Entry Games under Private Information

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June 29, 2019

Introduction

1. **Private information** is crucial in modern econ analysis, but is has not been fully explored in entry models
   - **Auctions**: Firms are privately informed about their valuation before participating
   - **Oligopolistic Markets**: Firms are better informed than competitors about their own costs before deciding whether to enter

2. We study entry into oligopolistic markets under private info
   - **Strategic interaction** post-entry relates to pre-entry decisions
   - **General** forms of market **competition** & firm **heterogeneity**

3. Goal: Characterize firms entry decisions
   - Given market characteristics, which firms are more likely to enter?
   - Are there conditions to guarantee a unique equilibrium?

Motivation: Theory

1. Our motivation is both **theoretical** and **empirical**
2. From the theory standpoint
   - **Market Design**
     - Entry fees or subsidies (Moreno & Wooders, 2011)
     - Optimal mechanism design (Jehiel & Lamy, 2015)
   - **Competition Policy**
     - Is entry efficient (Mankiw & Whinston, 1986)
     - Entry effects of merger (Marshall & Parra, 2019)
   - **Trade Policy**
     - Which firms enter international markets (Melitz, 2003)
3. **Dynamic** models of entry
4. All question above have their empirical counterpart

Motivation: Empirics

1. To incorporate entry into empirical analysis conveys **challenges**
   - Multiplicity of equilibrium & Lack of theory
2. **Equilibrium Multiplicity**
   - Weak identification (Tamer 2003; Ciliberto & Tamer, 2009).
   - (Im)possibility of making counterfactual analysis
   - These are solved via assumptions: e.g., assuming an entry order (Berry 1992, Mazzeo 2002, Jia 2008, ...)
3. **Current theories**
   - Symmetric oligopoly (Bresnahan & Reiss, 1990, 1991).
   - Market-symmetric firms with entry-cost heterogeneity (Berry).
   - Assume away post-entry strategic interaction (Hopenhayn, 1992; Melitz, 2003)
4. Relevant empirical work (Athey et al, 2011; Seim, 2006; Krasnokutskaya & Seim, 2011; Roberts & Sweeting, 2016; Ciliberto, Murray & Tamer, 2018)

Our Contribution

1. General model of entry with private information
   - **Heterogeneous Firms**: Firms may differ in profit functions π_i and distribution of private information F_i
   - **Strategic interaction**: Post-entry profits depend on private info, entry decisions, and private info of participating firms
2. We show that every equilibrium is in **cutoff** strategies
3. We develop a notion of **strength** of a firm. We rank firms according to their strength.
   - Strength is a measure based on the fundamentals of the model
4. We show that a **herculean** equilibrium always exists: stronger players play lower cutoffs
   - Focal equilibrium in markets with asymmetries
   - Reduces a combinatorial problem to solving a system of equations

Our Contribution

1. When the elasticity of profits with respect of the private information is not too elastic, the *herculean* equilibrium is the unique equilibrium of the game
2. These results open the door to a richer empirical/structural assessments of market entry
   - Richer forms of competition
   - Explicit modeling of strategic interaction
   - Wider variety of applications

Talk: Road Map

1. Two potential Firms
   - Model
   - Examples
   - Preliminary Results
   - Main Result
   - Intuition
   - Implications
2. Concluding Remarks

Two potential firms: Model

In the paper, we deal with n. Today, $n = 2$

1. Each firm i draws its private info v_i from F_i (an atomless distribution on $\mathbb{R}_+$
2. After observing v_i, firms decide whether to enter the market
3. Payoffs:
    1. i only entrant $\pi_i(v_i) \in \mathbb{R}$
    2. both firms enter $\pi_i(v_i, v_j) \in \mathbb{R}$
4. The tuples (π_i, F_i) are common knowledge

Assumptions

A1 **Monotonicity:** $\pi_i(\cdot)$ is strictly increasing an differentiable in v_i

A2 **Competition:** $\pi_i(v_i) \geq \pi_i(v_i, v_j)$ for all v_j and $\pi_i(v_i, v_j)$ is weakly decreasing in v_j

A3 **Entry:** There exists $\underline{v}_i < \bar{v}_i$ such that $\pi_i(\underline{v}_i) = 0$ (entry is costly) and

$$\int_0^\infty \pi_i(\bar{v}_i, s) f_j(s) > 0$$

If draw is good enough, every firm would like to enter

Examples

The model accommodates most models of competition

1. **Firms are privately informed about their entry costs**
   Model most used in empirical analysis of entry (Seim 2006, Grieco 2014)
   $$\pi_i = X_i\beta_i - \mathbb{I}_j\delta + v_i$$
   j's private information does not directly affects i's payoffs.

2. **Bertrand Competition under Logit Demand**
   Let e be vector of entry by firms
   $$\pi_i(v_e) = (p_i(v_e) - c_i)\frac{\exp(v_i - p_i(v_e))}{\sum_{k\in e}\exp(v_k - p_k(v_e)) + \lambda} - K_i$$
   where
   - $p_i(v_e)$ is the equilibrium price under v_e
   - c_i is marginal cost of i
   - λ is consumer outside option
   - K_i entry cost

Examples (cont.)

3 **Selective Entry to Auctions**

- Before entering the auction, bidders receive a signal v_i about their valuation
- Upon entry, pay a participation cost
- After entry but before bid, firms learn type $V_i = v_i \varepsilon_i$ where $\varepsilon_i \sim G_i$ is independent noise ($\mathbb{E}(\varepsilon_i) = 1$).

$$\pi_i(v_i) = v_i$$

$$\pi_i(v_i, v_j) = \int_0^\infty \left(\int_{-\infty}^{v_i \varepsilon_i} (v_i \varepsilon_i - \max\{0, s\}) dG_j \left(\frac{s}{v_j} \right) \right) dG_i(\varepsilon_i)$$

Preliminaries

A strategy is a mapping from the valuation v_i to a probability of entering the market $p_i(v_i)$.

Definition (Cutoff strategy)

A strategy $p_i(v_i)$ is called *cutoff* if there exists a threshold $x > 0$ such that

$$p_i(v_i) = \begin{cases} 1 & \text{if } v_i \geq x \\ 0 & \text{if } v_i < x \end{cases} .$$

To be clear:

1. x_i represents i's cutoff.
2. We denote strategies with the cutoff itself

Preliminaries (cont.)

Proposition (Existence and cutoff equilibrium)

In any entry game there exists an equilibrium. Every equilibrium of the game is in cutoff strategies; i.e., a pair x_1, x_2 that jointly solve:

$$\pi_i(x_i)F_j(x_j) + \int_{x_j}^{\infty} \pi_i(x_i, y)dF_j(y) = 0.$$

Explain Cutoff! What is the problem we want to solve?

Definition (Strength)

Strength of firm i is the unique number $s_i \in \mathbb{R}_+$ that solves

$$\pi_i(s_i)F_j(s_i) + \int_{s_i}^{\infty} \pi_i(s_i, y)dF_j(y) = 0.$$

Strength

Definition (Strength (cont.))

The *strength* of firm i is the unique number $s_i \in \mathbb{R}_+$ that solves

$$\pi_i(s_i)F_j(s_i) + \int_{s_i}^{\infty} \pi_i(s_i, y)dF_j(y) = 0.$$

We say that player i is *stronger* than player j if $s_i \leq s_j$.

- Strength is always well defined.
- ranks firms by building upon two ideas: that firms play cutoffs strategies and *symmetry*

Definition (Herculean Equilibrium)

An equilibrium is called *herculean* if the equilibrium cutoffs are ordered by *strength*, with stronger players playing lower cutoffs.

Entry: Main Result

Proposition

A herculean equilibrium always exists (no conditions!). Moreover, it is the unique equilibrium of the entry game if for all $v_i > \underline{v}_i$ *and* $v_j > \underline{v}_j$

$$\frac{f_i(v_i)}{F_i(v_i)} \frac{\Delta_i(v_i, v_j)}{\pi_i'(v_i)} < 1.$$

where $\Delta_i(v_i, v_j) = \pi_i(v_i) - \pi_i(v_i, v_j)$.
Actually, we can also use a stronger condition

$$\eta = \frac{f_i(v_i)}{F_i(v_i)} \frac{\pi_i(v_i)}{\pi_i'(v_i)} < 1.$$

Intuition

Strength of player i is the unique number s_i satisfying

$$\sigma_i(s_i) \equiv \pi_i(s_i)F_j(s_i) + \int_{s_i}^{\infty} \pi_i(s_i, y)dF_j(y)$$

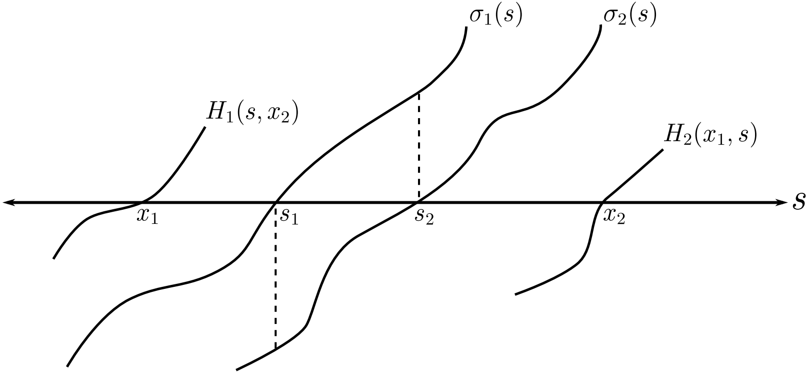

Figure: Strength and Herculean equilibrium

Examples

1 **Private info is entry costs** Recall

$$\pi_i = X_i\beta_i - \mathbb{I}_j\delta + v_i$$

condition for uniqueness becomes: for all $v_i > X_i\beta_i$

$$\frac{f_i(v_i)}{F_i(v_i)} < \delta^{-1}$$

Bounded inverted-hazard rate!
Berry and Tamer (2006) observe that, when $v_i \sim N(\mu, \sigma)$ and $\delta > \mu$: $\sigma = 0$ implies multiple equilibria and $\sigma = \infty$ implies unique eq.
We can provide tighter bound. Take for instance $X_i\beta_i = 0$, $\mu = 1$ and $\delta = 4$. Unique equilibrium exists whenever $\sigma > 3.876$.

Examples

2 **Differentiated Bertrand with logit demand** Condition for uniqueness becomes: for all $v_i > \underline{v}_i$

$$\frac{f_i(v_i)}{F_i(v_i)} < \frac{\lambda}{\exp(v_i - p_i(v_i)) + \lambda}$$

Market share of outside option!

3 **Selective entry to auction** Condition for uniqueness becomes: for all $v_i > K_i$

$$v_i f_i(v_i) < F_i(v_i)$$

Weak Concavity!

Implications of the Result

Entry is an $n!$ combinatorial problem.
Strength reduces it to computing n numbers and solving a system of equations. The system is non-linear, but always has a solution!

Herculean equilibrium is focal. Asymmetric analogue of symmetric equilibrium in symmetric games.

Advantage: one number summarizes all information

1. Optimal auctions: Virtual valuation
2. Multi armed bandit: Gittins index
3. Entry Games: Strength

More importantly, result aids structural analysis of markets with entry.

Also in the Paper

1. When is there a relation between cutoff and profit order?
2. We discuss the limitation of strength when dealing with n firms.
3. Similar conditions for uniqueness in:
   - n symmetric firms. (Bresnahan and Reis)
   - n market-symmetric firms, with different entry costs (Berry 1992)
   - Two groups of within-group symmetric firms, but different among groups (Athey et al. in auctions)
4. Extends to multi-product firms when demand can be written as an aggregative game (Shultz Nocke, 2018)

Thanks!

Comments and Suggestions Welcome