

# SEQUENTIAL INNOVATION, PATENT POLICY AND THE DYNAMICS OF THE REPLACEMENT EFFECT

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# MOTIVATION

- ① How does a more protective patent policy affect R&D investments?
  - Classic single-innovation literature: R&D increase.
  - Empirically, this seems not to be true.
  - Intuitively, in a sequential context, more protective patents may discourage incumbents from innovating.
  
- ② How the different tools in patent policy affect R&D investments, leadership persistence, and the number of firms in the market?
  - Patent length – For how long an innovator is protected.
  - Forward Protection – How strongly we protect an innovator from futures breakthroughs.
  - Finite patents leads to non-stationarity.

# MY CONTRIBUTION

## ① (Non) Stationarity Matters

- Non-stationarity is difficult to solve. To my knowledge, first paper to deal the non-stationarity induced by finite patents (Doraszelski, 93).
- Dynamic version of Arrow replacement effect.
- **Arrow's Reversal:** followers may invest less than leader.

## ② Patents that last too long **discourage innovation** and (possibly) **entry**.

- Longer patents **delay** leader's R&D and (possibly) the followers'.
- Cost of a protective policies lies beyond the DWL of the monopoly that the patent grants.

## ③ The effectiveness of each tool **depends** on the **market's characteristics**.

- Patent length and forward protection **substitute** for one another.
- Long but weak patents in markets that innovate slowly.
- Short but protective patents in markets that innovate fast.

# MY CONTRIBUTION IN PERSPECTIVE

- ① Two-periods models - Good intuitions but cannot fully capture dynamics (countless papers, Scotchmer, Denicolò).
- ② Infinite sequence of innovations - stationarity
  - Innovations occur exogenously (O'Donoghue et al., 1998; Hopenhayn et al., 2006; Hopenhayn and Mitchell, 2013).
  - Restrict the policy space to patents of infinite length (O'Donoghue, 1998; Bessen and Maskin, 2009; Acemoglu and Cao, 2010).
  - Patents terminate stochastically in a Poisson fashion (Acemoglu and Akcigit, 2012).
  - Restrict agents performing R&D to only potential entrants (Riis and Shi, 2012)
  - or only incumbents (Horowitz and Lai, 1996).

# TALK: ROAD MAP

- ① A model of sequential innovation.
- ② Solving the dynamic game.
- ③ Understanding investments dynamics.
- ④ Optimal Policy.
- ⑤ Back to the full model
- ⑥ Extensions (time permitting).

# THE MODEL: SETUP

- Consider an infinite ladder of innovations  
(Grossman Helpman, 91; Aghion Howitt, 92; Aghion et al., 01; etc.).
- Time is continuous and future payoffs discounted at a rate  $r$ .
- The (technology / market) *leader* ( $l$ ):
  - Firm with the latest technology.
  - Protected by a patent an active patent.
  - Gets a monopoly flow of  $\pi$ , while patent is active.
  - Invest in R&D to extend leader status.
- The followers ( $f$ ):
  - There are  $n$  endogenously-determined followers.
  - Compete using obsolete technologies (zero profit flow).
  - Invest in R&D to leapfrog and become the new leader.

# THE MODEL: PROBABILISTIC PATENTS

- A patent is described by the tuple  $(T, b)$ .
  - $T \geq 0$ : Patent length – How long.
  - $b \in [0, 1]$ : Forward protection – How strongly.
  - **Forward Protection:** With probability  $b$ , an entrant's innovation is considered an infringement of the active patent. In which case, she has to pay as a license fee the damages caused to the leader.  
(*Probabilistic Patents*: Lemley Shapiro, 05; Farrell Shapiro, 08)
- $t$  denotes how much time has passed *since the last innovation*.
  - When an innovation arrives clock is reset to  $t = 0$ .
  - When  $t \geq T$  no patent protection. Imitation drives profits to zero.
- Let  $v_t$  and  $w_t$  (resp.) be the value of being the leader and a follower.
  - At  $t = 0$ , followers pay entry costs  $K$ .
  - 'Free entry' drives  $w_0 = K$ .

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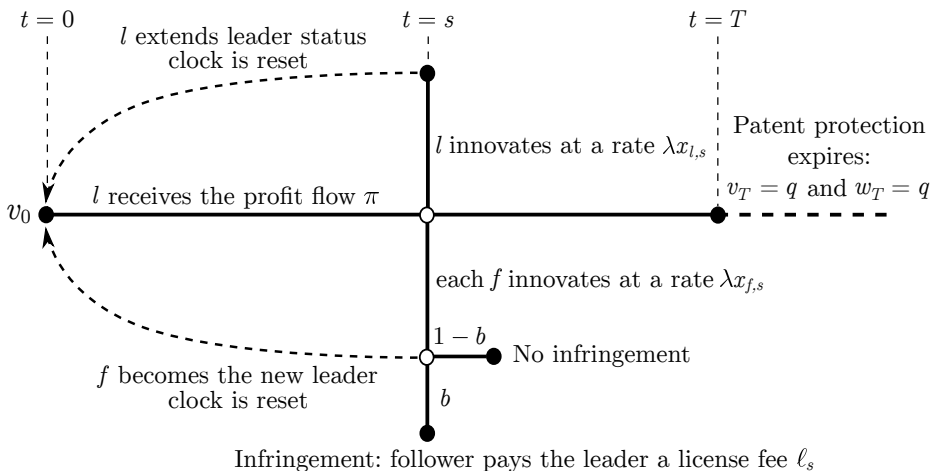


# THE MODEL: R&D AND INNOVATION

- $x_{k,t} \geq 0$  is the R&D intensity of firm  $k \in \{l, f\}$  at  $t$ .
- Cost of R&D at instant  $t$  is given by  $c(x_{k,t}) = (x_{k,t})^2/2$ .
- Firm  $k \in \{l, f\}$  innovations follow a non-homogeneous Poisson process with arrival rate of  $\lambda x_{k,t}$ , independent among firms.
- $\lambda \in \mathbb{R}_+$  is the market's natural innovation rate (cost!).

# TIMING OF THE GAME

Recall  $v_t$  and  $w_t$  are the values of a being the *leader* and a *follower* at  $t$ .



# PAYOFFS: AFTER PATENT PROTECTION

After a patent expires  $t \geq T$ :

- The race becomes symmetric and stationary.  
(Loury, 79; Lee Wilde, 80; Reinganum, 82; etc).
- There are  $n + 1$  firms competing.
- Let  $q$  be the value of competing in such a race. For generic firm  $i$ :

$$rq = \max_{x_i} \left\{ \lambda x_i (v_0 - q) + n \lambda \hat{x}_i (w_0 - K - q) - \frac{(x_i)^2}{2} \right\}$$

- At any instant  $t$ :
  - Pay R&D cost  $(x_i)^2/2$ .
  - Firm  $i$  innovates at a rate  $\lambda x_i$ . Gets net reward  $v_0 - q$
  - Competitors succeed at a rate  $n \lambda \hat{x}_i$ .
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# PAYOFFS: LEADERS

- **License fees:**

- Define  $q_t$  the (expected discounted) value of  $q$  at instant  $t$ .
- When infringement occurs, compulsory license fees equal to damages (Tandom, 82):

$$\ell_t = v_t - q_t$$

- Using the principle of optimality:

$$rv_t = \max_{x_{l,t} \geq 0} \left\{ \pi - \frac{x_{l,t}^2}{2} + \lambda x_{l,t}(v_0 - v_t) - n\lambda x_{f,t}(bq_t + (1-b)v_t) + v'_t \right\}$$

with boundary condition  $v_T = q$ .

- At any instant  $s > t$ .
  - Gets monopoly flow  $\pi$  and pays R&D costs.
  - The leader innovates at a rate  $\lambda x_{l,t}$ , gets net reward  $v_0 - v_t$
  - The followers innovate at a rate  $n\lambda x_{f,t}$ .
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# PAYOFFS: FOLLOWERS

Recall that  $w_t$  denotes the value of being a follower:

- Using the principle of optimality:

$$rw_t = \max_{x_{f,t} \geq 0} \left\{ -\frac{x_{f,t}^2}{2} + \lambda x_{f,t} (v_0 - w_t - b\ell_t) - \lambda (x_{l,t} + (n-1)\hat{x}_{f,t})w_t + w'_t \right\}$$

with boundary condition  $w_T = q$ .

- At any instant  $t$  followers:
  - Pay R&D costs.
  - Innovate at a rate  $\lambda x_{f,t}$  and get  $v_0 - w_t - \ell_t$
  - Opponents innovate at a rate  $\lambda (x_{l,t} + (n-1)\hat{x}_{f,t})$ .
  - The patent changes value  $v'_t$ .

# R&D DYNAMICS

Taking first order condition we obtain

$$x_{l,t}^* = \lambda(v_0 - v_t) \ ; \ x_{f,t}^* = \lambda(v_0 - w_t - b\ell_t)$$

The incremental rent!

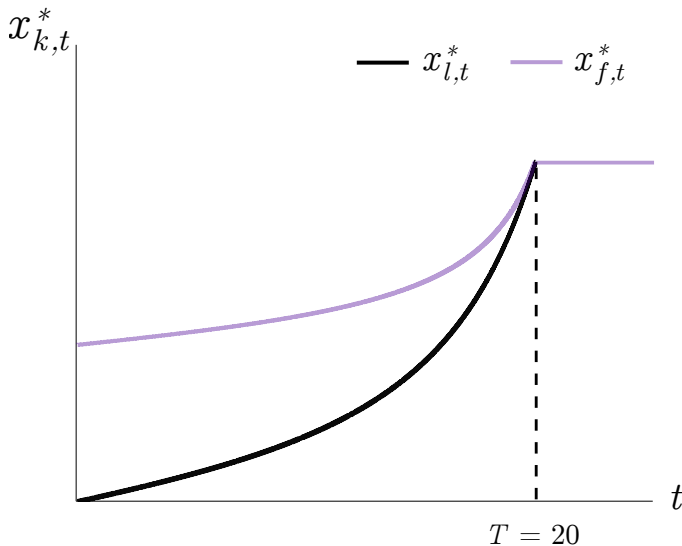
## Proposition (R&D Dynamics)

*At the beginning of a patent race ( $t = 0$ ), leaders do not invest in R&D. As an active patent approaches its expiration date, both types of firms perform increasing investments over time. When patent protection expires, leader's and followers' investments converge.*

Dynamic version of Arrow's Replacement Effect (1962)



# R&D Dynamics: Arrow's Replacement Effect



Recall:

$$x_{l,t}^* = \lambda(v_0 - v_t) ; \quad x_{f,t}^* = \lambda(v_0 - w_t - b\ell_t)$$

# INTERNALIZATION OF REPLACEMENT EFFECT

## Theorem (Arrow's Reversal)

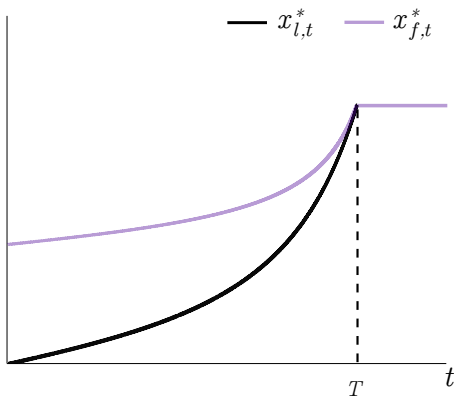
*Depending on forward protection, followers internalize the cost of replacing the leader. In particular, when forward protection is sufficiently strong, followers do not invest at the beginning of the patent life and then invest at a lower rate than the leader.*

Recall:  $x_{f,t}^* = \lambda(v_0 - w_t - b\ell_t)$ . When  $b = 1$  we can write

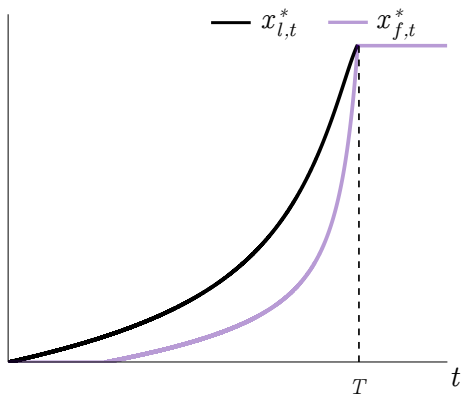
$$x_{f,t}^* = x_{l,t}^* - \lambda(w_t - q_t)$$

With dynamics there is two replacements effects (option value).  
Under strong forward protection, follower internalizes both.

(a) R&D Dynamics:  
Arrow's Replacement Effect



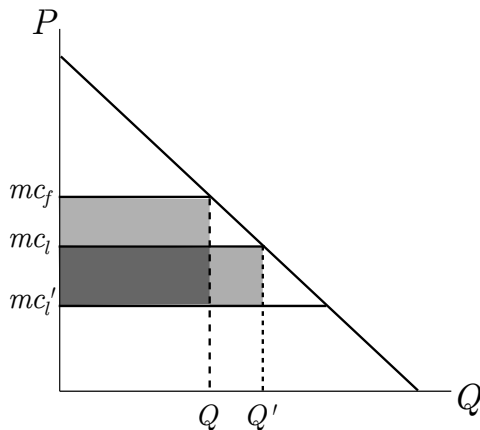
(b) R&D Dynamics:  
Arrow's Reversal



**Figure:** Leader may invest less in R&D at every  $t < T$ .

# REVERSAL IN CONTEXT

**Figure: Arrow's 62 Argument**



$$x_{f,t} = \lambda \left[ \underbrace{(v_0 - w_t)}_{f\text{'s replacement } \hat{\pi}_d} - \underbrace{b(v_t - q_t)}_{l\text{'s loss } \hat{\pi}_M - \hat{\pi}_d} \right]$$

**Gilbert Newbery's 82 Argument**

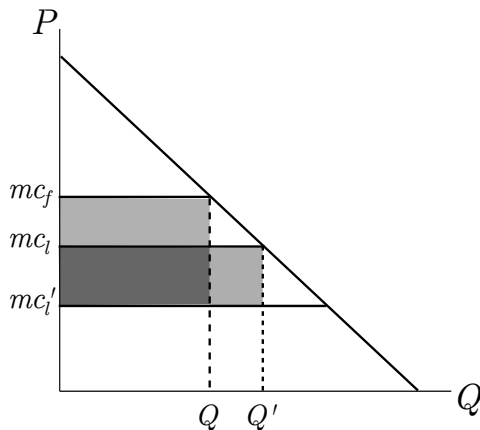
$$\hat{\pi}_M \geq 2\hat{\pi}_d$$

Thus, leader's incremental loss  $\hat{\pi}_M - \hat{\pi}_d \geq \hat{\pi}_d$  follows incremental gain.

**Reingaunum 83:** GN's result is not robust to patent races

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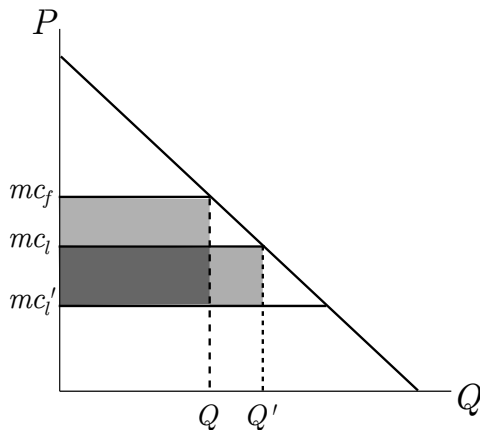
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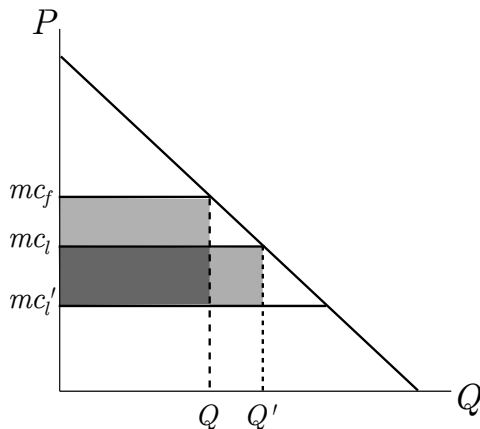
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## OTHER RESULTS II

The result does rely in the non-stationarity of the game!

### Lemma (Stationarity)

*If  $T = \infty$ , the values  $v_t$  and  $w_t$  become independent of  $t$ .*

*Firms investments become stationary. The leader performs no R&D and followers' invest at a positive rate of  $x_f = \lambda(v_\infty(1 - b) - K)$  whenever  $\pi(1 - b) > rK$ .*

- In stationary games there is no reversal.
- Forward protection simply discourages R&D.  
(O'Donoghue and Zweimller, 04; Denicolò and Zanchettin, 12)



# TALK: ROAD MAP

- ① A model of sequential innovation.
  - R&D dynamics - Arrow's Reversal
- ② Solving the dynamic game.
- ③ Understanding investments dynamics.
- ④ Optimal Policy.
- ⑤ Back to the full model
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# SOLVING

Replacing back the FOC, we obtain the following system of ODE:

$$\begin{aligned}rv_t &= v'_t + \pi + \frac{\lambda^2}{2}(v_0 - v_t)^2 - \lambda^2 n(bq_t + (1 - b)v_t)(v_0 - w_t - b\ell_t) \\rw_t &= w'_t + \frac{\lambda^2}{2}(v_0 - w_t - b\ell_t)^2 - \lambda^2(v_0 - v_t + (n - 1)(v_0 - w_t - b\ell_t))w_t.\end{aligned}$$

Unfortunately, there no closed-form solution.

Two options: Do it numerically or find a different way!

Long run - short run players (Fudenberg et al., 90).

- The leader faces a sequence of short-run followers
- Followers maximize spot payoff (no dynamic considerations)

$$x_{f,t} = \lambda(v_0 - bv_t)$$

- Reduces problem to single agent.

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# LEADER'S PROBLEM

Suppose the value of a new patent,  $v_0$ , is known.

The previous system of ODEs simplifies to:

$$rv_t = \pi + \frac{\lambda^2}{2}(v_0 - v_t)^2 - \lambda^2(1 - b)(v_0 - v_t)v_t + v_t'$$

This is a Riccati separable differential equation.

- Unique solution through the boundary condition  $v_T = 0$ .

The solution  $v_t$  depends on the initial condition  $v_0$ . An actual solution must satisfy:

$$v_{t=0} = v_0.$$

# SOLUTION

## Proposition (Existence and uniqueness)

*There always exists a unique fixed-point  $v_{t=0} = v_0$ . The value of a patent*

$$v_t = \frac{(2\pi + (\lambda v_0)^2) (e^{\phi(T-t)} - 1)}{\theta(e^{\phi(T-t)} - 1) + \phi(e^{\phi(T-t)} + 1)}$$

*decreases as its expiration date approaches, where  $\theta$  and  $\phi$  are known.*

The value of an active patent  $v_t$  increases with:

- ① Larger discounted expected profits.
- ② Longer patents
- ③ Forward protection
- ④ An increase in leader's productivity
- ⑤ A decrease in followers' productivity

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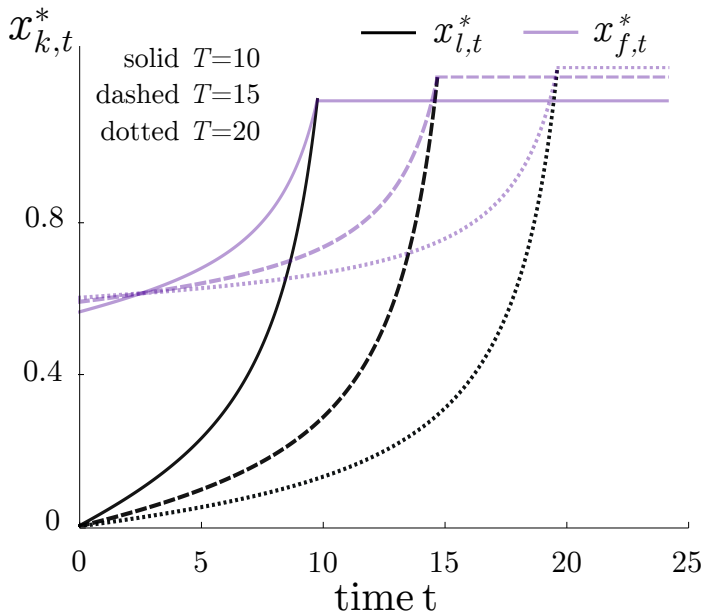
# R&D INVESTMENTS

## Theorem (Patent length and R&D)

*An increase in the statutory length  $T$ :*

- ① *delays the leader's investment, and;*
- ② *when forward protection is large enough, delay the followers' investments.*

**Figure:** R&D and patent length  $T$ .





# R&D INVESTMENTS: INTUITION

Recall the leader's first-order condition:

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Intuitively, how does patent length effect  $v_0$  and  $v_t$ ?

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For the followers simply notice:

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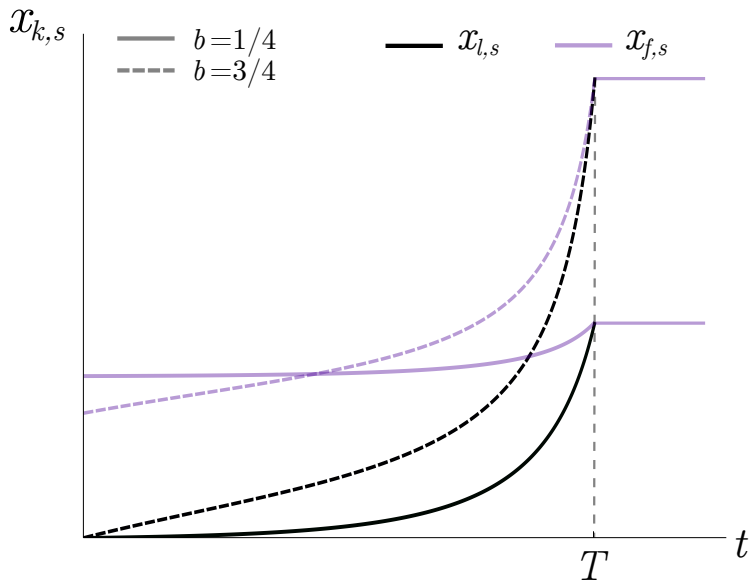
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# FORWARD PROTECTION



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# THE ECONOMY'S INNOVATION RATE

- **Goal:** measure the speed of the innovative activity.
- Why innovation pace and not welfare?
- Let  $x_t = x_{i,t} + x_{n,t}$ . Define the economy's innovation rate as *the inverse of the expected waiting time between innovations*

$$\hat{\lambda} = \mathbb{E}[t]^{-1} = \left( \int_0^{\infty} t \lambda x_t e^{-\int_0^t \lambda x_s ds} dt \right)^{-1}$$

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  - i.e. the arrival rate of an Exponential distribution with parameter  $\bar{\lambda}$ .
  - Thus, the larger  $\hat{\lambda}$ , the faster innovations occur.



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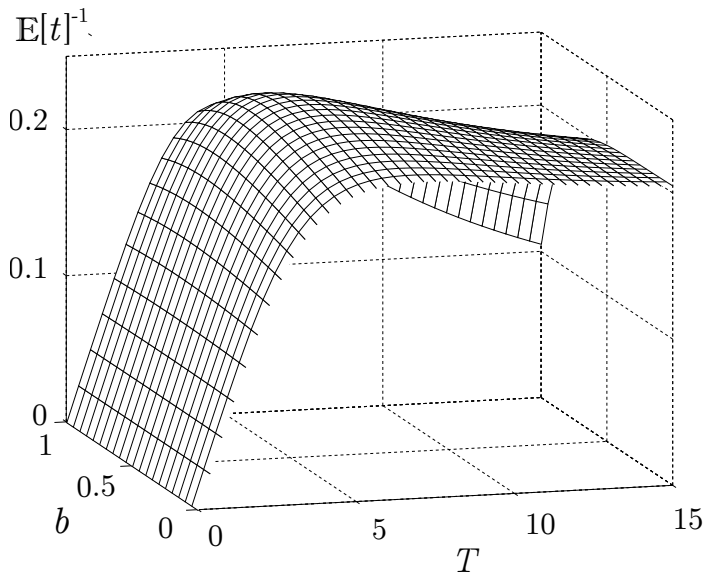
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## Theorem (Long patents discourage R&D)

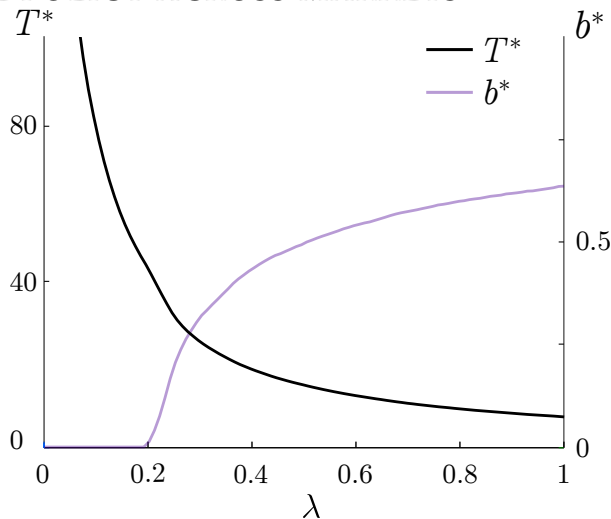
*The optimal policy  $(T^*, b^*)$  consists of a finite patent length.*

# OPTIMAL PATENT LENGTH IS FINITE



Note. Parameters used:  $r = 5\%$ ,  $K = 1/30$ ,  $\lambda = 1$  and  $\pi = 1/20$ .

# OPTIMAL POLICY ACROSS MARKETS



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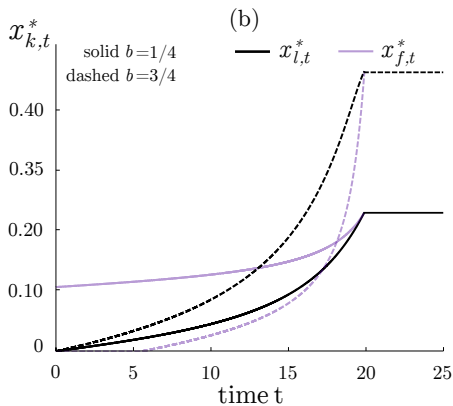
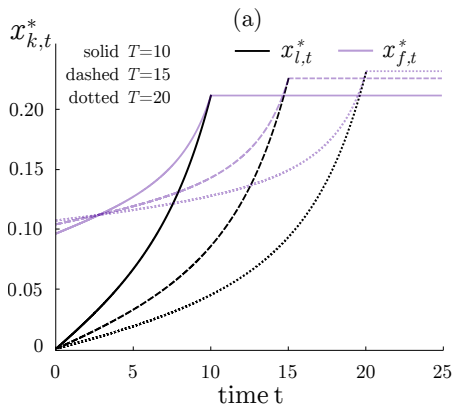
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O'Donoghue and Zweimller, 04; Denicolò and Zanchettin, 12.

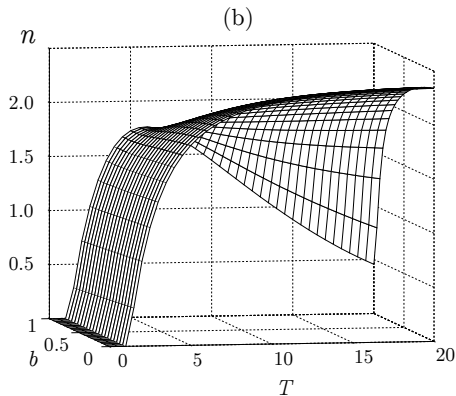
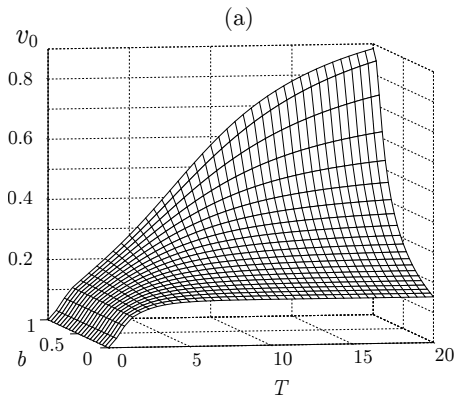
# TALK: ROAD MAP

- ① A model of sequential innovation.
  - R&D dynamics - Arrow's Reversal
- ② Solving the dynamic game.
- ③ Understand incentives and investments.
  - ① Longer patents delay investments.
  - ② Forward protection deters entrants.
- ④ Optimal Policy.
  - ① Slow markets: High  $T$  little  $b$ .
  - ② Fast markets: High  $b$  short  $T$ .
- ⑤ Back to the full model
- ⑥ Extensions (time permitting)

Figure: R&D over time



**Figure:** The value of a patent and the number of entrants



When forward protection is sufficiently strong, **longer patents discourage entry!**

**Table:** Optimal patent under different  $\lambda$ .

$\lambda$	$T^*$	$b^*$	$\mathbb{E}[t]^*$	$n^*$	$T = 10$		$T = 20$	
					$b = 1/3$	$b = 2/3$	$b = 1/3$	$b = 2/3$
0.5	33.6	0	6.26	3.10	19.2%	23.8%	6.6%	21.4%
0.75	14.4	0	4.42	2.57	2.4%	8.6%	5.8%	18.5%
1.0	9	0.02	3.48	2.18	1.7%	8%	7%	17.8%
1.25	5.7	0.22	2.87	1.81	3.8%	9.7%	9.36%	19.2%
1.50	4.1	0.24	2.45	1.55	6.6%	11.8%	12.2%	19.6%
1.75	3.2	0.25	2.14	1.35	9.6%	14.1%	15.1%	23.6%

Note: Parameters used:  $r = 5\%$ ,  $K = 1/30$  and  $\pi = 1/20$ .  $\mathbb{E}[t]/\mathbb{E}[t]^* - 1$  quantifies (in percentage points) the delay of implementing an inefficient policy.

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- ⑤ Back to the full model
  - ① Previous results are replicated.
  - ② Longer patents may deter entry.
- ⑥ Extensions



## LEADER EXTENDS ITS LEAD

Previous result assumed that only benefits of leader innovation were:

- Extending its patent clock
- Deterring Entrant R&D (in equilibrium).

Let  $m$  be the number of consecutive innovations by the leader.

- $\pi_{m+1} \geq \pi_m$ .

As a consequence

- $v_{m+1,t} \geq v_{m,t}$ ,  $\ell_{m,t} = v_{m,t} - q_t$ , and  $w_{m+1,t} \leq w_{m,t}$

In this context investments become:

$$x_{l,m,t} = \lambda(v_{m+1,0} - v_{m,t}); \quad x_{f,m,t} = \lambda(v_{1,0} - w_{m,t} - b\ell_{m,t})$$

Arrow Reversal get's reinforced with larger gap.

Delay effect of longer patent persists.

# TO WRAP UP AND CONCLUDE

## Relevance:

- Sequentiality + Non stationarity does play a role.
- Some important intuitions change with respect the one-shot case.
  - Arrow's result may reverse.
  - Longer patents may delay investments.
  - Protective patent policy may slow the economy's innovation rate.
- Optimal policy varies with market characteristics.
  - Policy should be market dependent.
  - More importantly, it seems there is room for self-selection.

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Thanks!