preserving this position, βv_{t+1}, weighted by the probability of surmounting the potential entry of an innovator at $t+1$. The terms in square brackets take into account that entry occurs with probability e_{t+1}, involves an obvious or a genuine innovator with probabilities $1-\alpha$ and α, respectively, and the probabilities $\lambda\mu_0$ and μ_1 with which each entrant obtains both a favorable assessment by the patent office and a positive court ruling.

As a result of the entry flow and the competition that it might entail, we can write the law of motion governing the proportion of monopolized niches, x_t, as

$$x_{t+1} = [1 - e_{t+1}(1-\alpha)\lambda\mu_0]x_t + \alpha e_{t+1}(1-x_t). \tag{3}$$

Monopolized niches at $t+1$ are those already monopolized at t that do not experience the successful entry of obvious innovators (as such niches become competitive) plus the previously competitive niches that experience the entry of genuine innovators and become monopolized.

The flow of innovating firms e_t is determined by a free-entry condition. Attempting entry has a cost that we normalize to 1 and ends up being profitable only if the entrant becomes a monopolist in the corresponding niche. This requires engendering a genuine innovation, which occurs with probability α, and either landing in a competitive niche, which occurs with probability $1-x_t$, or otherwise surmounting the opposition of the incumbent in court, which occurs with probability μ_1. The combination of the previous events yields a probability of becoming a monopolist of

$$p_t = \alpha[1 - x_t(1-\mu_1)]. \tag{4}$$

The free-entry condition can be written as $-1+p_t v_t \leq 0$, which in an equilibrium involving an interior entry flow $e_t \in (0,1)$ in period t will hold with equality:

$$p_t v_t = 1. \tag{5}$$

3.1 Steady State Equilibrium Analysis

Our analysis will focus on the interior-entry equilibrium of the model. Equations (2)-(5) characterize the dynamic equilibrium of the industry under exogenously given values of μ_0 and μ_1. They determine four key endogenous variables at each date t: the proportion of monopolized niches, x_t, the probability that a genuine innovator becomes a monopolist, p_t the entry flow, e_t, and the value of being a monopolist, v_t. To ease notation, we will denote the corresponding steady-state value of the above variables simply by x, p, e, and v, respectively.

The following assumption restricts the profit parameter π so that the steady-state equilibrium of the model involves an interior entry flow $e \in (0,1)$. Lemma 1 shows the necessity and sufficiency of the restriction and provides close-form expressions for the steady-state value of the key variables of such an equilibrium.

Assumption 1. $\pi \in \left(\underline{\pi}, \underline{\pi} + \frac{\beta[\alpha+(1-\alpha)\lambda\mu_0]}{\alpha}\right)$, *where* $\underline{\pi} = \frac{(1-\beta)(\alpha+(1-\alpha)\lambda\mu_0)}{\alpha(\alpha\mu_1+(1-\alpha)\lambda\mu_0)}$.

Lemma 1. *The model has a unique steady-state equilibrium with* $e \in (0,1)$ *if and only if Assumption 1 holds. This equilibrium is given by*

$$x = \frac{\alpha}{\alpha+(1-\alpha)\lambda\mu_0}, \tag{6}$$

$$p = \alpha\frac{\alpha\mu_1+(1-\alpha)\lambda\mu_0}{\alpha+(1-\alpha)\lambda\mu_0}, \tag{7}$$

$$e = \frac{\pi p-(1-\beta)}{\beta\left[\alpha\mu_1+(1-\alpha)\lambda\mu_0\right]}, \tag{8}$$

$$v = \frac{1}{p}. \tag{9}$$

In the above equilibrium, entry occurs until the expected value of developing an innovation, pv, equals the unit entry cost. It is worth to notice that the proportion of monopolized niches and the value of incumbency in steady state are not affected by the profitability parameter π. That is, an increase in monopoly profits is completely offset by increased entry, which raises producers' turnover within the invariant fraction of monopolized niches (reducing the duration of incumbency), allowing the values of x and v to remain unchanged. Notice also that parameters λ and μ_0 always appear in combination, as $\lambda\mu_0$, which represents the rate at which obvious innovations succeed in entering a niche. The next proposition summarizes the comparative statics of this equilibrium.

Lemma 2. *The effects of marginal changes in the parameters on the steady-state equilibrium values of x, p, e, and v have the signs shown in the following table:*

	π	β	α	$\lambda\mu_0$	μ_1
x	0	0	+	−	0
p	0	0	+	+	+
e	+	+	+	?	+
v	0	0	−	−	−

The proportion of niches operating under monopoly, x, is increasing in the probability that an innovation is genuine, α, and decreasing in the rate at which obvious innovations succeed in entering, $\lambda\mu_0$. Intuitively, the higher the probability that a firm with an obvious patent arises and it is allowed to produce, $(1-\alpha)\lambda\mu_0$, the more often monopolist incumbents will be challenged and defeated in court. In contrast, the probability with which genuine innovators succeed in court vis-a-vis an incumbent patentholder, μ_1, does not affect x since ruling in favor of the entrant implies replacing one monopolist with another.

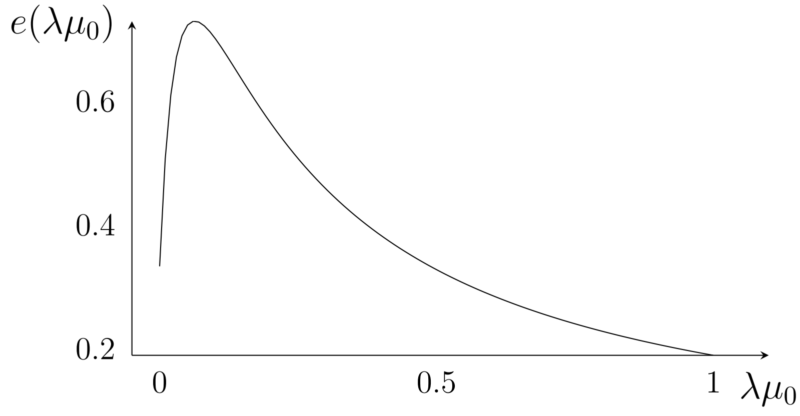
The value of incumbency v is, due to the free-entry condition, inversely related to the probability with which entrants become successful incumbents, p. Such probability is increasing in both the probability that the innovation is genuine, α, and that courts rule in its favor when confronting an incumbent, μ_1. More surprisingly, p is also increasing in the rate at which obvious innovations (which do not directly give rise to incumbency) enter successfully, $\lambda\mu_0$. This occurs because the entry of obvious innovators contributes to decrease the proportion of monopolized niches x, reducing the probability that an entrant with a genuine innovator faces the opposition of an incumbent monopolist. This force will make entry not necessarily decreasing in $\lambda\mu_0$ as we will show below.

As expected, entry is increasing in the flow of profits, π, and the discount factor, β. An increase in the judges' probability of ruling in favor of a genuine innovator, μ_1, or in the probability of obtaining a genuine innovation, α, also fosters entry, as it increases the probability of being successful. However, the effect of $\lambda\mu_0$ on entry is in general ambiguous as characterized in the next proposition.

Lemma 3. *The relationship between the rate at which obvious innovators successfully enter the market, $\lambda\mu_0$, and the steady-state equilibrium entry flow, e, can be increasing, decreasing or inverted-U shaped. In particular, it is decreasing when $\mu_1 = 1$.*

This result uncovers an interesting non-monotonic relationship between entry and the protection that incumbents receive against obvious innovations. The main driver of this result is that a change in $\lambda\mu_0$ engenders two effects of opposite sign. On the one hand, an increase in $\lambda\mu_0$ fosters entry — through the decrease in x — as it reduces the proportion of niches in which genuine innovators are challenged in court. On the other hand, an increase in $\lambda\mu_0$ decreases the value of incumbency, v, as monopolists are more likely to see their rents competed away by obvious innovators.

When the probability of success in court of an entrant with a genuine innovation, μ_1, is close to one, the second effect dominates and entry monotonically decreases with $\lambda\mu_0$. Intuitively, with $\mu_1 = 1$, the innovation-enhancing pro-competitive effect disappears since genuine innovators are always granted access, regardless of whether they land in monopolized or competitive niches. As illustrated by Figure 1, however, when the entry of genuine innovators is not guaranteed, the pro-competitive effect is relevant and may dominate when $\lambda\mu_0$ is low. In those cases, the innovation flow is maximized at some interior value $\lambda\mu_0$. These results will have non-trivial implications for the discussion on the socially optimal level of protection against imitation, $1 - \lambda\mu_0$, and its link to the socially optimal level of protection against a genuine innovation, $1 - \mu_1$.



Note: Parameter values are $\alpha = 0.1$, $\pi = 3.6$, $\beta = 0.8$, and $\mu_1 = 0.6$.

Figure 1: Entry flow and the protection against obvious innovators. This figure shows a case in which entry is maximized at an interior value of the probability with which obvious innovators are allowed to enter, $\lambda\mu_0$.

3.2 Optimal Patent Screening

To gain intuition about the effects of changing the patent office's screening rate λ, we characterize its socially optimal level. Screening affects welfare through the entry rate and the rate at which obvious innovations enter successfully the market as well as through its direct costs.

To rely on a properly microfounded welfare metric, we need to specify the demand side of the industry. We do this within a quality-ladder framework with limit pricing.[14] In particular, we assume that there is a unit mass of infinitely-lived homogeneous consumers willing to buy at most one unit of the product from each niche $j \in [0, 1]$ at each date t. Utility is additive across goods and dates, the intertemporal discount factor is $\beta < 1$, and the net utility flow from buying good j at price P_{jt} is $U_{jt} = Q_{jt} - P_{jt}$, where Q_{jt} is the quality of the good. For simplicity, production costs are assumed to be zero.

The successful entry of a genuine innovator in a given niche improves the quality of the best good available in that niche by π units. The genuine innovator, however, is able to charge a price $P_{jt} = \pi$ that captures the full quality advantage of its product vis-a-vis

[14]In section 5.1 we discuss an alternative market environment where firms invest in cost-reducing innovations. Unlike in the quality-ladder model, that case exhibits a deadweight loss derived from market power and we analyze its implications.

the best competing alternative. In contrast, the successful entry of an obvious innovation does not increase consumers' valuation of the good but it introduces competition for the latest technology, decreasing the market price P_{jt} to zero and transferring the gains to the consumers.

Let $\Pi \equiv \pi/(1-\beta)$ represent the social present value generated by an innovation. Then, the total per-period net addition to welfare in steady state is equal to

$$W = e\big[p\Pi - 1 - \kappa(\lambda)\big], \tag{10}$$

where $\kappa(\lambda)$ represents the cost of screening a patent application, which we assume to be a continuously differentiable, decreasing, and convex function of the fraction of obvious innovations that do not get detected.

The interpretation of (10) is as follows. In steady state, every period e innovations originate at a cost of one. An innovation contributes to social welfare if it is genuine and ends up being produced, either because it lands in a niche not occupied by a monopolist or because the entrant wins the patent-infringement case. These events occur with a total probability p. The rents associated with a successful genuine innovation are the present discounted value of a perpetual increase in quality π. These rents are split between firms and consumers. When a genuine innovation arrives, the rent is initially appropriated by the innovating firm. However, when a subsequent innovation, either genuine or obvious, is implemented in the niche, the rents of the incumbent are competed away and the benefits of the prior increase in quality accrue to consumers. In the absence of screening costs (i.e., $\kappa(\lambda) = 0$ for all λ), the free-entry condition (5) implies that the parenthesis in (10) is always positive.[15] More generally, the assumption $\kappa(1) = 0$ guarantees that, under the welfare-maximizing choice of λ, welfare remains positive.

Under exogenous courts, the derivative of the welfare function with respect to the screening rate λ is

$$\frac{\partial W}{\partial \lambda} = \frac{\partial e}{\partial \lambda}\big(p\Pi - 1 - \kappa(\lambda)\big) + e\left(\frac{\partial p}{\partial \lambda}\Pi - \kappa'(\lambda)\right). \tag{11}$$

[15] Specifically, we have $\Pi > v$ and $pv = 1$.

The level of screening directly affects welfare through two channels: it determines entry and the probability of success. From Lemma 2 we know that p is increasing in λ. That is, for a given entry flow e, a lower level of screening reduces the proportion of monopolized market niches, reducing the number of genuine innovations challenged in court and, consequently, the number of innovations that go to waste. From Lemma 3 we know that the net effect on entry can be ambiguous if $\mu_1 < 1$. This explains the next result, which focuses on a benchmark case with zero screening costs.

Proposition 1 (Rational ignorance)**.** *In the absence of screening costs ($\kappa(\lambda) = 0$ for all λ), there exists a threshold $\hat{\mu}_1 \in (0, 1)$ such that if $\mu_1 \geq \hat{\mu}_1$, it is socially optimal to fully screen out obvious innovations ($\lambda = 0$), and; if $\mu_1 < \hat{\mu}_1$ it is socially optimal to allow some obvious innovations. In this case, the optimal proportion of obvious innovations allowed is decreasing in* Π *and given by*

$$\hat{\lambda} = \begin{cases} 1 & \textit{if } \Pi < K, \\ \alpha \frac{2+(1-3\mu_1)\alpha\Pi+(1-\mu_1)\sqrt{\alpha\Pi(\alpha\Pi+8)}}{2\mu_0(1-\alpha)(\alpha\Pi-1)} & \textit{otherwise.} \end{cases} \tag{12}$$

where K is a known positive constant.

Clearly, if screening is costly and, in particular, if the screening cost function satisfies Inada-type conditions (specifically, $\kappa'(0) = -\infty$ and $\kappa(1) = \kappa'(1) = 0$), the optimal value of λ is interior. Proposition 1 goes further by saying that, even in the absence of screening costs, there may be circumstances (when $\mu_1 < 1$) in which it is optimal to grant a patent to some obvious innovations. The intuition behind this result is that obvious patents may foster entry by increasing the number of competitive niches, thus increasing the probability of success of future genuine innovators. When $\mu_1 = 1$, however, no genuine innovation goes to waste and the benefit of increasing p is nil. Consequently, only the effect of screening on steady-state entry matters. From Lemma 3, we know that entry is decreasing in λ so that the optimal solution is full screening.

To conclude this section, we solve as a benchmark the problem of a planner that can control both patents' ex-ante screening as measured by λ and their ex-post enforcement in court as represented by μ_0 and μ_1.

Corollary 1. *In the absence of screening and enforcement costs, a social planner that can decide on patents' ex-ante screening (λ) and on their enforcement (μ_0 and μ_1) chooses to fully screen out obvious innovations, either at the patent office or in court, and to always rule in favor of new genuine innovators.*

This result arises from a combination of the previous results. As shown in Lemma 2, higher values of μ_1 yield an increase in the probability that an entrant is successful, p, and, consequently, an increase in total entry, e. Both effects contribute to increase social welfare, as indicated in equation (11). Hence, a planner that could regulate the behavior of courts at no cost should choose $\mu_1 = 1$. Using Proposition 1 we know that obvious entrants should not receive any protection in that case, that is, it would be optimal to set $\lambda\mu_0 = 0$.

4 Endogenous Courts

In the previous sections we treated as exogenous the probabilities μ_0 and μ_1 that determine the result of court trials against obvious and genuine innovators, respectively. In this section we endogenize these probabilities as the result of an evidence gathering process carried out by the judge that oversees each case. We first analyze the decision of an individual judge involved in a single case. We examine how this judge best responds to different ex-ante screening rates by the patent office and the expectations on the behavior of judges that will rule on future cases. We then aggregate the decisions of all judges in the (Markov perfect) steady-state equilibrium of the model and analyze the effects of changes in the screening rate λ.

4.1 Type I and Type II Errors

We assume that when a case reaches a judge, she decides how much evidence-gathering effort s to exert in order to maximize the social welfare created in that niche. The judge takes the screening rate λ and other judges' effort as given. This means that the judge only considers the impact of that *specific* ruling on total welfare. A judge only reviews a case after entry has occurred, thus ignoring (in contrast with (10)) the already sunk cost of entry. The judge, however, takes into account the effect of the ruling on future entry. Because we focus on symmetric equilibria we assume that all future judges will exert the same effort level $\hat{s}$.

The impact of a judge on welfare is the result type I and type II statistical errors. We define the cost of type I error, E_I, as the welfare loss from precluding the production of an entrant that holds a genuine innovation. This cost is easy to assess, since it reduces social welfare by π on a permanent basis, so $E_I = \Pi$, where, as defined earlier, $\Pi = \pi/(1-\beta)$.

The type II error consists on failing to protect an incumbent monopolist against an obvious innovator, turning the niche into a competitive one. Making the niche competitive affects the probability of future entry and, consequently, the stream of future innovations. The welfare losses due to the type II error can be written as $E_{II} = \beta(w_M - w_C)$, where w_M and w_C are the present value of the future welfare realized in the niche, from the following period onwards, when starting in the state of monopoly and competition, respectively. The values w_M and w_C can be found by solving the following system of equations:

$$\begin{aligned} w_C &= \beta w_C + e\left[\alpha\left(\Pi + \beta(w_M - w_C)\right) - 1\right], \\ w_M &= \beta w_M + e\left[(1-\alpha)\lambda\mu_0\beta(w_C - w_M) + \alpha\mu_1\Pi - 1\right]. \end{aligned} \tag{13}$$

Because judges are atomistic, they take as given the enforcement decisions of future judges, $\hat{s}$. These decisions also determine the future entry rate e and the probabilities with which these genuine and obvious innovators will prevail in court, μ_1 and μ_0, respectively.

The social value of a competitive niche before entry takes place, w_C, depends on the likelihood and quality composition of the prospective entry. Without entry, the niche remains unchanged, generating a present value of welfare βw_C. Whenever entry occurs, the unit entry cost is incurred. With probability α, the entrant is a genuine innovator which directly contributes a discounted social surplus of Π and turns the niche into a monopolized one starting next period. This transition generates a capital gain $\beta(w_M - w_C)$ relative to the continuation of the niche in its competitive state.

Similarly, the social value of a monopolized niche, w_M, also depends on whether the incumbent faces entry or not, and the identity of the entrant. Without entry, the niche remains unchanged, generating a present value βw_M. When entry occurs, the unit entry cost is incurred. With probability $(1-\alpha)\lambda\mu_0$, a successful entrant with an obvious innovation turns the niche competitive, generating the capital gain $\beta(w_C - w_M)$. With probability $\alpha\mu_1$ a genuine entrant succeeds, producing a direct increase in social surplus of Π and no change in the monopolized status of the niche.

Lemma 4 shows, by solving (13), that the type II error, E_{II} has a negative sign. Intuitively, the type II error generates a net welfare gain because the continuation social surplus increases when the niche becomes competitive: eliminating the monopoly increases the probability that a future genuine innovator enters successfully. This perceived benefit from the type II error is a re-statement of the classical time-inconsistency associated to patent policy. Whereas the ex-ante promise of protection spurs innovation and increases social welfare, once the innovation has taken place, it is optimal to prevent the exercise of the market power that a patent allows, which in our model does not directly produce a deadweight loss but it is detrimental to the entry of subsequent innovators and the net value they engender. This bias against incumbents naturally arises here because courts are asked to rule in favor of either an entrant with an obvious innovation, which fosters future innovation by eliminating the litigation risk faced by future entrants, or an incumbent, whose genuine innovation has already materialized. The value of the this type II error is computed next.

Lemma 4. *The steady-state net welfare cost associated with type II error is negative and equal to*

$$E_{II}(\hat{s},\gamma) = -\Pi \frac{(1-\gamma)\alpha\beta(1-\mu_1(\hat{s}))e(\hat{s},\gamma)}{(1-\gamma)(1-\beta)+\alpha\beta\left(1-\gamma+\gamma\mu_0\left(\hat{s}\right)\right)e\left(\hat{s},\gamma\right)} \tag{14}$$

where

$$\gamma \equiv \frac{(1-\alpha)\lambda}{\alpha+(1-\alpha)\lambda} \in [0, 1-\alpha] \tag{15}$$

measures the probability that a judge faces an obvious entrant.

To simplify notation, we have implemented a change of variable from the screening quality λ to the proportion of obvious entrants faced by a judge, γ, as defined in (15). Observe that γ is increasing in λ and decreasing in the proportion of genuine innovations, α. To avoid confusion in the analysis that follows, in equation (14) we made explicit the dependency of μ_0 and μ_1 on $\hat{s}$, and of the entry rate and the cost of the type II error on $\hat{s}$ and γ. For now, we assume that judges face a given γ and take decisions based on it. The value of γ is endogeneized in Section 4.4.[16]

4.2 An Individual Judge's Problem

We can now characterize the optimal evidence-gathering decision of a given judge, $s \in [0,1]$. It is immediate that maximizing social welfare in a niche is equivalent to minimizing the expected social cost of both types of error plus the cost of the effort required to reduce such errors, $c(s)$. The overall cost to minimize can be expressed as

$$J(s, \hat{s}, \gamma) = (1-\gamma)(1-\mu_1(s))E_I + \gamma\mu_0(s)E_{II}(\hat{s}, \gamma) + c(s), \tag{16}$$

since, with probability $1-\gamma$ the judge faces an entrant with a genuine innovation, leading to a type I error with probability $1-\mu_1(s)$ and with probability γ the judge faces an entrant with an obvious innovation, leading to a type II error with probability $\mu_0(s)$.

The analysis of the decisions emerging from the minimization of (16) in the case in which s is a continuous variable and $c(s)$ is an increasing and convex cost function is rather involved. So we will first convey intuitions by considering, in the remaining of this section, the case in which the judge's decision is binary $s \in \{0,1\}$. We address the case in which s is continuous in Section 5.2. We can normalize the cost of no effort to 0, $c(0) = 0$, and define $c(1) = c > 0$. Using (1), we have $\mu_0(0) = \mu_1(0) = \frac{1}{2}$ and $\mu_1(1) = 1 > 0 = \mu_0(1)$, which greatly simplifies the analysis.

We now turn to a judge's optimal effort decision. Under full effort, $J(1, \hat{s}, \gamma) = c$ for any value of γ and $\hat{s}$. That is, when the judge's effort results in fully-informative evidence,

[16] In the expressions for w_C and w_M we have ignored the cost of screening a patent, $\kappa(\lambda)$. Introducing this cost would have no impact on E_{II} as, from the perspective of a judge, this cost is sunk.

type I and type II errors do not arise and the overall social cost of the judge's decision is only the cost of her effort, c. The social cost when a judge chooses to exert no effort is characterized in the next lemma.

Lemma 5. *When a judge exerts no effort, the social cost of her decision is given by*

$$J\left(0,\hat{s},\gamma\right)=\Pi(1-\gamma)\Phi(\hat{s},\gamma)/2. \tag{17}$$

where $\Phi\left(\hat{s},\gamma\right)$ *is a factor related to the present value of the innovation-reducing effects of type I error net of the innovation-enhancing effects of type II error, and is given by*

$$\Phi\left(\hat{s},\gamma\right)=\frac{\left(1-\gamma\right)\left[\left(1-\beta\right)+\alpha\beta e\left(\hat{s},\gamma\right)\right]}{\left(1-\beta\right)\left(1-\gamma\right)+\beta\alpha\left(\left(1-\gamma\right)+\gamma\hat{\mu}_0\right)e\left(\hat{s},\gamma\right)}\in[0,1]. \tag{18}$$

This factor is decreasing in $e(\hat{s},\gamma)$*,* γ*,* α *and* Π*.*

That is, for a given effort by the other judges, $\hat{s}$, and probability of facing an obvious innovator, γ, the social cost $J(0,\hat{s},\gamma)$ when a judge exerts no effort depends on the probability of facing a genuine innovator, $1-\gamma$, times the probability $1/2$ of taking the wrong decision (see (1) under $s=0$), times the magnitude of the perpetuity loss Π. The term $\Phi(\hat{s},\gamma)\in[0,1]$ measures the net intertemporal detrimental effect of type I and type II errors on the entry of genuine innovations. A larger $\Phi(\hat{s},\gamma)$ is associated with a stronger type I error and a weaker type II error.

The previous lemma also tells us how the net intertemporal detrimental effect of type I and type II errors changes with the parameters of the model. Observe that $\Phi\left(\hat{s},\gamma\right)\geq 0$, meaning that the benefit of the type II error never overcomes the cost of a type I error. However, an increase in the (endogenous) entry flow raises the probability that the niche will be occupied by a genuine innovator in the future, which increases the benefits of a type II error, decreasing $\Phi\left(\hat{s},\gamma\right)$. An increase in the probability of facing an obvious entrant, γ, also increases the benefits of a type II error. Intuitively, this occurs because an increase in the proportion of obvious patents decreases the chance of future litigation, raising the probability of successful entry by a genuine innovator $p(\hat{s},\gamma)$ which, in turn, increases future entry. Similarly, an increase in the probability of obtaining a genuine innovation α and in the discounted profits obtained by firms Π also boost entry, raising the benefits of a type II error and decreasing $\Phi(\hat{s}, \gamma)$.

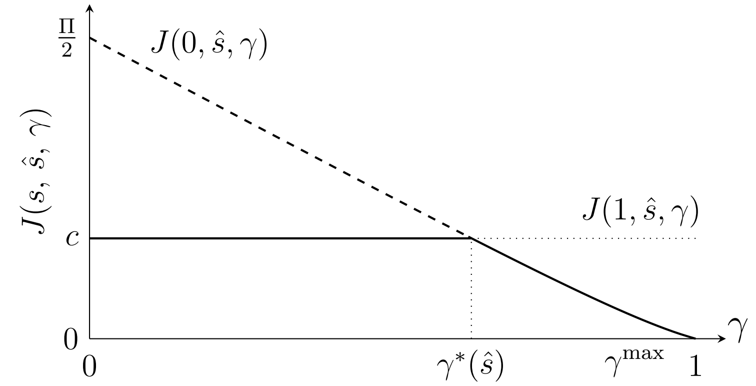
Proposition 2 (Screening complements enforcement)**.** *For any value of other judges' enforcement effort* $\hat{s} \in [0, 1]$*:* i) *If* $c < \Pi/2$*, there exists a threshold value for the probability of facing an obvious innovator,* $\gamma^*(\hat{s}) \in (0, 1)$ *such that the judge exerts effort if and only if* $\gamma \leq \gamma^*(\hat{s})$*.* ii) *If* $c \geq \Pi/2$*, a judge does not exert effort regardless of the value of* $\gamma > 0$*.*

The main implication of the previous proposition is that the patent office's screening rate (which reduces λ and hence γ) and an individual judge's enforcement effort are complementary. Intuitively, if the patent office screens out a larger proportion of obvious innovations (lower λ), the social cost of making a judicial error increases (as it is more likely to be type I error in this case), which encourages the judge to exert effort. Mathematically, (17) decreases in γ both directly and indirectly through $\Phi(\hat{s}, \gamma)$ as observed in Lemma 5.

Figure 2 illustrates the trade-offs behind this result. It depicts the social cost internalized by an individual judge as a function of the probability of facing an obvious innovator, γ, under the two possible enforcement effort choices, $s = 0$ and $s = 1$. The social cost under full effort is flat and equal to c, whereas the social cost under no effort declines with γ. The cutoff $\gamma^*(\hat{s})$ separates the ranges of γ for which, given other judges' enforcement effort, an individual judge will or will not exert effort. The complementarity between screening rate (low γ) and enforcement effort (choice of $s = 1$) will become important when we characterize the equilibrium judge behavior.

In order to focus on the case in which full enforcement effort is possible in equilibrium, throughout the rest of the paper we make the following assumption:

Assumption 2. $c < \Pi/2$.



Note: Parameter values are $\alpha = 0.1$, $\beta = 0.8$, $\pi = 3.6$, $c = 3$, and $\hat{s} = 0.6$.

Figure 2: The decision of an individual judge. This figure shows the social cost $J(s, \hat{s}, \gamma)$ minimized by each individual judge as function of the probability γ with which the confronted entrant holds an obvious innovation if all other judges choose effort $\hat{s}$. The maximum possible value of γ is $\gamma^{\max} = 1 - \alpha$.

4.3 Judges' Equilibrium Enforcement Effort

The analysis in the previous section refers to an individual judge, taking the symmetric behavior of future judges, $\hat{s}$, as given. From the perspective of a judge's objective function in a niche, $\hat{s}$ only affects the type II error and it does so through two channels. First, it affects the expected entry flow, $e(\hat{s}, \gamma)$. Second, and contingent on a given future entry flow, it affects the prospects of transition between monopoly and competition in the specific niche to which the ruling will apply. These transitions are affected by $\hat{s}$ through $\mu_0(\hat{s})$ and $\mu_1(\hat{s})$.

In a symmetric steady-state equilibrium, judges' enforcement effort s^* is given by the effort level of an individual judge s^* that is consistent with having $\hat{s} = s^*$ as the enforcement effort exerted by future judges. To characterize s^* with binary enforcement effort, it is important to understand how the threshold value of the probability of facing an obvious innovator that determines an individual judge's effort depends on $\hat{s}$.

Lemma 6. *The thresholds $\gamma^*(\hat{s})$ determining how a judge's effort depends on other judges' effort $\hat{s} \in \{0, 1\}$ satisfy:* $\gamma^*(0) < \gamma^*(1) \equiv (\Pi - 2c)/\Pi$.

In words, the range of values of γ over which a judge exerts effort expands when future judges are also expected to exert effort, meaning that the effort of subsequent judges are strategic complements. This result, illustrated in Figure 3, yields the equilibrium configurations described in the next proposition.

Proposition 3 (Enforcement equilibria)**.** *In a pure-strategy symmetric steady-state equilibrium, judges' effort in the patent enforcement game is given by:*

$$s^* = \begin{cases} 1 & \text{if } \gamma \leq \gamma^*(0), \\ \{0,1\} & \text{if } \gamma \in (\gamma^*(0), \gamma^*(1)], \\ 0 & \text{if } \gamma > \gamma^*(1). \end{cases}$$

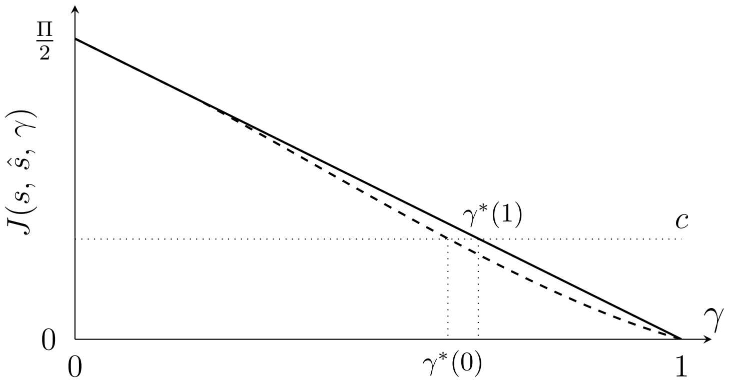
In the multiple equilibrium region, entry is higher with full enforcement effort ($s^* = 1$) than with no enforcement effort ($s^* = 0$).

The underlying complementary implies that a choice $\hat{s} = 0$ ($\hat{s} = 1$) by other judges strengthens the incentives for a given judge to also choose $s = 0$ ($s = 1$). With low effort some obvious innovators will end up replacing existing monopolists, which contributes to increase social welfare by facilitating the entry of future genuine innovators. However, under $\hat{s} = 1$ this entry-facilitating effect of $s = 0$ (and, thus, the convenience of the type II error) disappears because future judges always allow genuine innovators to enter. Without the rationale for $s = 0$ coming from the convenience of type II error, whether $s = 0$ or $s = 1$ is optimal only depends on comparing the gains from reducing type I error with the enforcement effort cost c.

We can now characterize how the equilibrium enforcement effort changes with the parameters of the model.

Proposition 4 (Comparative Statics)**.** *In the symmetric steady-state equilibrium, judges' enforcement effort s^* is increasing in the value of a genuine innovation Π, increasing in the screening quality of the patent office (decreasing in λ), and decreasing in the enforcement effort cost, c.*

In the previous section we established the complementarity between the effort carried out by an individual judge and the ex-ante screening of patents. This result naturally



Note: Parameter values are $\alpha = 0.1$, $\beta = 0.8$, $\pi = 3.6$, and $c = 3$.

Figure 3: Judges' equilibrium enforcement effort. The solid and dashed line represent $J(0, 1, \gamma)$ and $J(0, 0, \gamma)$, respectively. An equilibrium with $s^* = 1$ ($s^* = 0$) exists whenever the probability of facing an obvious innovator is below (above) $\gamma^*(1)$ ($\gamma^*(0)$)

extends to the equilibrium enforcement effort level. A lower λ, arising from a more thorough application review process by the patent office and fewer patents granted for obvious innovations, implies a lower value of γ and a higher enforcement effort by all judges.

This result also allows us to understand under which circumstances the level of patent enforcement that maximizes entry and innovation (characterized in Proposition 1 for the case in which courts were assumed to be exogeneous) can be attained once courts are endogenized. As it turns out, the complementarity between ex-ante screening and ex-post judge effort implies that λ has to be sufficiently small so that $\gamma \leq \gamma^*(1)$ in order to induce $\mu_1(1) = 1$ and $\mu_0(1) = 0$. In the next section we build on this result and explore the socially optimal choice of λ once its impact on judges' enforcement effort is taken into account.

The previous proposition also establishes a positive relationship between the value of an innovation and judges' equilibrium enforcement effort. For each individual judge, and given $\hat{s}$, an increase in π or β generates two effects that operate in opposite directions. On the one hand, and as established in Lemma 5, if the value of the (future) innovation increases, allowing obvious innovations today decreases the cost of $s = 0$ due to the higher social value of type II error. On the other hand, the higher the discounted value of the innovation, the higher the cost of mistakenly preventing the production of a genuine innovation in the current period (type I error). When π or β increase, this second effect dominates, implying an upward shift in the function $J(0, s^*, \gamma)$ for both $\hat{s} = 0$ and $\hat{s} = 1$. In terms of Figure 3 this means that both $\gamma^*(0)$ and $\gamma^*(1)$ increase, expanding the region over which an equilibrium with $s^* = 1$ is sustainable. As genuine innovations become more prevalent (lower λ) and their social value increases (higher Π), investing in enhancing the quality of enforcement becomes more valuable and an equilibrium with full enforcement is more likely to emerge.[17]

4.4 Screening and Enforcement Equilibrium

In this section we characterize the socially optimal patent screening rate, λ^*, taking into account the endogenous response of courts. Given the equilibrium enforcement effort decision of judges, $s^*(\lambda, c)$ characterized in the previous section, a social planner chooses λ to maximize[18]

$$W(\lambda; s^*, c) = e\left(s^*, \lambda\right)\left[p\left(s^*, \lambda\right)\Pi - 1 - \kappa(\lambda) - c(\alpha + (1-\alpha)\lambda)x\left(s^*, \lambda\right)s^*\right] \tag{19}$$

where $x\left(s^*, \lambda\right)$, $p\left(s^*, \lambda\right)$ and, $e\left(s^*, \lambda\right)$ are the equilibrium values for the proportion of monopolized niches, the probability that an entrant obtains a novel innovation, and number of entrants, respectively (see equations (6), (7) and, (8)). Social welfare differs from the patent's office problem (10) in two ways. First, equation (10) is not regarded as the endogenous enforcement effort of judges. Second, social welfare in (19) also takes into account that judges incur in a cost of $s^*(\lambda, c)c$ per case reviewed and that the mass of cases reviewed is the proportion $(\alpha + (1-\alpha)\lambda)\, x\left(s^*, \lambda\right)$ of entrants that obtain a patent and land in a monopolized niche.

[17]The effect of α is more difficult to assess. It is true that, given γ, the cost of poor enforcement decreases as α increases (see Lemma 5). However, the value of γ negatively depends on α. Numerical calculations indicate that, as in the case of Π, the overall effect of α on s^* is positive, although an analytical characterization of this result is elusive.

[18]For parsimoniousness, we omit λ and c from $s^*(\lambda, c)$ as arguments in equation (19).

To solve the previous problem it is convenient to define the socially optimal values of λ conditional on each of the possible levels of judge-enforcement effort:

$$\lambda_0 \in \arg\max_{\lambda\in[0,1]} W(\lambda; s, c), \quad \text{s.t. } s^*(\lambda, c) = 0,$$

which does not depend on c, since when judges exert no effort this cost does not affect social welfare, and

$$\lambda_1(c) \in \arg\max_{\lambda\in[0,1]} W(\lambda; s, c), \quad \text{s.t. } s^*(\lambda, c) = 1.$$

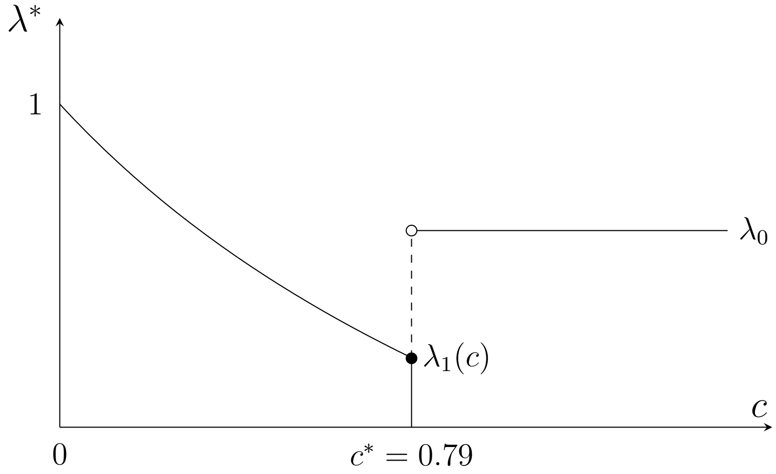
Finally, we define c_{01} to be the unique value of c that solves $W(\lambda_0; 0) \equiv W(\lambda_1(c_{01}); 1, c_{01})$; that is, c_{01} is the maximum enforcement cost under which inducing judge effort might be optimal.

To solve the planner's problem, we make two simplifying assumptions. First, we assume $\kappa(\lambda) = (1-\lambda)^2/\lambda$. Assuming a functional form allows us to simplify exposition in the proof, but the proposition holds more generally. We also assume that $\lambda_1(c_{01}) < \lambda_0$; that is, when the enforcement cost is such that the optimal screening with and without effort leads to identical welfare, better screening by the patent office is required when judges exert effort.[19]

Proposition 5 (Optimal Screening)**.** *Assume $\kappa(\lambda) = (1-\lambda)^2/\lambda$ and $\lambda_1(c_{01}) < \lambda_0$. Then, there exists a threshold c^* such that the socially optimal ex-ante patent screening rate, $\lambda^*(c)$, is decreasing in c and equal to $\lambda_1(c)$ (which induces full judge effort) whenever $c \leq c^*$, and equal to λ_0 (which induces no effort) otherwise. Furthermore, $\lambda^*(c)$ has a upward discontinuity at c^*.*

The previous result is illustrated in Figure 4. The social planner faces the following tradeoff: a low screening level (high λ) saves on the cost of reviewing patent applications by the patent office but reduces judges' incentives to exert enforcement effort, so it is only compatible with inducing such effort for when the enforcement cost c is low. As

[19]This assumption holds in every numerical simulation of the model under the adopted functional form for κ.



Note: Parameter values are $\alpha = 0.1$, $\beta = 0.8$, and $\pi = 3.6$.

Figure 4: Optimal screening as a function of the enforcement cost. As a result of substitutability, screening increases (λ^* declines) with the enforcement cost c while it is optimal to induce high enforcement effort by judges. When $c > c^*$, judges perform low enforcement effort and, as a result of complementarities, screening jumps down (λ^* jumps to λ_0).

c increases, the optimal patent screening increases (λ^{so} falls). Beyond the threshold c^*, it is socially superior to give up on inducing a high level of enforcement. The upward jump of λ^* at c^* (that is, the fall in screening at that point) reflects the complementarity between ex-ante screening and ex-post enforcement. As explained in prior sections, the lower screening rate removes some monopolies, replacing them with obvious innovators, reducing the hurdle to future entry and mitigating the discouragement to innovation that a low enforcement effort would otherwise generate. As we shall see below, the effects of this complementarity also tend to prevail in the case in which we allow judges' enforcement effort to vary continuously.

5 Robustness and Extensions

Here we analyze two extensions of the main model and show that the main insights from previous sections go through in alternative or more general settings. First, we study how the existence of static inefficiency in a context of cost-saving innovations affects the judges' decisions and the outcome of the patent enforcement game. We then examine the case in which judges' enforcement effort is continuous. The technical details are relegated to Appendix B.

5.1 Cost-saving Innovations and Static Inefficiency

In the baseline model we consider quality-improving innovations in an environment with inelastic demands. In that context, genuine innovators extract all the surplus from consumers, avoiding deadweight losses and simplifying the welfare analysis. We now provide a tractable framework that extends our analysis to a case of cost-reducing innovations, where market power involves welfare losses.

In this setup there is also a continuum of niches of size 1. In each niche the good produced is homogeneous. Firms compete in prices and face a demand function $q = a/p$. Each genuine innovation decreases the existing marginal cost by a proportion $1 - \delta$ where $\delta \in (0, 1)$. That is, if z represents the baseline marginal cost, after m genuine innovations the marginal cost becomes $z_m = \delta^m z$.[20]

Lemma 7. *In a cost-saving innovations setup, the profit flow π and the deadweight loss ℓ generated by a genuine innovation are independent of the baseline marginal cost z and the cumulative number of innovations m. In particular, as illustrated in Figure 5, they are equal to $\pi = a(1-\delta)$ and $\ell = a\left(\ln(\delta^{-1}) - (1-\delta)\right) > 0$.*

Social Welfare Because profits are invariant to the cumulative number of innovations, firm behavior and industry dynamics described in Section 3 go through without alterations. The objective functions of the patent office and courts, however, need to be reformulated to account for the deadweight loss ℓ. In particular, the main difference with respect to the baseline model is that now, allowing entry into a monopolized niche converts the existing deadweight loss into consumer surplus. Entry, whether from a genuine or obvious entrant, increases the social welfare by ℓ on a permanent basis, $L \equiv \ell/(1-\beta)$.

[20] This demand and the proportional cost-saving innovation are also used in Marshall and Parra (2019).

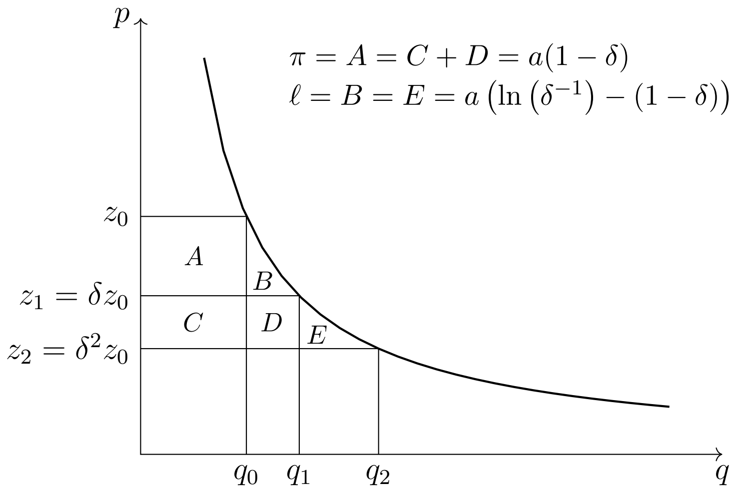


Figure 5: Monopoly profits and deadweight losses with cost-saving innovations. This figure identifies the profits (π) and deadweight losses (ℓ) associated with a successful genuine cost-saving innovation as areas under the relevant demand curve.

As carefully shown in Appendix B.1. the objective function of the patent office is now

$$W = e\left[p(\Pi + L) - 1 - \kappa(\lambda)\right].$$

This expression is analogous to that in (10) except for the new term in L, which captures the deadweight loss recouped with the arrival of an innovation. Because the behavior of e and p with respect to λ remains unchanged with respect to the baseline model, it is immediate that the message of Proposition 1 extends to this environment. When courts' behavior, as represented by μ_0 and μ_1, is exogenous, the optimal patent screening rate may be interior even in the absence of screening costs.

The Judges' Problem We can now analyze how a judge's enforcement decision changes when innovations are cost reducing and monopoly involves a deadweight loss. Recall that entry is only opposed by the incumbent in a monopolized niche. Therefore, a ruling in favor of the entrant will always increase welfare by (at least) L regardless of the entrant's innovation quality. As in the benchmark case, a type I error arises whenever a genuine innovation is excluded from the market. Since this error can only occur in already monopolized niches, it now leads to a loss of $E_I^{CS} = \Pi + L$, where the superindex CS stands for the cost-saving innovation setup.

The cost of type II error represents the reduction (in fact a gain, since its sign is negative as in the benchmark model) in social welfare that occurs when a firm with an obvious innovation is allowed to replace an active monopolist. This error has now two components. First, each time a monopolist is replaced by another firm, the deadweight loss associated to its innovation is eliminated, leading to an immediate gain of L. Second, as in the benchmark case, the dynamic effect of enhanced entry increases the future social value of the niche. Altogether, the benefit of type II error is now larger compared to the benchmark model; that is, for every $\hat{s}$ and γ, $E_{II}^{CS}(\hat{s},\gamma) < E_{II}(\hat{s},\gamma)$.

As before, an individual judge decides her enforcement effort s taking as given the enforcement effort decisions of other judges $\hat{s}$. We denote as $J^{CS}(s,\hat{s},\gamma)$ the cost minimized by the individual judge in this case. As in the benchmark case, when a judge chooses $s = 1$ no error is made, so the only cost is that related to her effort, $J^{CS}(1,\hat{s},\gamma) = c$. When a judge chooses effort $s = 0$, however, the cost is

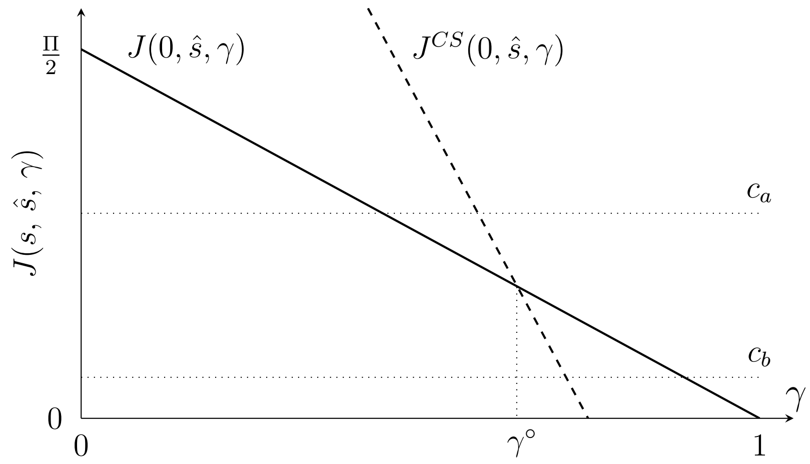
$$J^{CS}(0,\hat{s},\gamma) = J(0,\hat{s},\gamma) + L(1-\gamma)\Delta(\hat{s},\gamma)/2,$$

where $J(0,\hat{s},\gamma)$, defined in (17), is the judge's loss function in the model without deadweight losses and

$$\Delta(\hat{s},\gamma) = \frac{(1-2\gamma)(1-\beta) + \alpha\beta(1-\gamma)e(\hat{s},\gamma)}{(1-\gamma)(1-\beta) + \alpha\beta\left(1-\gamma+\gamma\mu_0(\hat{s})\right)e(\hat{s},\gamma)} \leq 1$$

is a function which, given the effort of other judges and the level of screening by the patent office, measures the net intertemporal contribution of type I and type II errors to the occurrence of the deadweight losses represented by L.

Similarly to the factor $\Phi(\hat{s},\gamma)$ that measures the net detrimental impact of judicial errors on the entry of genuine innovators, the factor $\Delta(\hat{s},\gamma)$ is decreasing in the probability that the confronted entrant is an obvious innovator, γ, which implies that the presence of the deadweight loss L reinforces the complementarity between patent screening (which reduces γ) and patent enforcement (which avoids the cost $J^{CS}(0,\hat{s},\gamma)$). Observe that, for values of γ close to one $\Delta(\hat{s},\gamma)$ can be negative, whereas when γ is low $\Delta(\hat{s},\gamma)$ is positive. The stronger dependence of $J^{CS}(0,\hat{s},\gamma)$ on γ explains the following result:



Note: Parameter values are $\alpha = 0.1$, $\beta = 0.8$, $a = 4$, $\delta = 0.1$, $c_a = 5$, $c_b = 1$ and $\hat{s} = 1$.

Figure 6: Costs minimized by an individual judge with and without static inefficiency. The solid and dashed lines represent $J(0, \hat{s}, \gamma)$ and $J^{CS}(0, \hat{s}, \gamma)$, respectively. The parallel dotted lines show two possible illustrative levels of the cost of high enforcement effort.

Proposition 6 (Enforcement incentives with deadweight losses)**.** *For every $\hat{s} \in [0, 1]$, there exists $\gamma^\circ(\hat{s}) \in (0, 1)$, increasing in $\alpha\Pi$, such that: i) when $\gamma > \gamma^\circ(\hat{s})$ a judge has less incentives to exert effort relative to a situation without static inefficiency; ii) when $\gamma < \gamma^\circ(\hat{s})$ a judge has more incentives to exert effort relative to a situation without static inefficiency.*

Thus, as shown in Figure 6, whether a judge exerts more or less effort relative to the model without deadweight losses depends on the values of γ and c. As in the baseline case, provided c is not too large, the judge exerts effort if and only if γ is low enough. For high values of the enforcement cost, such as c_a, the range of values of γ leading to maximum enforcement effort is wider in the situation with deadweight losses. But when the cost is low enough, such as c_b, the result is reversed and, in the presence of deadweight losses, maximum effort prevails over a narrower set of values of γ.

From here, the final characterization of the possible outcomes of judges' enforcement game would be analogous to that in Proposition 3. Relative to the model without deadweight losses, the ranges of values of γ over which an equilibrium with high enforcement effort can be sustained expand with c.

5.2 Continuous Enforcement Effort

From Section 4.2 onwards, we streamlined the analysis by focusing on the case in which judge enforcement effort can take only two values, $s \in \{0, 1\}$. In this section we explore the case where effort is continuous, $s \in [0, 1]$. As expected, the main results carry through. To ease the exposition, we assume that the cost of exerting effort is quadratic. In particular $c(s) = \tilde{c}s^2/2$ where $\tilde{c} > 0$ is a scale parameter. In line with Assumption 2 above, in this section we focus on the case in which effort is interior, that is, $\tilde{c} > \Pi/2$. We provide details for this case and for the case where $\tilde{c} \leq \Pi/2$ in Appendix B.2.

For a given enforcement effort by other judges $\hat{s}$ and screening rate by the patent office, γ, a judge's best response is unique an given by

$$s(\hat{s}, \gamma) = \Pi(1 - \gamma)\Phi(\hat{s}, \gamma)/2\tilde{c} \in (0, 1)$$

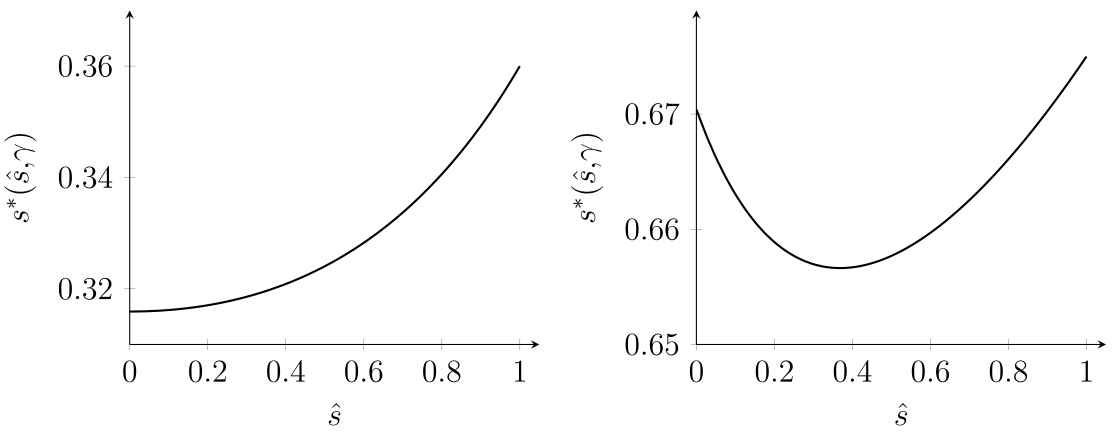
where $\Phi(\hat{s}, \gamma)$ is the function defined in Lemma 5. It also follows from this lemma that the best response of an individual judge, $s(\hat{s}, \gamma)$, increases in the quality of the screening of the patent office (decreases in γ). That is, ex-ante patent screening remains complementary to ex-post enforcement under continuous enforcement effort.

Proposition 7 (Revisiting judges' effort complementarity). *In the continuous enforcement effort case, a judge's best response is increasing in the enforcement effort of other judges $\hat{s}$ if and only if*

$$\Pi > \frac{3 - \hat{s} - 2\gamma}{\alpha(1 + \hat{s}(1 - 2\gamma))} \tag{20}$$

This condition always holds in a neighborhood of $\hat{s} = 1$ or $\gamma = 1$.

Proposition 7 shows that, unlike in the binary effort scenario, where judge enforcement effort decisions are complementary, here judges' enforcement efforts can be strategic complements or strategic substitutes. An increase in other judges' effort $\hat{s}$ induces two opposing effects. As in the baseline model (see Lemma 2), the proportion of monopolized niches increases, making the frequency of type I errors higher, relative to type II



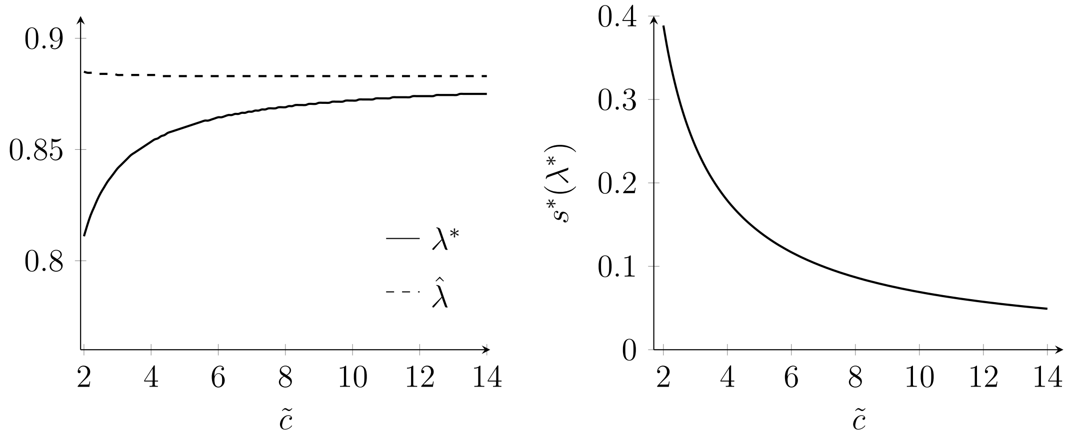
(a) Strategic complements ($\gamma = 0.6$) **(b)** Substitutes and complements ($\gamma = 0.25$)

Note: Parameter values are $\alpha = 0.1$, $\beta = 0.8$, $\pi = 3.6$, and $\tilde{c} = 14$.

Figure 7: Complementarity in the continuous enforcement effort case. Panels show the best response of an individual judge to other judges' enforcement effort $\hat{s}$ for two different levels of the screening by the patent office, as (inversely) measured by γ. In panel (a) judges' efforts are strategic complements, while in panel (b) they can be strategic substitutes or complements depending on the size of $\hat{s}$.

errors. This effect increases the factor $\Phi(\hat{s},\gamma)$, calling for more effort, that is, pushing for strategic complementarity. On the one hand, in the continuous effort scenario a new free-riding effect arises. Better (but far from perfect) rulings by other judges increase the probability of future successful entry, $p(\hat{s},\gamma)$. The prospect of higher entry increases the benefits of a type II error, decreasing $\Phi(\hat{s},\gamma)$ (see Lemma 5) and, through it, the effort of an individual judge's best response. This effect (which vanishes in the proximity of $\hat{s} = 1$) pushes towards strategic substitutability.

The necessary and sufficient condition (20) tells us that strategic complementarity tends to occur when incumbency profits are sufficiently high. The condition becomes weaker when the patent office screens less (higher γ) and when other judges exert more effort (they are already making good rulings). Consistent with the binary effort case, regardless of the parameters of the model, strategic complementarity always occurs when other judges' enforcement effort is high enough. Complementarity also occurs when the patent office screens little and/or most innovations are obvious (that is, when γ is high); see Figure 7a. In contrast, as depicted in Figure 7b, substitutability can emerge under



(a) Optimal screening **(b)** Optimal enforcement

Note: parameter values are $\alpha = 0.1$, $\beta = 0.8$, and $\pi = 3.6$.

Figure 8: Optimal screening and induced level of enforcement in continuous effort case. Panel (a) depicts optimal ex-ante screening when enforcement effort is endogenous and exogenous (λ^* and $\hat{\lambda}$, respectively), as a function of the enforcement cost $\tilde{c}$. Panel (b) shows the enforcement effort $s^*(\lambda^*)$ induced by the optimal ex-ante screening quality λ^*, also as a function of $\tilde{c}$.

higher ex-ante screening.

In Appendix B we show that the multiplicity of equilibria in the enforcement game is not present when judges' enforcement effort is continuous. That is, there is a unique symmetric steady-state equilibrium with an effort level s^* satisfying $s(s^*, \gamma) = s^*$. This means that, with continuous enforcement efforts, the judges' enforcement game no longer entails a coordination problem.[21]

To conclude this section we numerically explore the socially optimal screening (see problem (19)) under continuous enforcement effort. Figure 8 shows a case in which, consistent with the complementarity discussed in Proposition 5, an increase in the enforcement costs decreases both judges' enforcement effort (panel (b)) and patent office's ex-ante screening (panel (a)). That is, despite the substitution effect calling for better screening when the cost of enforcement goes up, the impact of a decrease in enforcement

[21]This suggests that the coordination problem in the binary efforts case is related to the manner in which the prospects of perfect enforcement by subsequent judges ($\hat{s} = 1$) fully removes the social value of type II error, reinforcing the incentive of an individual judge to choose $s = 1$. For $\hat{s} < 1$, type II error by an individual judge is still valuable at the margin.

effort dominates, inducing less screening in equilibrium.

To illustrate this complementarity in a different way, we also compare the socially optimal ex-ante screening behavior λ^*, just described, with what would be the optimal screening in the hypothetical case in which court behavior remains exogenously fixed at the equilibrium value induced by λ^*, $s^*(\lambda^*)$. We call the optimal screening rate in the hypothetical scenario $\hat{\lambda}$. As shown in Figure 8a, when courts' endogenous responses are taken into account, it is socially efficient to screen more (to set a lower λ): the possibility to affect court's behavior, induces more screening in equilibrium.

6 Concluding Remarks

Innovation is considered key to industry dynamics. Entry, exit, and innovation are complex interrelated phenomena in every industry, and especially so in those that are youngest and more technology-intensive. Many of these industries rely on IP as the source of temporary monopoly power that allows the successful innovators to obtain a return from their previous R&D investments. IP protection, however, is a double-edged sword for the dynamics of innovative industries, as the protection of incumbent innovators may be an obstacle to the success of genuine innovators.

This paper contributes to the growing literature that analyzes the role of IP protection by embedding it in an industry-dynamics framework in which the value of innovation is stochastic and new firms replaced existing incumbents. We find that innovation and welfare are maximized when incumbents receive maximal protection against small improvements but as little as possible against large innovations. However, if incumbents receive some protection against large innovation, allowing for some small improvements to succeed in the market may be socially beneficial.

We also delve into the interaction between the mechanisms that implement the level of protection of incumbents against each kind of innovation, namely the ex-ante screening of the patent office and the ex-post enforcement by courts. The patent office can expend resources in trying to screen out small innovations before their very entry. Courts enter the game once an incumbent claims that its IP has been infringed by an entrant and can at that stage invest resources to improve the quality of their rulings. We show that courts provide better enforcement the more the patent office engages in the ex-ante screening. This complementarity shapes the choice of the socially optimal level of screening by the patent office.

Our model focuses on the interaction between the patent office's ex-ante screening and courts' enforcement of intellectual property but remains silent about other dimensions of patent policy. Future work could extend this type of analysis to cover aspects such as patent breadth or patentability standards, which until now have been studied in the literature without accounting for judges' endogenous enforcement decisions.

References

Aghion, Philippe and Peter Howitt, "A Model of Growth through Creative Destruction," *Econometrica*, 1992, *60* (2), pp. 323–351.

Allison, John R. and Mark A. Lemley, "Empirical evidence on the validity of litigated patents," *AIPLA Quarterly Journal*, 1998, *26* (3), p. 185.

Bessen, James and Eric Maskin, "Sequential Innovation, Patents, and Imitation," *Rand Journal of Economics*, 2009, *40* (4), pp. 611–635.

Daughety, Andrew F. and Jennifer F. Reinganum, "Keeping Society in the Dark: On the Admissibility of Pretrial Negotiations as Evidence in Court," *Rand Journal of Economics*, 1995, *26* (2), pp. 203–221.

Daughety, Andrew F and Jennifer F Reinganum, "Appealing judgments," *Rand Journal of Economics*, 2000, *31* (3), pp. 502–525.

—, "Informal sanctions on prosecutors and defendants and the disposition of criminal cases," *Journal of Law, Economics, and Organization*, 2016, *32* (2), pp. 359–394.

—, "Reducing Unjust Convictions: Plea Bargaining, Trial, and Evidence Disclosure," *Journal of Law, Economics, and Organization*, 2020, *36* (2), pp. 378–414.

Denicolò, Vincenzo, "Two-Stage Patent Races and Patent Policy," *Rand Journal of Economics*, 2000, *31* (3), pp. 488–501.

Denicolò, Vincenzo and Piercarlo Zanchettin, "How should forward patent protection be provided?" *International Journal of Industrial Organization*, 2002, *20* (6), pp. 801–827.

Denicolò, Vincenzo and Piercarlo Zanchettin, "What Causes Over-investment in R&D in Endogenous Growth Models?" *Economic Journal*, 2014, *124* (581), pp. 1192–1212.

Farrell, Joseph and Robert P. Merges, "Incentives to challenge and defend patents: Why litigation won't help reliably fix patent office errors and administrative patent review might help," *Berkeley Technology Law Journal*, 2004, *19* (3), pp. 943–970.

Farrell, Joseph and Carl Shapiro, "How Strong Are Weak Patents," *American Economic Review*, 2008, *98* (4), pp. 1347–1369.

Frakes, Michael D. and Melissa F. Wasserman, “Is the Time Allocated to Review Patent Applications Inducing Examiners to Grant Invalid Patents? Evidence from Microlevel Application Data,” *Review of Economics and Statistics*, 2017, *99*(3), pp. 550–563.

—, “Irrational Ignorance at the Patent Office,” *Vanderbilt Law Review*, 2019, *72*(3), pp. 975–1030.

Galasso, Alberto and Mark Schankerman, “Patents and cumulative innovation: Causal evidence from the courts,” *Quarterly Journal of Economics*, 2015, *130*(1), pp. 317–369.

Gilbert, Richard J and David M G Newbery, “Preemptive Patenting and the Persistence of Monopoly,” *American Economic Review*, 1982, *72*(3), pp. 514–526.

Green, Jerry R. and Suzanne Scotchmer, “On the Division of Profit in Sequential Innovation,” *Rand Journal of Economics*, 1995, *26*(1), pp. 20–33.

Grossman, Gene M and Elhanan Helpman, “Quality Ladders in the Theory of Growth,” *Review of Economic Studies*, 1991, *58*(1), pp. 43–61.

Grossman, Gene M and Michael L Katz, “Plea bargaining and social welfare,” *American Economic Review*, 1983, *73*(4), pp. 749–757.

Heller, Michael A. and Rebecca S. Eisenberg, “Can Patents Deter Innovation? The Anticommons in Biomedical Research,” *Science*, 1998, *280*, pp. 698–701.

Hopenhayn, Hugo, Gerard Llobet and Matthew Mitchell, “Rewarding Sequential Innovations: Prizes, Patents and Buyouts,” *Journal of Political Economy*, 2006, *104*(6), pp. 1041–1068.

Landeo, Claudia M, Maxim Nikitin and Scott Baker, “Deterrence, lawsuits, and litigation outcomes under court errors,” *Journal of Law, Economics, & Organization*, 2007, *23*(1), pp. 57–97.

Lee, Tom and Louis L Wilde, “Market Structure and Innovation: A Reformulation,” *Quarterly Journal of Economics*, 1980, *94*(2), pp. 429–36.

Lei, Zhen and Brian D. Wright, “Why weak patents? Testing the examiner ignorance hypothesis,” *Journal of Public Economics*, 2017, *148*, pp. 43–56.

Lemley, Mark A and Bhaven Sampat, "Is the patent office a rubber stamp," *Emory Law Journal*, 2008, *58*, p. 181.

Lemley, Mark A and Carl Shapiro, "Probabilistic patents," *Journal of Economic Perspectives*, 2005, *19* (2), pp. 75–98.

Lemley, Mark D., "Rational Ignorance at the Patent Office," *Northwestern University Law Review*, 2001, *95* (1495-1532).

—, "Ignoring Patents," *Michigan State Law Review*, 2008, pp. 19–34.

Loury, Glenn C, "Market Structure and Innovation," *Quarterly Journal of Economics*, 1979, *93* (3), pp. 395–410.

Marshall, Guillermo and Álvaro Parra, "Innovation and competition: The role of the product market," *International Journal of Industrial Organization*, 2019, *65*, pp. 221–247.

O'Donoghue, Ted, "A Patentability Requirement for Sequential Innovation," *Rand Journal of Economics*, 1998, *29* (4), pp. pp. 654–679.

O'Donoghue, Ted, Suzanne Scotchmer and Jacques-François Thisse, "Patent Breadth, Patent Life, and the Pace of Technological Progress," *Journal of Economics & Management Strategy*, 1998, *7* (1), pp. 1–32.

Parra, Álvaro, "Sequential innovation, patent policy, and the dynamics of the replacement effect," *Rand Journal of Economics*, 2019, *50* (3), pp. 568–590.

Reinganum, Jennifer F, "Plea bargaining and prosecutorial discretion," *American Economic Review*, 1988, pp. 713–728.

Schankerman, Mark and Florian Schuett, "Patent Screening, Innovation, and Welfare," 2020, mimeo.

Scotchmer, Suzanne, *Innovation and Incentives*, MIT Press, Cambridge, MA, 2004.

Scotchmer, Suzanne and Jerry Green, "Novelty and Disclosure in Patent Law," *Rand Journal of Economics*, 1990, *21* (1), pp. pp. 131–146.

Segal, Ilya and Michael D. Whinston, "Antitrust in Innovative Industries," *American Economic Review*, 2007, *97* (5), pp. 1703–1730.

Spier, Kathryn E, "Pretrial bargaining and the design of fee-shifting rules," *Rand Journal of Economics*, 1994, pp. 197–214.

Spier, Kathryn E., "Litigation," in A. Mitchell Polinsky and Steven Shavell, eds., *Handbook of Law and Economics*, volume 1, Elsevier, 2007 pp. 259–342.

Appendix

A Proofs

Proof of Lemma 1. To show existence we start by imposing the steady condition $x_t = x$ for all t in eq. (3), obtaining eq. (6). Replacing x into (4) we obtain eq. (7). Using the free-entry condition (5), we obtain $v = p^{-1}$. Finally, using condition (5) again and in the steady state of (2), we find the expression for the entry flow (8). Because all these equations are linear, we have a unique solution.

To determine when the entry flow is positive, observe that (8) is positive if and only if $\pi > \underline{\pi}$ and lower than one if and only if $\pi < \underline{\pi} + \beta[\alpha + (1-\alpha)\lambda\mu_0]/\alpha$. ■

Proof of Lemmas 2 and 3. Most comparative statics are direct, as they follow from direct differentiation. The derivative of the entry flow with respect to $\lambda\mu_0$ is

$$\frac{de}{d\left(\lambda\mu_0\right)} = \frac{\left(1-\alpha\right)\left(1-\beta\right)}{\beta\left(\alpha\mu_1 + \left(1-\alpha\right)\lambda\mu_0\right)^2}\left[1 - \Pi\frac{p^2}{\alpha}\right] \tag{21}$$

where $\Pi = \pi/(1-\beta)$ is the present social value of a genuine innovation. The sign of the derivative is given by the sign of the term in square brackets. This term is a monotonically decreasing function of Π. To show that the derivative can be positive for any value of $\lambda\mu_0$, take the maximum value of Π for which there is no entry; i.e., take $\Pi = \underline{\pi}/(1-\beta) \equiv p^{-1}$. In this case, the bracket term becomes $1 - p/\alpha$, which is positive whenever $\mu_1 < 1$, as $p < \alpha$. By continuity, the results holds for values of $\Pi > p^{-1}$ for which there is entry.

To show that the derivative is negative when $\mu_1 = 1$, observe that in this case $p = \alpha$ and the parenthesis becomes $1 - \Pi\alpha$. In this scenario, Assumption 1 is equivalent to $\Pi > p^{-1} = \alpha^{-1}$ and the derivative is negative for any value of $\lambda\mu_0$. Finally, an example of an inverted-U shape relation is given in the main text. ■

Proof of Proposition 1. For an interior solution of $\hat{\lambda}$ we compute the derivative (11) under the assumption $\kappa(\lambda) = 0$ and solve for the values of λ such that (11) is equal to zero. Because the first order condition corresponds to a third degree polynomial, we obtain three candidate solution. Call them $\{0, -, +\}$. The first solution,

$$\lambda_0 = \frac{-\alpha\left(\Pi\alpha\mu_1 - 1\right)}{\mu_0\left(1-\alpha\right)\left(\Pi\alpha - 1\right)} < 0$$

where $\Pi = \pi/(1-\beta)$, corresponds to a value for which the welfare function is zero ($W(\lambda_0) = 0$). This occurs as $e(\lambda_0) = 0$ and $p(\lambda_0) = \Pi^{-1}$ hold. Using these facts, we show that the second order condition at λ_0 is positive. Start by observing

$$\frac{d^2W}{d\lambda^2} = \frac{d^2e}{d\lambda^2}\left(\Pi p - 1\right) + 2\frac{de}{d\lambda}\frac{dp}{d\lambda}\Pi + e\frac{d^2p}{d\lambda^2}\Pi = 2\frac{de}{d\lambda}\frac{dp}{d\lambda}\Pi$$

Proposition 2 implies that $dp/d\lambda > 0$. Similarly, using $p(\lambda_0) = \Pi^{-1}$ and Assumption 1 in (21) we can verify $de/d\lambda > 0$. Therefore, the critical point corresponding to a (local) minimum and a feasible maximum (i.e., with $\hat{\lambda} \in [0,1]$) must lie to the right of λ_0.

The second and third solutions are given by

$$\lambda_{-,+} = \lambda_0 + \frac{\alpha\left(1-\mu_1\right)\left(\Pi\alpha \pm \sqrt{\alpha\Pi\left(\Pi\alpha + 8\right)}\right)}{2\mu_0\left(1-\alpha\right)\left(\Pi\alpha - 1\right)}$$

Given Assumption 1, the denominator above is positive and the sign of the fraction depends on solution. For the negative solution the numerator is negative, and positive for the positive solution. Therefore, we have that $\lambda_- < \lambda_0 < \lambda_+$. Thus, given the analysis above, λ_+ is the only feasible solution. It can be readily verified, through differentiation, that λ_+ decreases in Π and that it is less than one whenever

$$\Pi \leq K \equiv \frac{(\alpha + (1-\alpha)\mu_0)^2}{\alpha(\alpha\mu_1 + (1-\alpha)\mu_0)(\alpha(2\mu_1 - 1) + (1-\alpha)\mu_0)}.$$

Finally, it can be verified that λ_+ decreases in μ_1 and $\lambda_+ < 0$ when $\mu_1 = 1$. Consequently, there exists $\hat{\mu}_1$ such that, $\mu_1 > \hat{\mu}_1$ implies that $\hat{\lambda} = 0$, which proves the result. ■

Proof of Lemma 4. Subtracting the value functions w_M and w_C we obtain

$$w_M - w_C = -e\alpha\Pi\,(1-\mu_1) + [1 - e\,(\alpha + (1-\alpha)\lambda\mu_0)]\,\beta(w_M - w_C),$$

Solving for $w_M - w_C$ delivers (14). ■

Proof of Lemma 5. Fix a triplet $(s,\,\hat{s},\,\gamma)$. Using $E_I = \Pi$, E_{II} given by equation (14), and equation (1) we can rewrite $J\,(s, \hat{s}, \gamma)$ as:

$$\frac{\Pi}{2}\,(1-\gamma)\,(1-s)\underbrace{\left(1 - \frac{\gamma\alpha\beta\,(1-\mu_1\,(\hat{s}))\,e\,(\hat{s},\gamma)}{(1-\beta)\,(1-\gamma) + \beta\alpha\,((1-\gamma) + \gamma\mu_0\,(\hat{s}))\,e\,(\hat{s},\gamma)}\right)}_{\Phi(\hat{s},\gamma)} + c\,(s)\,.$$

Working the parenthesis and using that $\mu_0(s)+\mu_1(s) = 1$ for all s, we obtain the expression in (18). Differentiating $\Phi(\hat{s},\gamma)$ with respect to $e\,(\hat{s},\gamma)$ we obtain

$$\frac{d\Phi(\hat{s},\gamma)}{de} = -\frac{\alpha\beta\,(1-\beta)\,(1-\gamma)\,\gamma\mu_0(\hat{s})}{[(1-\beta)\,(1-\gamma) + \beta\alpha\,((1-\gamma)+\gamma\hat{\mu}_0)\,e\,(\hat{s},\gamma)]^2} < 0.$$

Because it will be useful for the comparative statics and to prove other propositions, we replace $e\,(\hat{s},\gamma)$ (see equation (8)) into $\Phi(\hat{s},\gamma)$ to obtain

$$\Phi\,(\hat{s},\gamma) = \frac{(1-\gamma)\,[\Pi\alpha k_1 - k_2] + k_1 k_2}{k_2\,(\Pi\alpha k_1 - (1-\mu_1(\hat{s}))\,(1-\gamma))} \tag{22}$$

where $k_1 = (1-\gamma)\,\mu_1(\hat{s}) + \gamma\mu_0(\hat{s})$ and $k_2 = 1 - \gamma + \gamma\mu_0(\hat{s})$ are functions of $\hat{s}$ and γ. We omit this dependence for parsimonious notation. Observe that, for a given γ, α and Π are always together in (22). Thus,

$$\frac{d\Phi(\hat{s},\gamma)}{d\Pi\alpha} = -\frac{\gamma\mu_0(\hat{s})k_1^2}{k_2(\Pi\alpha k_1 - (1-\mu_1(\hat{s}))(1-\gamma))^2} < 0$$

which proves the result for α and Π. Differentiating (22) with respect to γ we obtain

$$\frac{d\Phi\,(\hat{s},\gamma)}{d\gamma} = -\mu_0\frac{\Pi\alpha k_1\,(\Pi\alpha k_1 - [k_2 + (1-\mu_1(\hat{s}))\,(1-\gamma)]) + k_2^2(1-\mu_1(\hat{s}))}{k_2^2\,(\Pi\alpha k_1 - (1-\mu_1(\hat{s}))\,(1-\gamma))^2}.$$

Observe that the denominator is always positive. Assumption 1 implies $\Pi > k_2/(\alpha k_1) = p^{-1}$. Because the numerator is increasing in Π, we can replace the minimal feasible profit in the numerator to obtain a upper bound for the derivative

$$\frac{d\Phi\,(\gamma,\hat{s})}{d\gamma} < -\frac{\gamma\mu_0^2\,(1-\mu_1(\hat{s}))}{k_2\,(\Pi\alpha k_1 - (1-\mu_1(\hat{s}))\,(1-\gamma))^2} \leq 0$$

which proves the last comparative statics result. ■

Proof of Proposition 2. To show existence of $\gamma^*(\hat{s})$ simply observe that, when $\gamma = 0$, $J(0,\hat{s},0) = \Pi/2$. Also, when $\gamma = 1$, $J(0,\hat{s},1) = 0$. Because J is continuous in γ, if $c < \Pi/2$, Intermediate Value Theorem implies there exists $\gamma^*(\hat{s}) \in (0,1)$ such that $J(0,\hat{s},\gamma^*(\hat{s})) = c$. To prove that $\gamma^*(\hat{s})$ is unique, we show that $J(0,\hat{s},\gamma)$ is decreasing in γ. Computing the derivative of $J(0,\hat{s},\gamma)$ with respect to γ we obtain

$$\frac{dJ(0,\hat{s},\gamma)}{d\gamma} = \frac{\Pi}{2}\left(-\Phi(\hat{s},\gamma) + (1-\gamma)\frac{d\Phi(\hat{s},\gamma)}{d\gamma}\right),$$

which, by Lemma 5, is negative. Thus, $dJ/d\gamma < 0$ and uniqueness follows. If $c \geq \Pi/2$, J decreasing in γ implies that the cost of effort is always higher than no exerting effort. ■

Proof of Lemma 6. It can be readily verified that, when $\hat{s} = 1$, $\Phi(1,\gamma) = 1$. Consequently, solving for $J(0,1,\gamma) = c$ we obtain that $\gamma^*(1) = (\Pi - 2c)/\Pi$. Also, when $\hat{s} = 0$, $\Phi(0,\gamma) < 1$ for all $\gamma > 0$. Thus, $J(0,1,\gamma) > J(0,0,\gamma)$ for all $\gamma \in (0,1)$. Starting from $\gamma^*(0) \in (0,1)$. The previous observation implies $c = J(0,0,\gamma^*(0)) < J(0,1,\gamma^*(0))$. Then, because J is decreasing in γ (see Proposition 2), $\gamma^*(0) < \gamma^*(1)$. ■

Proof or Proposition 3. The characterization of the equilibrium is immediate from Lemma 6 and the discussion in the text. Regarding the equilibrium entry, observe

$$e(0,\gamma) = \frac{2(1-\gamma)(1-\beta)}{\alpha\beta}\left(\frac{\Pi\alpha}{2-\gamma} - 1\right) \qquad e(1,\gamma) = \frac{1-\beta}{\alpha\beta}(\Pi\alpha - 1).$$

If $\Pi \leq (2-\gamma)/\alpha$, then $e(0,\gamma) = 0$ and $e(1,\gamma) \geq 0$. If $\Pi > (2-\gamma)/\alpha$ then $e(0,\gamma) > 0$ and $e(1) > e(0)$ if and only if

$$\Pi > \frac{(2-\gamma)}{\alpha}\frac{(2\gamma-1)}{\gamma}.$$

Since $(2\gamma-1)/\gamma < 1$ for $\gamma < 1$ and, in equilibrium, $\gamma^*(\hat{s}) < 1$, the result holds. ■

Proof of Proposition 4. As explained in the main text the result arises from the complementarity of the decisions of individual judges. Hence, to conduct the comparative statics exercise we only need to show that the effort level of a judge is monotonically decreasing in c and α and increasing in Π. The effect c is immediate, higher effort costs decreases, on the margin, a judge's effort. Regarding the effects of an increase in Π and α, we need to compute the sign of

$$\frac{dJ}{d\Pi} = \frac{1-\gamma}{2}\left(\Phi + \Pi\frac{d\Phi}{d\Pi}\right) \text{ and } \frac{dJ}{d\alpha} = \frac{\Pi(1-\gamma)}{2}\frac{\partial\Phi}{\partial\alpha},$$

respectively. From Lemma 5 we know that $\Phi(\hat{s},\gamma)$ is decreasing in α for all $\hat{s}$ which is enough to show the effect of this parameter. For Π we have that

$$\frac{dJ}{d\Pi} = \frac{1-\gamma}{2}\frac{\left[(1-\gamma)(\Pi\alpha k_1 - k_2) + k_1k_2\right]\left[\alpha\Pi k_1 - (1-\mu_1(\hat{s}))(1-\gamma)\right] - \gamma\mu_0(\hat{s})\Pi\alpha k_1^2}{k_2(\Pi\alpha k_1 - (1-\mu_1(\hat{s}))(1-\gamma))^2},$$

where k_1 and k_2 are defined in Lemma 5. We show that a lower bound of this derivative is positive. Notice that positive entry (assumption 1) implies that $\Pi > k_2/(\alpha k_1)$. As a result both $\Pi\alpha k_1 - k_2 > 0$ and $\Pi\alpha k_1 - (1-\mu_1(\hat{s}))(1-\gamma) > 0$. This means that a lower bound for the first square bracket is $k_1 k_2$. Substituting and rearranging we obtain

$$\frac{dJ}{d\Pi} > \frac{(1-\gamma)\left[\Pi\alpha k_1^2 - (1-\mu_1(\hat{s}))k_1k_2\right]}{k_2(\Pi\alpha k_1 - (1-\mu_1(\hat{s}))(1-\gamma))^2} > \frac{(1-\gamma)\mu_1(\hat{s})k_1k_2}{k_2(\Pi\alpha k_1 - (1-\mu_1(\hat{s}))(1-\gamma))^2} > 0,$$

where we used $k_2/(\alpha k_1)$ as a lower bound for Π, proving the result. ■

Proof of Proposition 5. For a given c, we want to solve $\lambda^*(c) \in \arg\max W^{sp}(\lambda, c)$ where

$$W^{sp}(\lambda, c) = \begin{cases} W(\lambda; 1, c) & \text{if } \lambda \le \lambda_J(c) \\ W(\lambda; 0) & \text{if } \lambda > \lambda_J(c) \end{cases} \quad \text{and} \quad \lambda_J(c) = \frac{\alpha}{1-\alpha}\frac{\Pi - 2c}{2c} \tag{23}$$

is the minimum screening quality that induces enforcement effort by the judges; $\lambda_J(c)$ comes from replacing $\gamma^*(1) = (\Pi - 2c)/\Pi$ into equation (15). This function is decreasing in c, takes the value of zero when $c = \Pi/2$ and diverges when the enforcement cost approaches zero (see Figure 9).

We need to find $\lambda^*(c)$ and study how it changes with the cost of screening c. To start, we characterize the welfare function under no enforcement effort.

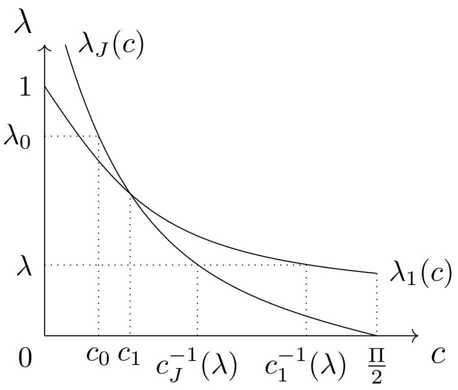


Figure 9: Enforcement effort constraint.

Lemma A.1. *The function $W(\lambda; 0)$ is single peaked when $\lambda \in [0, 1]$.*

Proof. Recall $W(\lambda; 0) = e(\lambda)[\Pi p(\lambda) - 1 - \kappa(\lambda)]$ and observe that $W(1; 0) > 0$ and $\lim_{\lambda\to 0} W(\lambda, 0) = -\infty$. We show that there is a unique $\lambda_0 \ge 0$ in the $\arg\max W(\lambda; 0)$ satisfying $W(\lambda_0; 0) > 0$. The first order condition for an interior maximum is given by

$$0 = e'(\lambda_0)\left(\Pi p(\lambda_0) - 1 - \kappa(\lambda_0)\right) + e(\lambda_0)\left(\Pi p'(\lambda_0) - \kappa'(\lambda_0)\right).$$

Observe that $e(\lambda)$ and $\Pi p'(\lambda) - \kappa'(\lambda) > 0$ for all λ . Also, because $W(0; 0) > 0$, $\Pi p(0) - 1 - \kappa(0) > 0$ and, consequently, $\Pi p(\lambda_0) - 1 - \kappa(\lambda_0) > 0$ is necesary to have positive welfare. Thus, $e'(\lambda_0) < 0$. The second order condition is given by

$$e''(\lambda_0)\left(\Pi p(\lambda_0) - 1 - \kappa(\lambda_0)\right) + 2e'(\lambda_0)\left(\Pi p'(\lambda_0) - \kappa'(\lambda_0)\right) + e(\lambda_0)\left(\Pi p''(\lambda_0) - \kappa''(\lambda_0)\right).$$

Substituting in $\Pi p(\lambda_0) - 1 - \kappa(\lambda_0) = e(\lambda_0)\left(\Pi p'(\lambda_0) - \kappa'(\lambda_0)\right)/(-e'(\lambda_0))$ from the first order condition, we obtain

$$\left(e''(\lambda_0)e(\lambda_0) - 2e''(\lambda_0)^2\right)\left(\Pi p'(\lambda_0) - \kappa'(\lambda_0)\right)/(-e'(\lambda_0)) + e(\lambda_0)\left(\Pi p''(\lambda_0) - \kappa''(\lambda_0)\right),$$

which is negative, as $\Pi p''(\lambda) - \kappa''(\lambda) < 0$ and $e''(\lambda)e(\lambda) - 2e''(\lambda)^2 < 0$ for $\lambda > 0$. Therefore, every point satisfying the FOC is a maximum and, at most, one maximum exist. Thus, the function is single peaked in $\lambda \in [0, 1]$ (where λ_0 is either interior or 1). ■

We now turn to examine the scenario when judges exert full enforcement effort; that is, we study the $\arg\max W(\lambda; 1, c)$, which has a unique solution, called $\lambda_1(c)$, implicitly given by the first order condition $\kappa'(\lambda) = (1-\alpha)c$. Given the Inada conditions for $\kappa(\lambda)$, the function $\lambda_1(c)$ is decreasing in c, takes the value of 1 when $c = 0$ and approaches zero from above as c goes to infinity.[22] This implies that the functions $\lambda_1(c)$ and $\lambda_J(c)$ cross at least once.

Lemma A.2. *If $\kappa(\lambda) = (1-\lambda)^2/\lambda$, then $\lambda_1(c) = 1/\sqrt{1 + c(1-\alpha)}$ and the functions $\lambda_1(c)$ and $\lambda_J(c)$ single cross.*

Proof. The functional form for $\lambda_1(c)$ follows from the first order condition. The problem $\lambda_1(c) = \lambda_J(c)$ is cubic in c. Thus, it has three solutions, two of which are imaginary. Consequently, the functions cross only once in the real plane. ■

We define c_1 to be the unique cost satisfying $\lambda_1(c_1) = \lambda_J(c_1)$. The previous analysis implies that $\lambda_1(c) < \lambda_J(c)$ for all $c < c_1$. Similarly, define c_0 to be the unique cost satisfying $\lambda_J(c_0) = \lambda_0$. The cost c_0 can be above or below c_1 (see Figure 9).

Lemma A.3. *The function $W(\lambda_1(c); 1, c)$ is strictly decreasing in c and $W(\lambda_J(c); 1, c)$ is strictly decreasing in c whenever $c \geq c_1$.*

Proof. For $W(\lambda_1(c); 1, c)$, using the Envelope Theorem, we have

$$\frac{dW}{dc} = -(\alpha + (1-\alpha)\lambda_1(c)) < 0.$$

For $W(\lambda_J(c); 1, c)$, differentiate with respect c to obtain

$$\frac{dW}{dc} = -\left(\kappa'(\lambda_J(c)) + c(1-\alpha)\right)\frac{d\lambda_J(c)}{dc} - (\alpha + (1-\alpha)\lambda_J(c))$$

Because $c \geq c_1$, we know $\lambda_J(c) \leq \lambda_1(c)$ and, consequently, $0 < -\kappa'(\lambda_1(c)) \leq -\kappa'(\lambda_J(c))$. Then, $0 = -\left(\kappa'(\lambda_1(c)) + c(1-\alpha)\right) \leq -\left(\kappa'(\lambda_J(c)) + c(1-\alpha)\right)$ and the result follows from observing $d\lambda_J/dc < 0$. ■

Lemma A.4. *There is a unique cost $c_{01} > 0$ such that $W(\lambda_1(c_{01}); 1, c_{01}) = W(\lambda_0; 0)$. Consequently, $W(\lambda_1(c); 1, c) > W(\lambda_0; 0)$ for $c < c_{01}$ and $W(\lambda_1(c); 1, c) < W(\lambda_0; 0)$ if $c > c_{01}$.*

Proof. It can be readily verified that, when $c = 0$, $W(\lambda_1(0); 1, 0) > W(\lambda_0; 0)$. Then, because the function $W(\lambda_1(c); 1, c)$ is strictly decreasing in c and $\lim_{c\to\infty} W(\lambda_1(c); 1, c) = -\infty$, there is a unique value of c, call it c_{01}, such that $W(\lambda_1(c_{01}); 1, c_{01}) = W(\lambda_0; 0)$. ■

Figure 10 shows, in solid lines, $W(\lambda_1(c); 1, c)$ when c is 0 and c_{01}, and $W(\lambda, 0)$. The values of λ_0 and $\lambda_1(c_{01})$ are also depicted. Define $c_1^{-1}(\lambda)$ to be the inverse of $\lambda_1(c)$. The dashed line going through the points A and B depicts the pairs $(\lambda, W(\lambda; 1, c_1^{-1}(\lambda)))$; that is, the line shows the maximum welfare attainable when judges exert full effort and the screening costs $c_1^{-1}(\lambda)$ is such that λ is the optimal screening quality. It is clear that, without the constraint of $\lambda_J(c)$, the optimal screening quality $\lambda^*(c)$ would be equal to $\lambda_1(c)$ for $c \leq c_{01}$ and equal to λ_0 otherwise. To solve for the socially optimal screening rate we need to study three cases: whether $\lambda_J(c_{01})$ is below, in, or above the interval $[\lambda_1(c_{01}), \lambda_0]$ (which is non-empty by assumption).

[22] Using implicit differentiation we obtain $\lambda_1'(c) = -(1-\alpha)/\kappa''(\lambda_1(c)) < 0$.

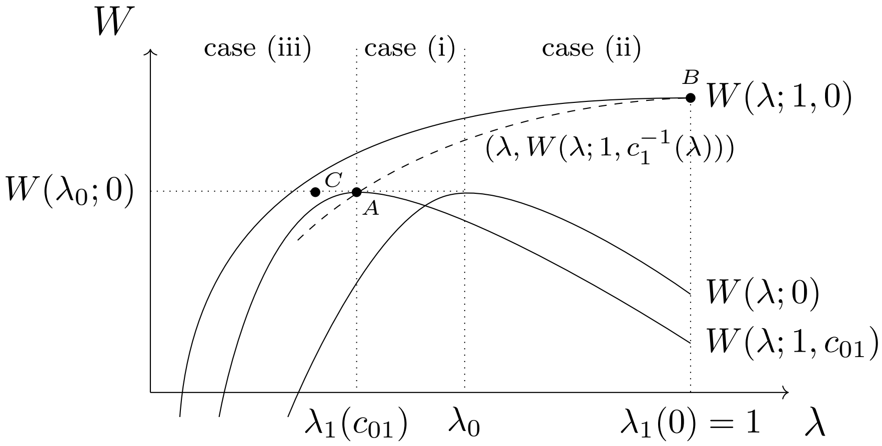


Figure 10: $W(\lambda;0)$ and $W(\lambda;1,c)$ as functions of λ.

Case (i) $\lambda_J(c_{01}) \in [\lambda_1(c_{01}), \lambda_0]$: Because $\lambda_J(c_{01}) \geq \lambda_1(c_{01})$, we know by Lemma A.2 that $\lambda_J(c) > \lambda_1(c)$ for all $c < c_{01}$. That is, $\lambda_J(c)$ is not binding for $\lambda_1(c)$ in this region and $\lambda^*(c) = \lambda_1(c)$ for all $c \leq c_{01}$. Also, as $\lambda_J(c_{01}) \leq \lambda_0$ and $\lambda_J(c)$ being decreasing in c, we know $\lambda_J(c)$ is not binding for λ_0 for $c < c_{01}$ and $\lambda^*(c) = \lambda_0$ in this region. Consequently, the optimal screening is quality is:

$$\lambda^*(c) = \begin{cases} \lambda_1(c) & \text{if } c \leq c_{10} \\ \lambda_0 & \text{if } c > c_{10} \end{cases}. \tag{24}$$

Because $\lambda_1(c_{01}) < \lambda_0$, the function $\lambda^*(c)$ is decreasing in c until c_{01} and jumps to λ_0, remaining constant afterwards.

Case (ii) $\lambda_0 < \lambda_J(c_{01})$: The condition $\lambda_1(c_{10})(< \lambda_0) < \lambda_J(c_{01})$ implies that $\lambda_1(c) < \lambda_J(c)$ for all $c < c_{01}$; i.e., the $\lambda_J(c)$ constraint is not binding in that range and $\lambda^*(c) = \lambda_1(c)$ for all $c < c_{01}$. The condition $\lambda_0 < \lambda_J(c_{01})$ also implies that $c_{01} < c_0$, as $\lambda_J(c)$ is decreasing in c. Define c_{J1} to be the unique cost satisfying $W(\lambda_J(c_{J1}), 0) = W(\lambda_1(c_{J1}); 1, c_{J1})$. This cost exists and $c_{J1} \in [c_{01}, c_0]$ as, starting from c_{01}, the function $W(\lambda_1(c); 1, c)$ decreases in c from a value equal to $W(\lambda_0, 0)$ and the function $W(\lambda_J(c), 0)$ increases towards $W(\lambda_0, 0)$, thus the functions cross once. Then, in the range $c \in [c_{01}, c_{J1}]$, $W(\lambda_1(c); 1, c) > W(\lambda_J(c), 0)$ and $\lambda_1(c)$ would remain the socially optimal screening if $\lambda_J(c)$ remains not binding for $\lambda_1(c)$ (or equivalently if $c_1 > c_{J1}$). This last steps follows from observing that: $\lambda_1(c_{J1}) < \lambda_1(c_{01}) < \lambda_0 < \lambda_J(c_{J1})$ and, consequently, $\lambda_J(c)$ cannot bind $\lambda_1(c)$ for $c \in [c_{01}, c_{J1}]$. Then,

$$\lambda^*(c) = \begin{cases} \lambda_1(c) & \text{if } c \leq c_{J1} \\ \lambda_J(c) & \text{if } c \in (c_{J1}, c_0] \\ \lambda_0 & \text{if } c > c_0 \end{cases} \tag{25}$$

Because $\lambda_1(c_{J1}) < \lambda_0 < \lambda_J(c_{J1})$, the function $\lambda^*(c)$ is decreasing in c until c_{J1} where it jumps. Then it decreases in the interval $(c_{J1}, c_0]$, remaining constant afterwards.

Case (iii) $\lambda_J(c_{01}) < \lambda_1(c_{01})$: Because $\lambda_J(c_{01}) < \lambda_1(c_{01})$ we can conclude that $c_1 < c_{01}$. Also, we know that $\lambda_J(c)$ does not bind when $c \leq c_1$ and, consequently, $\lambda^*(c) = \lambda_1(c)$ in that interval. Because $c_1 < c_{01}$, at or in a neighborhood of c_1 we have $W(\lambda_J(c); 1, c) > W(\lambda_0; 0)$. By Lemma A.3, we know that the function $W(\lambda_J(c); 1, c)$ is decreasing in c for $c > c_1$ and diverges to $-\infty$ when the enforcement cost is unboundedly large. Thus, $W(\lambda_J(c); 1, c)$ single crosses $W(\lambda_0; 0)$. Define c_{0J} to be such cost. We show that $c_{0J} > c_0$. Define $c_J^{-1}(\lambda)$ to be the inverse of $\lambda_J(c)$; i.e., for any λ and c such that $\lambda_J(c) = \lambda$ we have $W(\lambda; 1, c_J^{-1}(\lambda)) = W(\lambda_J(c); 1, c)$. Then observe that, for any $\lambda < \lambda_J(c_1)$, $W(\lambda; 1, c_J^{-1}(\lambda)) > W(\lambda; 1, c_1^{-1}(\lambda))$ as, by construction, $c_J^{-1}(\lambda) < c_1^{-1}(\lambda)$ (see Figure 9). Then,

$$W\left(\lambda_1(c_{01}); 1, c_J^{-1}\left(\lambda_1(c_{01})\right)\right) > W\left(\lambda_1(c_{01}); 1, c_1^{-1}\left(\lambda_1(c_{01})\right)\right) = W(\lambda_0; 0)$$

and, because $W(\lambda_J(c); 1, c)$ decreasing in c for $c > c_1$ implies $W(\lambda; 1, c_J^{-1}(\lambda))$ increasing in λ, the function $W(\lambda; 1, c_J^{-1}(\lambda))$ crosses $W(\lambda_0; 0)$ at a $\lambda_J(c_{J0}) < \lambda_1(c_{01}) < \lambda_0$; i.e., to the left of point A in Figure 9 (e.g., point C). Consequently, $c_{0J} > c_0$. Then,

$$\lambda^*(c) = \begin{cases} \lambda_1(c) & \text{if } c \leq c_1 \\ \lambda_J(c) & \text{if } c \in (c_1, c_{0J}] \\ \lambda_0 & \text{if } c > c_{0J} \end{cases} \tag{26}$$

Because $\lambda_J(c_{0J}) < \lambda_1(c_{01}) < \lambda_0$, the function $\lambda^*(c)$ is decreasing in c until c_{0J} where it jumps to λ_0, remaining constant afterwards. ■

Proof of Lemma 7. Let $m > 1$ be an arbitrary number of innovations in the quality ladder and $z_m = \delta^m z$ for any $z > 0$. In a monopolized niche, due to the unit elasticity of demand, the incumbent wants to charge the highest price feasible. In this case $p = z_{m-1}$. Then, the incumbent profits are given by $\pi = (p - z_m)q = a(1-\delta)$ which is independent of the number of innovations and the baseline cost z. The dead-weight loss in the market is given by

$$\int_{z_m}^{z_{m-1}} q(p)dp - \pi = a\left(\ln\left(\delta^{-1}\right) - (1-\delta)\right)$$

also independent of m and z. ■

Proof of Proposition 6. Because $s^{CS}(\hat{s}, \gamma) = s^*(\hat{s}, \gamma) + (1-\gamma)L\Delta(\hat{s}, \gamma)/2\tilde{c}$ and $L > 0$, whether s^{CS} is higher than s^* depends on the sign of $\Delta(\hat{s}, \gamma)$. It is easy to verify that $\Delta(\hat{s}, 0) = 1$ and $\lim_{\gamma \to 1} \Delta(\hat{s}, \gamma) = -\infty$. Because $\Delta(\hat{s}, \gamma)$ is strictly decreasing in γ and is continuous, $\Delta(\hat{s}, \gamma^\circ(\hat{s})) = 0$ exists and is uniquely defined. To show that $\gamma^\circ(\hat{s})$ is increasing in $\Pi\alpha$, observe that

$$\frac{d\Delta}{d\Pi\alpha} = \frac{\gamma k_1^2[(1-\gamma)(1-\mu_0(\hat{s})) + \gamma\mu_0(\hat{s})]}{(1-\gamma)k_2\left[\Pi\alpha k_1 - (1-\mu_1(\hat{s})))(1-\gamma)\right]^2} > 0.$$

Thus, because $\Delta(\hat{s}, \gamma)$ is decreasing in γ, higher $\Pi\alpha$ implies a higher $\gamma^\circ(\hat{s})$. ■

Proof of Proposition 7. Differentiating $s^*(\hat{s}, \gamma)$ with respect to $\hat{s}$ we obtain

$$\frac{d\Phi}{ds} = \frac{2\gamma\left(1-\gamma\right)\hat{k}_1\left(\hat{k}_1 + s + 2\gamma - 3\right)}{\hat{k}_2^2\left(\hat{k}_1 - \left(1-\gamma\right)\left(1-s\right)\right)^2} \tag{27}$$

where $\hat{k}_1 = \Pi\alpha(1 + s\,(1 - 2\gamma)) > 0$ and $\hat{k}_2 = 2 - \gamma\,(1 + s) > 0$ are the analogous expressions for k_1 and k_2 in Lemma 5 after replacing for the judge's beliefs using (1). To ease out notation, $\hat{k}_1$ also includes $\Pi\alpha$. The sign of this derivative is given by the term in parenthesis in the numerator, which is positive if and only if condition (20) holds. When $\hat{s} = 1$ or $\gamma = 1$ condition (20) becomes equal to $\Pi > p(\hat{s}, \gamma)^{-1}$ which is equivalent to the positive entry assumption (Assumption 1). ■

B Extensions: Technical Details

B.1 Cost-saving Innovations and Static Inefficiency

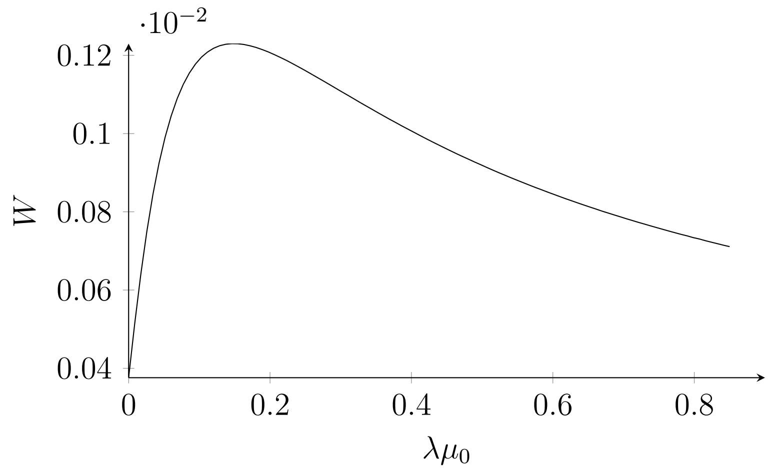
In this appendix we present the details of the cost-saving innovation model introduced in Section 5.1. We start deriving the objective function for the patent office and then move to the judges' problem.

Social Welfare Because profits are invariant to the cumulative number of innovations, the firm behavior and industry dynamics described in Section 3 go through without alterations. The objective functions of the social planer and courts, however, need to be reformulated to account for the deadweight loss ℓ. Figure 5 depicts the product market payoff associated with each innovation. Building on the expression for the deadweight losses obtained in Lemma 7, social welfare can be written as follows

$$W = e(\hat{s}, \gamma(\lambda)) \left[(1-\alpha)\lambda x \mu_0(\hat{s}) L + \alpha\big(x\mu_1(\hat{s})(\Pi + L) + (1-x)\Pi\big) - 1 - \kappa(\lambda)\right], \quad (28)$$

where $L \equiv \ell/(1-\beta)$ is the present value of a permanent dead-weight loss. Notice that in $e(\hat{s}, \gamma(\lambda))$ we are making explicit the dependence of γ on λ (see equation (15)). To explain this welfare expression consider Figure 5 when the latest technology available in the market attains a marginal cost z_1. The first term in square brackets captures the arrival of an obvious entrant, which occurs with probability $1-\alpha$ and who obtains a patent with probability λ. With probability $1-x$ the obvious entrant lands in a competitive niche, not affecting welfare. With probability x the obvious entrant lands in a monopolized niche and the incumbent takes it to court. The obvious entrant succeeds with probability μ_0, in which case, the market price goes down from z_0 to z_1. The area A depicts the profits of the replaced incumbent which are transferred to consumers as surplus. The area A brings no new social surplus. The area B, on the other hand, is the dead-weight loss associated with the patent protection given to the incumbent. With the arrival of the obvious innovation, this loss is transferred to consumers permanently, increasing welfare by L.

The second term in the expression represents the payoffs when the entrant is a genuine innovator and obtains a patent, which occurs with probability α. With probability x the entrant lands in a monopolized niche and gets challenged in court. The entrant wins in court with probability μ_1. In that case, the price goes down from z_0 to z_1. As before, the area A is transferred from the incumbent to consumers and the original dead-weight loss, B, is now captured by consumers permanently. In contrast, the entrant captures the area $C + D$ as profits. When a new innovation arrives in the future, these profits will be eventually transferred to consumers. That is, the welfare value created are the areas $B + C + D$ at perpetuity, or $\Pi + L$. Finally, when the genuine innovator lands in a competitive niche (that is, the existing price equals the marginal cost of the latest technology z_1), it does not get challenged in court and appropriates the area $C + D$ as profits. As before, these profits will eventually be transferred to consumers when a new innovation arrives. The entrant, thus, creates a welfare value Π. The area E is the new deadweight loss created, not appropriated by anyone until a new innovation successfully enters the market.



Note: Parameter values are $\alpha = 0.1$, $\pi = 2.4$, $\ell = 0.13$ (i.e., $a = 24$ and $\delta = 0.9$), $\beta = 0.8$ and $\mu_1 = 0.85$.

Figure 11: Welfare with deadweight loss; interior maximum when screening is free.

In order to understand better the welfare trade-offs in this model we can re-arrange (28) using the steady-state values of relevant objects provided in Lemma 1 we obtain

$$W = e\left[p(\Pi + L) - 1 - \kappa(\lambda)\right],$$

which is the expression presented in the main text. Figure 11 presents an example of an interior screening rate by the patent office when screening is free and the courts' behavior is exogenous.

The Judges' Problem We now analyze how a judge's endogenous decision changes when innovations are cost reducing and a dead-weight loss might arise. As explained in the main text a type I error arises whenever a genuine innovator is prevented from entering the market. In this scenario the type I error leads to a loss $E_I^{CS} = \Pi + L$.

The type II error represents the "loss" in social welfare when a firm with an obvious innovation is allowed to replace an active monopolist. This negative cost has now two components. First, there is a short run gain, derived from eliminating the deadweight loss that the incumbent generated. Second, there is the same dynamic effect explained in the benchmark case that increases the future value of the niche. That is,

$$E_{II}^{CS} = -L + \beta(w_M^{CS} - w_C^{CS}),$$

where the value of a monopolistic and competitive niche are respectively defined as

$$\begin{aligned} w_C^{CS} &= \beta w_C + e(\hat{s}, \gamma(\lambda))\left[\alpha\left(\Pi + \beta(w_M - w_C)\right) - 1\right], \\ w_M^{CS} &= \beta w_M + e(\hat{s}, \gamma(\lambda))\left[\alpha\mu_1(\hat{s})(\Pi + L) + (1-\alpha)\lambda\mu_0(\hat{s})\left(L + \beta(w_C - w_M)\right) - 1\right]. \end{aligned} \tag{29}$$

The difference with the benchmark case is that, now, the value of a monopolistic niche depends on the dead-weight loss L. Each time a monopolist is replaced by another firm the deadweight loss associated to its innovation is eliminated and a discounted surplus

L is transferred to consumers by means of a lower price. This effect increases w_M and narrows the difference between the value of a monopolistic and a competitive niche.

As a result, the introduction of a deadweight loss generates two opposing effects on the type II error. First, there is the static gain of eliminating the deadweight loss, enhancing the return from eliminating an existing monopolist; i.e., making the type II error more negative. The second effect, corresponds to the dynamic effect of reducing the incremental value of a competitive niche relative to a monopolized one (discussed in the previous paragraph). As the next lemma shows, the static effect dominates.

Lemma B.1. *For all $\hat{s}$ and γ, $E_{II}^{CS}(\hat{s},\gamma) < E_{II}(\hat{s},\gamma)$ where $E_{II}(\hat{s},\gamma)$ is given by* (14) *and*

$$E_{II}^{CS}(\hat{s},\gamma) = E_{II}(\hat{s},\gamma) - L\frac{(1-\gamma)[(1-\beta)+\alpha\beta(1-\mu_1(\hat{s}))e(\hat{s},\gamma)]}{(1-\beta)(1-\gamma)+\alpha\beta(1-\gamma+\gamma\mu_0(\hat{s}))e(\hat{s},\gamma)}.$$

Proof. Subtracting the value functions w_M and w_C in (29) and solving we obtain

$$\begin{aligned}\beta(w_M - w_C) &= \frac{[\beta(\alpha\mu_1(\hat{s})+(1-\alpha)\lambda\mu_0(\hat{s}))L - \alpha(1-\mu_1(\hat{s}))\Pi]e(\hat{s},\gamma)}{1-\beta+\beta(\alpha+(1-\alpha)\lambda\mu_0(\hat{s})e(\hat{s},\gamma)} \\ &= L\frac{\beta(\alpha\mu_1(\hat{s})+(1-\alpha)\lambda\mu_0(\hat{s}))e(\hat{s},\gamma)}{1-\beta+\beta(\alpha+(1-\alpha)\lambda\mu_0(\hat{s})e(\hat{s},\gamma)} + E_{II},\end{aligned}$$

where (14) was used in the last step. Replacing in $E_{II}^{CS} = -L + \beta(w_M - w_C)$ delivers the expression in the lemma. ■

We now formalize the details of the analysis presented in the main text for an individual judge's decision. Recall that when a judge chooses $s = 1$ no error is made, so the only cost is that related to her effort, $J^{CS}(1,\hat{s},\gamma) = c$. When a judge chooses effort $s = 0$, however, the cost is

$$J^{CS}(0,\hat{s},\gamma) = J(0,\hat{s},\gamma) + L(1-\gamma)\Delta(\hat{s},\gamma)/2,$$

where $J(0,\hat{s},\gamma)$ is defined in (17) and

$$\Delta(\hat{s},\gamma) = \frac{(1-2\gamma)(1-\beta)+\alpha\beta(1-\gamma)e(\hat{s},\gamma)}{(1-\gamma)(1-\beta)+\alpha\beta\left(1-\gamma+\gamma\mu_0(\hat{s})\right)e(\hat{s},\gamma)} \le 1. \tag{30}$$

Proposition B.1 (Complementarity in the model with DWL)**.** *For a given $\hat{s}$, $\Delta(\hat{s},\gamma)$ is decreasing in γ. Consequently, $J^{CS}(0,\hat{s},\gamma)$ is decreasing in γ. That is, the presence of the term in L in J^{CS} reinforces the negative effect of γ on the best response function of an individual judge.*

Proof. From Proposition 2 we know that $J(0,\hat{s},\gamma)$ decreases in γ. We need to show that $\Delta(\hat{s},\gamma)$ decreases in γ. Start by substituting for $e(\hat{s},\gamma)$ in (30) to obtain

$$\Delta(\hat{s},\gamma) = \Phi(\hat{s},\gamma) - \Omega(\hat{s},\gamma), \text{ where } \quad \Omega(\hat{s},\gamma) = \frac{\gamma k_1}{(1-\gamma)[\Pi\alpha k_1 - (1-\mu_1(\hat{s}))(1-\gamma)]},$$

$\Phi(\hat{s},\gamma)$ is given by (18), and k_1 and k_2 are defined in the proof of Lemma 5. By Lemma 5, $\Phi(\hat{s},\gamma)$ decreases in γ. Hence, it is sufficient to show that $\Omega(\hat{s},\gamma)$ increases in γ. Differentiating

$$\frac{d\Omega}{d\gamma} = \frac{\Pi\alpha k_1^2 - (1-\mu_1(\hat{s}))(1-\gamma)\left(k_1+\gamma\mu_0(\hat{s})\right)}{(1-\gamma)^2\left[\Pi\alpha k_1 - (1-\mu_1(\hat{s})))(1-\gamma)\right]^2}.$$

The denominator is always positive. We use Assumption 1 (i.e., $\Pi \geq k_2/(\alpha k_1)$) to construct the following lower bound for the numerator

$$k_1 k_2 - (1-\mu_1(\hat{s}))(1-\gamma)\left(k_1 + \gamma\mu_0(\hat{s})\right) = \gamma\mu_0(\hat{s})(1-\gamma)(4-5\mu_0(\hat{s})) + (\gamma + \mu_0(\hat{s}) - 1)^2$$

where we have used $\mu_1(s) = 1 - \mu_0(s)$. This expression is positive as $\mu_0(s) \leq 1/2$ for all s; proving that the derivative is positive. Therefore, $\Delta(\hat{s}, \gamma)$ and, consequently, $J^{CS}(0, \hat{s}, \gamma)$ decrease in γ. ■

B.2 Continuous Enforcement Effort

We previously assumed that a judge's effort, s, could take only two values. The judge could either exert no effort, 0, or full effort, 1. In this section we explore the case where effort is continuous; i.e., $s \in [0, 1]$. As expected, the main results carry through.

To ease the exposition, we assume that the cost of exerting effort is quadratic. In particular $c(s) = \tilde{c}s^2/2$ where $\tilde{c} > 0$ is a scale parameter.[23] Building on Lemma 5, the objective function that a judge minimizes with respect $s \in [0, 1]$ can be written as

$$J(s, \hat{s}, \gamma) = \Pi(1-\gamma)(1-s)\Phi(\hat{s}, \gamma)/2 + c(s), \tag{31}$$

where $\Phi(\hat{s}, \gamma) \in (0, 1]$ is given by equation (18). This objective function is the continuous-effort analogous of (17). The next lemma characterizes an individualjudge's best response taking the possibility of corner solutions into account.

Lemma B.2. *For a given enforcement effort by other judges $\hat{s}$ and screening rate by the patent office, γ, a judge's best response is unique an given by*

$$s(\hat{s}, \gamma) = \min\left\{1, \Pi(1-\gamma)\Phi(\hat{s}, \gamma)/2\tilde{c}\right\}. \tag{32}$$

If $\tilde{c} > \Pi/2$, the best response is always interior; i.e., $s(\hat{s}, \gamma) \in [0, 1)$. If $\tilde{c} \leq \Pi/2$ there exists values of $(\hat{s}, \gamma)$ for which the best response is $s(\hat{s}, \gamma) = 1$.

Proof. The proof that $s^*(\hat{s}, \gamma)$ is interior whenever $\Pi/2 < \tilde{c}$ was given in the main text. This solution is a minimum, as the second order condition is given by $\tilde{c} > 0$. For the case in which $\Pi/2 \geq \tilde{c}$, take $\gamma = 0$. Then, $(1-\gamma)\Phi(\hat{s}, \gamma) = 1$ and the first order condition of the judge's problem (31) satisfies $\tilde{c}s - \Pi/2 < 0$ for all $s \in [0, 1)$. Thus, $s^*(\hat{s}, 1) = 1$ is the unique solution to the minimization problem. ■

The objective function in (19) is strictly convex so when $s(\hat{s}, \gamma)$ is interior it characterizes the unique solution of the judge's problem. Additionally $s(\hat{s}, \gamma)$ is always positive so the only corner solution that may arise involves $s(\hat{s}, \gamma) = 1$. When the cost of effort is sufficiently high, $\tilde{c} > \Pi/2$, $s^*(\hat{s}, \gamma) < 1$ for all values of $(\hat{s}, \gamma)$. To see this observe that $\Pi/2\tilde{c} < 1$ and, by Lemma 5, $(1-\gamma)\Phi(\hat{s}, \gamma) \leq 1$.[24] When the cost of effort is not too high, $\tilde{c} < \Pi/2$, the corner solution $s(\hat{s}, \gamma) = 1$ arises for values of $(\hat{s}, \gamma)$ that make $(1-\gamma)\Phi(\hat{s}, \gamma)$ sufficiently close to one.

In the rest of this section we focus on the case $\tilde{c} > \Pi/2$ so that the solution is guaranteed to be interior. This assumption is analogous to Assumption 2 in the binary effort scenario.

[23] Results can be easily generalized to an environment with a convex cost function (i.e., $c'(s) > 0, c''(s) > 0$ for $s > 0$) satisfying $c(0) = c'(0) = 0$.

[24] Under a general cost function $c(s)$, the sufficient condition for an interior solution is $c'(1) > \Pi/2$.

Proposition B.2 (Screening complements enforcement)**.** *For a given screening rate by the patent office γ, there is a unique symmetric steady-state equilibrium in the enforcement game in which judges' effort is s^* such that $s(s^*,\gamma)=s^*$. In this equilibrium, an increase in the patent office's quality of screening increases judges' enforcement effort s^*. Thus, patent screening and patent enforcement are complementary.*

Proof. For a given γ, a symmetric interior equilibrium is given by

$$s^* = \Pi\left(1-\gamma\right)\Phi\left(\gamma, s^*\right)/2\tilde{c}.$$

We prove that the symmetric equilibrium is unique. Define the function $F\left(s,\gamma\right) = \Pi\left(1-\gamma\right)\Phi\left(\gamma,s\right)/2-\tilde{c}s$. Every symmetric equilibrium is a solution to $F\left(s^*,\gamma\right)=0$. To prove uniqueness, we show that dF/ds is negative; thus, for a given γ, F can only cross zero once. Differentiating

$$\frac{dF}{ds} = \frac{\Pi\left(1-\gamma\right)}{2}\frac{d\Phi}{ds} - \tilde{c} \leq \frac{\hat{\Pi}}{2}\left(\left(1-\gamma\right)\frac{d\Phi}{ds} - 1\right).$$

where $d\Phi/ds$ is given by (27). The inequality above follows from the assumption $\tilde{c} > \Pi/2$. We show that an upperbound of the parenthesis in the expression above is negative. The parenthesis is equal to

$$\frac{\hat{k}_1\left[\hat{k}_1\left(2\gamma\left(1-\gamma\right)^2-\hat{k}_2^2\right)+2\left(1-\gamma\right)\left(\gamma\left(1-\gamma\right)k_3+\hat{k}_2^2\left(1-s\right)\right)\right]-\left(\hat{k}_2(1-\gamma)(1-s)\right)^2}{\hat{k}_2^2\left(\hat{k}_1-\left(1-\gamma\right)\left(1-s\right)\right)^2}$$

where $k_3 = s+2\gamma-3 < 0$, and $\hat{k}_1$ and $\hat{k}_2$ are defined in the proof of Proposition 7. Because the denominator is positive, we need to show that an upperbound of the numerator is negative. The last term of the numerator is a subtracting a positive term. Thus, it is sufficient to show that the term in square brackets is non-positive. The first term of the square brackets, $2\gamma\left(1-\gamma\right)^2-\hat{k}_2^2$, is negative. This can be readily verified using that $\hat{k}_2$ decreases in s. The second term of the square brackets is positive. Thus, the square brackets can be positive or negative. Using Assumption 1, we can use that the positive entry assumption implies $\hat{k}_1 > \hat{k}_2$, to find the following upper bound for the term in square brackets

$$\begin{aligned}&\hat{k}_1\left(2\gamma\left(1-\gamma\right)^2-\hat{k}_2^2\right)+2\left(1-\gamma\right)\left(\gamma\left(1-\gamma\right)k_3+\hat{k}_2^2\left(1-s\right)\right) < \\ &\quad -\left(2\gamma\left(1-\gamma\right)^3\left(1-s\right)+\left(2-\gamma\left(s+1\right)\right)^2\left(\left(1-s\right)\gamma+2s\left(1-\gamma\right)\right)\right) < 0\end{aligned}$$

Therefore, $dF/ds < 0$ and the symmetric equilibrium is unique.

To prove that the Patent office's effort is complementary to a Judge's effor we use the Implicit Function theorem; i.e.,

$$\frac{ds^*}{d\gamma} = -\frac{dF/d\gamma}{dF/ds}.$$

Observe that $dF/d\gamma = dJ/d\gamma$. From the proof of Proposition 2 we know $dJ/d\gamma < 0$. Consequently, we have that $ds^*/d\gamma < 0$, proving the result. ■

Proposition B.2 shows the robustness of the complementarity found in the case with binary enforcement effort. Better patent screening (a decrease in γ) increases the enforcement effort exerted by the judges. It is also interesting to observe that the multiplicity of equilibria is not present in this case. Specifically, with continuous enforcement efforts and after ruling out corner solutions with $s^* = 1$, we no longer have a coordination problem among judges.[25] Figure 8 depicts the equilibrium under various parameters and screening rates. The upper left panel shows the complementarity between patent screening and patent enforcement. The other two figures show how the equilibrium enforcement effort increases in Π and α.

[25]This suggests that the coordination problem in the binary efforts case is related to the manner in which the prospects of perfect enforcement by subsequent judges ($\hat{s} = 1$) fully removes the social value of type II error, reinforcing the incentive of an individual judge to choose $s = 1$. For $\hat{s} < 1$, type II error by an individual judge is still valuable at the margin.