Mergers in Innovative Industries: A Dynamic Framework*

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Abstract

We investigate how a merger with R&D efficiencies affects market outcomes over time. To this end, we propose a dynamic framework based on a patent race model of sequential innovations with endogenous market structure. We show that timely (but costly) entry into the patent race is sufficient to guarantee that mergers are welfare improving from an innovation standpoint. These results hold for all efficiency levels and despite the fact that mergers may reduce the number of firms performing R&D by more than one.

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1 Introduction

A key idea underlying merger policy is that greater competition—usually interpreted as low levels of market concentration—is at heart of a healthy economy. For instance, it is well accepted that competition generally lowers prices. In innovative industries, however, the role of competition on market outcomes is far less clear.[1] Although merger review is one of the main activities performed by competition policy authorities and innovation has been regarded by many as the engine of a growing economy, the key tradeoffs that arise in mergers in innovative industries are still not well understood. (Katz and Shelanski, 2005, 2007)

From a competition policy standpoint, the main question is whether innovations should factor into merger analysis, and if so, how. Some authors have noted several dynamic factors that complicate merger analysis in these industries. (Gilbert and Sunshine, 1995, Evans and Schmalansee, 2002, Katz and Shelanski, 2005, 2007) These factors include, firstly, that market leaders face a constant threat of becoming obsolete by the introduction of new products, making standard measures of market concentration uninformative regarding actual competition. Secondly, innovations may have a high impact on market structure (i.e., current market structure is a bad predictor of future market structure), making short-run price changes less relevant compared to changes in the expected time between innovations or *pace* of innovation. Thirdly, much of the R&D activity is often towards products that are yet to reach the market, making market definition a challenging task. Finally, mergers may involve firms without products in the market which may affect R&D outcomes without affecting the product market equilibrium.

In this work, we propose a dynamic framework with endogenous market structure to study the consequences of mergers in innovative industries and to analyze whether mergers are ever desirable from an innovation standpoint. We follow a "creative destruction" approach by considering a winner-take-all competition for a sequence of innovations, capturing the constant threat faced by market leaders of being replaced by the new "killer" product. We use the model to answer the question of how an R&D enhancing-merger affects market structure, the expected time between innovations, and the industry-wide R&D expenditure.

We find that timely (but costly) entry into the R&D race guarantees that mergers with R&D efficiencies are welfare improving. The welfare gains come from a merger either increasing the pace of innovation or keeping the pace of

[1] See, for instance, Aghion et al. (2005).

innovation constant while reducing overall R&D expenditure. The results hold true despite the finding that a merger may substantially reduce the number of firms performing R&D by inducing exit after the merger. An important implication of these results is that verifying efficiencies is unnecessary as the welfare gains hold true for all efficiency levels. We show that these results are robust to a series of extensions which include various forms of patent protection, patent infringement, firm heterogeneity, quality competition, and price competition. We do wish to remark that these welfare gains, however, are in contrast to potential price increases that may be experienced in the products market in the short-run.

In concrete terms, we develop a sequential extension to the classic patent race models of Loury (1979), Lee and Wilde (1980), and Reinganum (1982). We assume that each innovation has a high "innovation impact", with the most recent innovator becoming a monopolist in the product market in replacement of the previous market leader. As in these models, R&D is endogenous and determined by the number of competitors and the firms' productivity levels. In contrast to the literature, the value of an innovation and the number of firms competing to achieve the next breakthrough are endogenously determined by entry and exit conditions. These conditions relate the value of participating in the R&D race with start-up costs (e.g., setting up a lab) and each firm's opportunity cost.

The main driving force behind the positive effects of a merger is that entry and exit in the innovation market are intrinsically linked to the equilibrium value of being the market leader. When the value of being leader is "too high", entry of new firms speeds innovation up, shortening the lifespan of the leader, and decreasing the value of being the industry leader. Similarly, when the value of being leader is "too low", exit of firms slows down the innovation in the industry, increasing the incentives to become a leader. In this way, entry and exit of firms stabilize the value of being market leader to an equilibrium value that is a function of the opportunity cost of firms. A critical piece of our argument is that we show that there is a one-to-one mapping between the speed of innovation and the value of being market leader. Consequently, as long as a merger does not affect the entry or exit conditions of the marginal firm, the merger will make the industry more cost-efficient without affecting the industry's pace of innovation.

The entry and exit stabilizing forces, however, only operate as long as firms are willing to participate in the race. When the R&D efficiencies of the merged firm are large enough, the merged firm alone can achieve a pace of innovation that is higher than the pre-merger pace of innovation. In that case, the equilibrium value of being the leader is so low that only the merged firm chooses to stay in the race. Despite the massive exit of firms, the merger is welfare increasing as the pace of innovation strictly increases.

In the horizontal merger literature, Williamson (1968) first identified the classical merger tradeoff between an increment in market concentration and efficiency gains. In a static Cournot framework, Farrell and Shapiro (2008) found sufficient conditions for mergers to enhance consumer surplus. Nocke and Whinston (2010) identify conditions under which the rule proposed by Farrell and Shapiro (2008) is optimal for a sequence of endogenous mergers. Nocke and Whinston (2013) study the scenario where those conditions do not apply. Mermelstein et al. (2015) analyze how merger policy affects investment and industry dynamics in a model where firms can grow—either internally or through mergers—to exploit economies of scale. The dynamic merger review literature has also incorporated endogenous entry and exit into the analysis. Pesendorfer (2005) shows that while entry may decrease the profitability of mergers, profitable mergers still exist. In a companion paper, Marshall and Parra (2015) analyze mergers in innovative industries when market structure is fixed and discuss strategic motives behind mergers that are absent when market structure is endogenous.

This paper also belongs to a new but increasing literature on how changes in the institutional background affect R&D in the context of sequential innovation. Segal and Whinston (2007) study how antitrust regulation affects R&D investments by changing how profits are divided between a technology leader and innovating follower. Parra (2015) studies how different aspects of patent policy affect the timing of the firms R&D investment decision. Hopenhayn et al. (2006) study how to implement efficient levels of R&D through the implementation of a general buyout scheme. Our methodology also embodies many models used in the literature of growth through innovation, most notably Aghion and Howitt (1992), Aghion et al. (2005), and Acemoglu and Akcigit (2012).

Finally, on the empirical side of the literature, measurement and identification issues have limited the study of how mergers affect innovation. Entezarkheir and Moshiri (2015) and Ornaghi (2009), for instance, find evidence consistent with post-merger R&D efficiencies. Entezarkheir and Moshiri (2015) finds a post-merger increase in innovation activity among merged firms, while Ornaghi (2009) finds that the post-merger R&D expenditure decreases among merged firms.

The rest of the paper is organized as follows. Section 2 introduces the baseline model and derives our main results on the effect of mergers on market outcomes. Section 3 extends the model in several ways and shows that our results are robust to alternative modeling choices. Lastly, Section 4 discusses our results and concludes.

2 Mergers in Innovative Industries

The main purpose of this section is to develop a simple and stylized dynamic model of an innovative industry. We use the model to examine the effects of relaxing a binding merger policy on the pace of innovation, industry concentration, and total expenditure in R&D.

2.1 Setup

Consider a continuous-time economy consisting of firms competing in an infinitely-lived market. At every instant of time there is one technology leader and n endogenously-determined followers. The market leader obtains a monopoly profit flow $\pi > 0$, while the followers make no direct profits from participating in the race. Followers, denoted by i, perform R&D to leapfrog the incumbent and become the new market leader. Firms discount their future payoffs at a rate $r > 0$ and we assume that firms are protected by infinitely long patents.[2]

At each instant in time, each follower i chooses its R&D investment level which induces a Poisson arrival of innovations $x_i \geq 0$. The cost flow of this investment is given by $c(x_i)$, which we assume is strictly increasing, twice differentiable, and strictly convex. The Poisson processes are independent among firms and generate a stochastic process that is memoryless.

In order to enter the R&D race, followers have to incur a fixed cost K. We interpret K as the cost of setting up a laboratory or a research facility. In addition, we also assume that this investment is reversible, i.e., firms are able to recover this investment when quitting the race. The reversibility assumption is later relaxed and, as will be shown, relaxing this assumption only strengthens the results presented in this section. Under the reversibility assumption, the opportunity cost of performing R&D at any instant of time is given by the returns of investing K at market interest rate r. The decisions of entering and exiting the race are, therefore,

[2] Section 3 shows that the main results hold in environments in which patent protection expires and in which new innovations may infringe currently active patents.

based on a comparison between the value of being a follower and the alternative use of the start-up investment.[3]

For the analysis to be of economic interest, we impose assumptions that guarantee a positive number of followers. We assume that the profit flow is larger than the sum of the opportunity cost of a follower and the fixed cost of performing R&D, i.e., $\pi > rK + c(0)$. In addition, to guarantee positive investments, we require that $c'(0) < \pi/r - K$. For simplicity we treat n as a continuous variable, with the interpretation that each firm has measure one.

2.2 Equilibrium Analysis

We study symmetric and stationary Markov perfect equilibria of the game, using a continuous time dynamic programming approach. Our assumptions guarantee the concavity of the value functions, implying that all computed equilibria will be unique. Let V be the value of being a market leader and W be the value of being one of the followers. Fixing any instant of time t, we can write the payoffs of both types of firms as

$$V = \int_t^\infty (\pi + \hat{x}W)e^{-(r+\hat{x})(s-t)}ds$$

$$W = \max\left\{K, \max_{x_i} \int_t^\infty (x_i V + x_{-i}W - c(x_i))e^{-(r+\hat{x})(s-t)}ds\right\},$$

where $\hat{x} = \sum_i x_i$ is the industry-wide *pace* or *speed* of innovation, and $x_{-i} = \sum_{j \neq i} x_j$. To understand the payoffs of participating in this industry, fix any instant in time $s > t$. With probability $\exp(-\hat{x}(s-t))$, no innovation has arrived between t and s. At that instant of time, the leader receives the flow payoff π and an innovation replaces it at a rate $\hat{x}$, turning the leader into a follower receiving W. On the other hand, a follower can compete or quit the race, in which case it receives the value of its capital, K. If it competes, the follower innovates at a rate x_i, pays the flow costs of its R&D, $c(x_i)$, and faces innovation by other firms at a rate x_{-i}. All these payoffs are discounted by $\exp(-r(s-t))$.

To solve the problem above, we make use of the Principle of Optimality, which

[3] We assume that if a potential entrant is indifferent between entering and not entering the race, the potential entrant chooses not to enter the race.

implies that at every instant in time, the values for each type of firm must satisfy

$$rV = \pi - \hat{x}(V - W) \tag{1}$$

$$rW = \max\left\{rK,\ \max_{x_i}\ x_i\,(V - W) - c(x_i)\right\}, \tag{2}$$

where $\hat{x} = \sum_i x_i$ is the *pace* of innovation in the industry.[4] In words, the flow value of being the market leader at any instant of time, rV, is equal to the profit flow obtained in that instant plus the expected loss if an innovation occurs. Similarly, the instantaneous value of being a follower, rW, must be the maximum between quitting the race and obtaining rK, and staying and achieving the incremental value of becoming the leader, $V - W$, at the rate x_i, minus the cost of R&D.[5]

In equilibrium, free entry and exit of firms guarantee that $W = K$. Maximizing the value function (2) and imposing symmetry, we find that a follower's investment level is given by

$$c'(x^*) = V - K, \tag{3}$$

or $x^* = 0$ if $c(0) > V - K$. That is, at every instant of time the followers invest until the marginal cost of increasing the arrival rate is equal to the incremental rent of a successful innovation. Strict convexity implies that condition (3) can be inverted so that $x^* = f(V - K)$, where $f(z)$ is an strictly increasing function of z (this function is characterized in Lemma 1 in the Appendix). By replacing x^* into equation (2), we can solve the innovation game. We describe the equilibrium in the following proposition.

Proposition 1 (Pre-merger equilibrium)**.** *The equilibrium in the industry is char-*

[4]For illustration purposes we show how to apply the Principle of Optimality to the leader's value. Starting at an arbitrarily small time interval $[t, t + dt)$; the leader's value satisfies:

$$V = (\pi + \hat{x}W)dt + e^{-(r+\hat{x})dt}V$$

That is, the value of being the leader is equal to the payoff flow at that instant dt in time, plus its discounted expected continuation value. For sufficiently small dt, the discount factor $\exp(-(r + \hat{x})dt$ is equal to $1 - (r + \hat{x})dt$. Equation (1) is obtained by substituting in and rearranging. From now on, we will apply the Principle of Optimality directly to all payoff functions.

[5]Because of the infinite patent protection and the assumption that a new innovation completely replaces the old technology, Arrow's replacement effect implies that the incumbent performs no R&D. Both of these assumptions will be relaxed in Section 3. Also, we omit the leader's exit condition as in equilibrium $V > K$. Similar considerations will apply to the merged firm value equation (8) below.

acterized by $W = K$ and by the unique positive V that solves

$$rK = f(V - K)(V - K) - c(f(V - K)). \tag{4}$$

Each follower's investment is given by equation (3) evaluated at the equilibrium values of V and W. Finally, the equilibrium number of followers in the industry is given by

$$n = \frac{\pi - rV}{x^*(V - K)}. \tag{5}$$

We note that this simple model captures many of the features we expect from a model of innovation with endogenous market structure. For instance, the equilibrium number of followers, n, increases with a larger profit flow, a lower discount rate, or lower entry costs. Similar results can be shown for the speed of innovation. We summarize these results in the following proposition.

Proposition 2 (Comparative statics)**.** *The pace of innovation $\hat{x}$ and the equilibrium number of firms in the industry, n, are increasing in the profit flow, π, and decreasing in both entry costs, K, and the interest rate, r.*

2.3 Merger Analysis

In this section, we study the impact of relaxing a binding merger policy, allowing for an R&D-enhancing merger to take place. In particular, we explore the effect of a one-time unexpected merger on the degree of market concentration, pace of innovation, and total R&D expenditure. The one-time change in the merger policy assumption simplifies the analysis in the sense that firms do not have expectations of facing other mergers in the future. The unexpected change assumption guarantees that the characterization in Proposition 1 is the proper comparison for the post-merger scenario.

A merger consists of two firms coming together to exploit synergies in their R&D processes and form a new firm of size one. We capture these synergies by assuming that the merged firm, denoted by M, is more effective than other firms at the moment of translating its R&D investments into breakthroughs. In particular, we assume that the investment of the merged firm induces innovations at a rate $\phi y_M \geq 0$, where the parameter $\phi > 1$ captures the extent to which the merged firm is more effective than its rivals. To facilitate comparisons with the pre-merger scenario, we denote the arrival rate of all other firms by y_i and the total number of followers (including the merged firm) by m. To simplify exposition, we assume that the higher effectiveness of the merged firm (i.e., $\phi > 1$) lasts until the merged firm achieves a breakthrough. As we discuss in Section 3, our results will be robust to making the efficiency gains permanent.

Let L be the value of being a non-merged firm market leader, F be the value of being a non-merged firm follower, and M (with abuse of notation) be the value of being a merged-firm follower. By the Principle of Optimality, the values for each type of firm satisfy

$$rL = \pi - y_{-M}(L - F) - \phi y_M(L - W) \tag{6}$$

$$rF = \max\left\{rK,\ \max_{y_i}\ y_i\,(L - F) - c(y_i) - \phi y_M(F - W)\right\} \tag{7}$$

$$rM = \max_{y_M}\ \phi y_M\,(V - M) - c(y_M), \tag{8}$$

where $y_{-M} = \sum_{i \neq M} y_i$, and V and W are the pre-merger values of being a follower and a leader (see equations (1) and (2)). Equations (6) to (8) possess the same structure than the equation in the pre-merger scenario, but with two key distinctions: (i) the existence of a merged firm that is more efficient than the other followers, and; (ii) once the merged firm succeeds, the efficiencies expire and the industry returns to the pre-merger scenario with n firms.

In equilibrium, free entry and exit of firms guarantee that $F = K$. Optimizing with respect to the followers' arrival rates and using symmetry we find that the followers invest according to the incremental rent they obtain from participating in the race

$$c'(y^*) = L - K \qquad \text{and} \qquad c'(y_M^*) = \phi\,(V - M)\,. \tag{9}$$

To characterize the post-merger equilibrium, we split the equilibrium analysis for when the efficiency gains are small ($\phi < \bar{\phi}$) or large ($\phi \geq \bar{\phi}$), where the value of $\bar{\phi}$ is defined as the smallest value of ϕ such that the efficiency of the merged firms incentivizes all non-merged firms to exit (i.e., foreclosure): $m = 1$. $\bar{\phi}$ is implicitly defined by[6]

$$\bar{\phi} f(\bar{\phi}(V - M)) = \frac{\pi - rV}{V - K}. \tag{10}$$

Proposition 3 (Post-merger equilibrium: $\phi < \bar{\phi}$)**.** *The industry equilibrium is*

[6] Lemma 3 in the Appendix shows the existence and uniqueness of $\bar{\phi}$.

characterized by $F = K$, $L = V$, and by the unique positive M that solves

$$rM = \phi f(\phi(V - M))(V - M) - c(f(\phi(V - M))). \tag{11}$$

In equilibrium $M \in (K, V)$, $y^* = x^*$, and the merged firm's investment is given by condition (9) evaluated at the equilibrium value of M. The total number of followers in the industry is given by

$$m = n + 1 - \frac{\phi y_M^*}{y^*}. \tag{12}$$

An important consequence of Proposition 3 is that the merger policy is in fact restrictive. Under reversible investments, firms will always have incentives to merge when they have synergies to exploit. To see this, observe that once two firms have merged they gain $M - 2W$, plus the value K of the redundant capital after the merger. Thus, the incremental value of a merger is $M - K > 0$, proving that firms have incentives to merge as long as $\phi > 1$.[7]

Perhaps surprisingly, Proposition 3 states that the value of being the industry leader is not affected by a merger. As a consequence, the incremental rent of the non-merged followers is not affected by the merger, keeping the followers' R&D investments constant. This result is driven by the entry and exit conditions. Since in equilibrium the value of being a follower remains unchanged (i.e., $F = W = K$) and a follower's value is only a function of the leader's value (see equations (2) and (7)), the value of being the industry leader must also remain constant after the merger. The degree of freedom to accommodate the merger efficiencies is, therefore, the number of followers.

To further explore the implications of a the merger, it is convenient to write the leader's value (6) as

$$rL(\text{pace}) = \pi - \underbrace{((m-1)y_i + \phi y_M)}_{\text{pace}}(V - K),$$

which is illustrated in Figure 1.a. From the equation above we can observe that the equilibrium value of the industry leader is decreasing in the pace of innovation. Since in equilibrium $L = V$, the equilibrium pace of innovation is found by setting $L(\text{pace}) = V$. As can be noted, the solution to this equation is independent of

[7]See the proof of Proposition 3 for details.

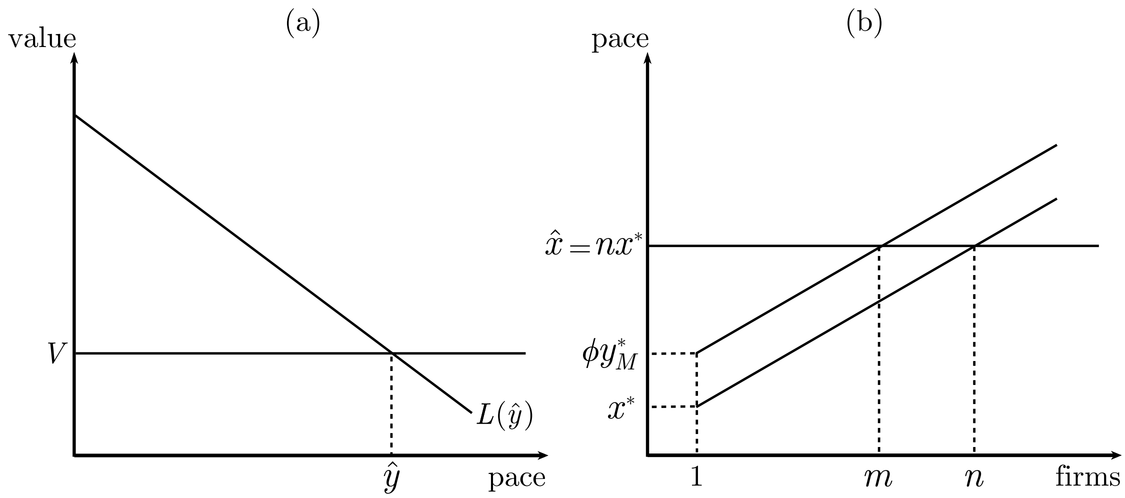

Figure 1: Equilibrium number of followers under different levels of ϕ.

ϕ as long as foreclosure does not take place (i.e., $\phi \leq \bar{\phi}$). This shows that a merger leaves the pace of innovation unchanged whenever the efficiency gains are sufficiently small.

To obtain a better understanding of how the merger affects the number of firms, observe that ϕy_M is increasing in ϕ (see Lemma 2 in Appendix) while x^* is unaffected by ϕ. Consequently, as illustrated in Figure 1.b, for a given pace of innovation, $\hat{x}$, a higher ϕ implies that the merged firms performs more of the R&D that the industry can accommodate, incentivizing inefficient firms to exit the market. In turn, the displacement of inefficient firms implies a decrease of R&D expenditure in the industry. This discussion is summarized in the following proposition.

Proposition 4 (Effect of a merger I)**.** *A merger with small efficiency gains ($\phi < \bar{\phi}$):*

1. *Increases market concentration ($m < n$).*
2. *Does not affect the speed of innovation.*
3. *Decreases overall R&D expenditure.*

When instead the efficiency gains are large (i.e., $\phi \geq \bar{\phi}$), the merged firm alone can achieve an innovation pace that is higher than the pre-merger pace of innovation and it induces the exit of all inefficient followers. This result is stated in the following proposition.

Proposition 5 (Effect of a merger II)**.** *If the efficiency gains are large ($\phi \geq \bar{\phi}$), the merged firm forecloses the market, becoming the unique follower (i.e., $m = 1$).*

In this case, the merger increases the pace of innovation.

In simple words, Proposition 4 and 5 imply that as long as there is timely (but costly) entry, mergers are always desirable from an innovation standpoint. Mergers either decrease the total cost of R&D without affecting the pace of innovation or directly increase the speed of innovation in the industry. Also noteworthy is that these results hold even for mergers with $\phi = 1$ (i.e., no efficiency gains) as the market equilibrium is unaltered by the merger. This implies that the efficiencies do not even need to be verified to assess the effect of a merger in this model.

3 Robustness

In order to check the robustness of the previous results, we show that the desirability of mergers in innovative industries hold in several extensions of the model. We consider the case when the efficiency gains are small enough that the merged firm cannot foreclose its rivals (i.e., $\phi \leq \bar{\phi}$). While restrictive, this shifts our attention to the policy relevant cases where a merger brings less social benefits.

3.1 Permanent Increase in Productivity

In the previous section we assumed that the merger efficiency gains lasted until the merged firm achieved its first innovation. Although it may be reasonable to think that in some industries the efficiency gains are transitory, the results described above do not rely on this assumption. To see this recall that the value of being the market leader, V, is determined by the inefficient followers' value in equilibrium (equations (2) and (7)) in conjunction with the entry and exit conditions. Therefore, V does not depend on whether the increase in productivity of the merged firm is permanent. More generally, having firms with different productivity levels will not affect the leader's equilibrium value as long as the marginal firm entering the industry is an inefficient follower. Moreover, since the pace of innovation is a function of the leader's value, V, the observation that the pace of innovation remains constant after a merger also extends to the scenario with heterogeneous firms.

3.2 Merger between a Leader and a Follower

The result in the previous section—the speed of innovation is constant even in presence of firm heterogeneity—carries through to a scenario in which the merging parties are the industry leader and a follower. This result is of interest in competition policy, as the merger guidelines put special attention on whether these type of mergers would delay the arrival of innovations.

The intuition for the result is as follows. As before, regardless of any efficiency gain, Arrow's replacement effect and the infinitely long patent protection implies that the—now merged—industry leader would still choose to not perform R&D while leader. In the meantime that the merged firm chooses not to perform R&D, an entrant compensates for the lost R&D, leaving market competition unchanged relative to the pre-merger scenario.

Once the merged firm loses its leadership position, the firm becomes a merged follower and the equilibrium is characterized by that in Section 2.3. That is, a merger between a leader and a follower has the same implications than one between two followers. The differences with Section 2.3 are that the merger efficiency gains are delayed until the merged firm starts to perform R&D and that the merged leader has the benefit of becoming a more efficient follower once losing its leadership position.

3.3 Patent Infringement

Thus far we have assumed away the possibility that an innovating follower may infringe the leader's patent. This section shows that the results in the previous section are robust to the inclusion of this possibility. We assume that the reward obtained from an innovation is represented by a continuously increasing function $\ell(v)$ satisfying $\ell(v) \leq v$ and $\ell(\pi/r) > K$, where $v \geq 0$ is the value of being the industry leader.[8] The function ℓ encompasses multiple forms of licensing schemes. For instance, $\ell(v) = \beta v$ corresponds to the follower's reward after Nash-bargaining with the leader, where $\beta \in [0, 1]$ represents its relative bargaining power (Green and Scotchmer (1995)); or $\ell(v) = v - \alpha r$ to the case with license fees consisting of royalties $r > 0$ incurred with probability $\alpha \in [0, 1]$, where α represents the strength of the patent (Farrell and Shapiro (2008)).

[8] That is, the follower's reward from an innovation increases with the value of being the leader, license fees are non-negative, and are not prohibitively high.

The baseline model, described in equations (1) and (2), is now modified in the following way

$$rV = \pi + nx(W - \ell(V))$$
$$rW = \max\left\{rK, \max_x x\left(\ell(V) - W\right) - c(x)\right\}.$$

There are two differences with respect to the baseline model. First, when the leader is replaced, the leader obtains license fees equal to $V - \ell(V)$ in addition to the incremental payoff of $W - V$. Second, when a follower innovates it receives the value of an innovation net of license fees, $\ell(V)$.

After a merger takes place, the value equations describing the behavior of the industry leader and the inefficient followers are analogous to the equations above but incorporating potential innovations coming from a merged firm (see equations (6) and (7)). Similar to an inefficient follower, the merged firm's value is now described by

$$rM = \max_{y_M} \phi y_M \left(\ell(V) - M\right) - c(y_M).$$

Proposition 6 (Patent Infringement)**.** *When patents may be infringed, a merger does not affect the pace of innovation, it concentrates the industry, and it reduces the total expenditure in R&D.*

The proof follows almost directly from Proposition 4. As before, the endogenous market structure guarantees that the value of being a follower is equal to K, regardless of whether a merger takes place. The followers' equilibrium condition then determines the value of an innovation net from license fees, $\ell(V)$, which in turn determines V. Since the value of being a leader is unaffected by a merger, the pace of innovation is also unaffected. Moreover, since the efficient merged firm displaces inefficient firms, overall R&D expenditure must fall.

3.4 Patent Protection

Previous results are also robust to the possibility that the protection granted by a patent expires. Following Parra (2015) we assume that patent protection lasts T years.[9] As we shall see below, finite patent protection breaks the stationarity of the

[9] Qualitatively identical results are obtained by following Acemoglu and Akcigit (2012) and assuming that patent protection terminates at an exogenous Poisson rate $\mu \geq 0$.

model providing leaders with incentives to perform R&D to extend their leadership position. These incentives increase as the patent expiration date approaches.

No active patent We start the analysis by studying a situation in which no patent is in place. When an industry leader loses its patent protection we assume that its innovation is imitated, which drives the leader's profit to zero and changes its status to being a follower. Denoting by z the investments when no patent is in place, we can write the followers' value of participating in the race as

$$r\hat{W} = \max\{rK, \max_{z_i} z_i(V - \hat{W}) - c(z_i) + z_{-i}(W - \hat{W})\}$$
$$r\hat{M} = \max_{z_M} \phi z_M(V - \hat{M}) - c(z_M) + z_{-M}(M - \hat{M})$$

where z_{-i} is the sum of the arrival rates of all rival followers —including the merged firm when applicable—, $z_{-M} = \sum_{i \neq M} z_i$, and V and W are the equilibrium values computed in Proposition 1 (see below for an explanation). The main difference of this scenario with respect to the baseline model is that now both types of firms take into consideration that they switch to the scenario with an active patent when a rival firm innovates.

Active patent We turn now to the scenario in which the leader is protected by a patent that lasts T years. As shown in the Online Appendix, followers are still characterized by equations (2) and (8). Hence, the value of being a non-merged follower and the equilibrium value of obtaining an innovation correspond to the same values that were computed in Proposition 1.

Denote by t the time that has passed since the last innovation.[10] The value of a leader evolves according to the following differential equation

$$rV(t) = \max_{x_{l,t}} \{\pi + x_{l,t}(V - V(t)) + \hat{x}(W - V(t)) - c(x_{i,t}) + V'(t)\}, \quad (13)$$

with boundary condition $v(T) = \hat{W}$, and where $x_{l,t}$ is the R&D investment performed by the leader at t and $\hat{x} = \sum_{i \neq l} x_i$ is the aggregate R&D performed by followers. Equation (13) can be interpreted as follows. At every instant in time t, the leader receives the profit flow of being the market leader; it innovates at a rate

[10]For instance, $t = 0$ means that an innovation just arrived and that the patent will not expire for another T years. More generally, $t < T$ means that there are $T - t$ years left before patent expiration.

$x_{l,t}$, earning an incremental rent $V - V(t)$; it is replaced at a rate $\hat{x}$; it pays the costs of its R&D, $c(x_{i,t})$; and it loses the value $V'(t)$ as its patent approaches the expiration date.[11] The boundary condition tells us that, once the patent expires, the leader becomes one of the many followers in the race.

The first order condition of problem (13) is $c'(x_{l,t}) = V - V(t)$. Arrow's Replacement effect tells us that firms invest according to the incremental rent they obtain from an innovation. Since the value of an active patent declines as its expiration date approaches, the incremental rent of the leader increases as the time goes by and so does its investment. Also, because of the boundary condition and $V(0) = V$ in equilibrium, the leader's investment is zero at $t = 0$ and increases to $V - \hat{W}$ when $t = T$—see Figure 2.a for a representation.

In order to obtain an analytic solution for the differential equation (13) we need to impose further structure to the cost function. In particular, we assume that $c(x) = x^2/2$. As shown in the Online Appendix, the solution to the problem above is

$$V(t) = \frac{(\hat{x}(2\hat{W} - W) + \theta_1)(e^{\varphi(T-t)} - 1) + \hat{W}\varphi(e^{\varphi(T-t)} + 1)}{(\hat{x} + \theta_2)(e^{\varphi(T-t)} - 1) + \varphi(e^{\varphi(T-t)} + 1)}, \tag{14}$$

where θ_1, θ_2, and φ are positive constants. It is not hard to verify that equation (14) is decreasing in t, and satisfies $v(T) = \hat{W}$. However, the boundary condition does not guarantee that $V(0) = V$. The equilibrium value of a newly issued patent, $V(0)$, is both a function of V (i.e., the value of an innovation that sets the followers market in equilibrium) and the followers' pace of innovation $\hat{x}$.

Proposition 7 (Patent Protection)**.** *There is a unique followers' pace of innovation, $\hat{x}^*$, that sets the market in equilibrium, i.e., $V(0) = V$. As a consequence, regardless of whether a patent is in place, a merger does not affect the pace of innovation, it concentrates the industry, and reduces total expenditure in R&D.*

Once again, the timely entry assumption guarantees that the value of being a follower is equal to K regardless of whether there is a merger or a patent in place. When no patent is in place, the value of being a follower $\hat{W}$ is a decreasing function of the pace of innovation in the industry. Consequently, the entry and exit conditions imply $\hat{W} = K$ which in turn determines the innovation pace.[12] A

[11] In equilibrium $V'(t) < 0$ for all $t < T$.

[12] Note also that when there is no patent in place, firms have no incentives to exit as the equation for W exactly matches the equation for $\hat{W}$ in equilibrium. Implying that the marginal firm has no incentives to exit when the market becomes leaderless.

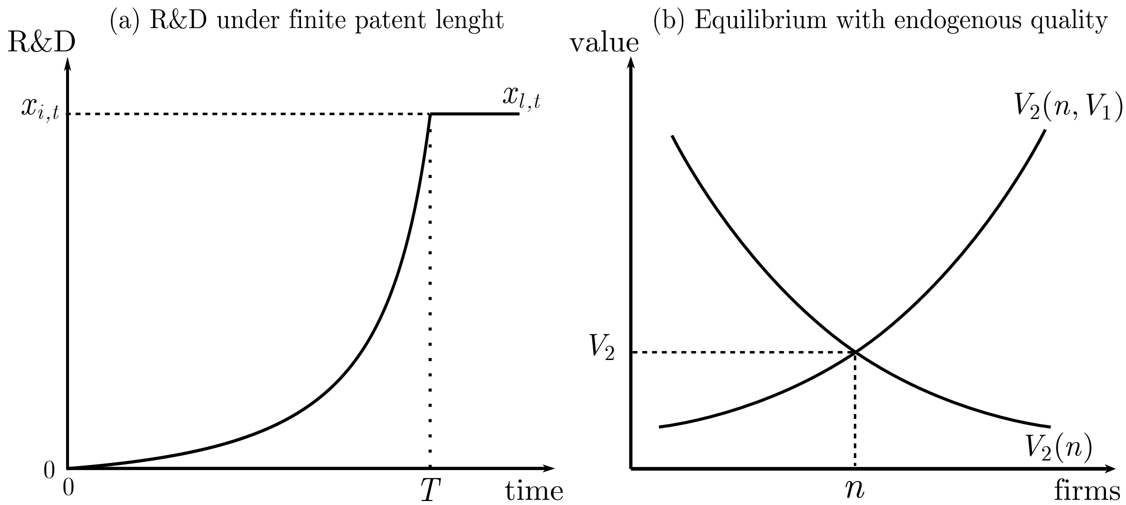

Figure 2: Robustness.

merger simply displaces inefficient followers to keep the innovation pace constant and reduces the industry expenditure in R&D.

When a patent is in place, a follower's value equation determines the reward V that an innovation has to give the marginal firm in order for it to be indifferent between participating in the race or not. From the leader's standpoint, the value of an innovation, $V(0)$, depends on the reward of a new innovation, V, but also on the level of competition it faces, $\hat{x}$. As shown in the Online Appendix, the leader's value is decreasing in $\hat{x}$. Thus if $V(0) > V$ (or $V(0) < V$) the industry faces entry (exit), until the market reaches the equilibrium $V(0) = V$ at $\hat{x}^*$. Once again, a merger simply incentivizes inefficient followers to exit the market until the equilibrium followers' pace of innovation is reached.

3.5 Leader Innovation with Endogenous Quality

In this section we extend the model in two dimensions. First, while we still assume that followers perform radical innovations that completely replace the technology in place, we allow for incumbents to improve the quality of their existing products (Acemoglu and Cao, 2015). In addition, we let the increase in quality to be endogenously determined.

As before, each follower introduces a radical innovation at a rate x_1 and flow cost $c(x_1)$. The industry leader can improve the quality of its current product in q units, increasing its profit flow by $d(q)$. We assume $d(q)$ to be an increasing, differentiable, and concave function of q. We also assume that $d(0) = 0$, meaning that with no investment, profits cannot increase. The quality increase q arrives at a Poisson rate x_2 attained at flow cost $c(x_2, q)$. We assume that this cost function is increasing and convex. To guarantee uniqueness of equilibrium, we further assume that the derivative with respect to quality $c_q(x, q)$ is convex in x.

Although our results apply to environments in which the leader may improve the quality of its product multiple times, for illustration purposes, we examine a situation in which the leader can increase the quality of its product only once. Let V_s be the value of being a leader that has innovated $s \in \{1, 2\}$ times in a row. The pre-merger equilibrium is characterized by the followers value equation (2) and by replacing the leader's value equation (1) with the next two equations

$$rV_1 = \max_{x_2, q} \pi + x_2 (V_2(q) - V_1) - c(x_2, q) + nx_1(W - V_1) \tag{15}$$

$$rV_2(q) = \pi + d(q) + nx_1(W - V_2(q)), \tag{16}$$

The first equation describes the value of a being a leader that has innovated only once and that is expending resources to increase the quality of its product by an endogenously determined magnitude q at rate x_2. The second equation describes the value of a leader that has increased the quality of its innovation in q units and enjoys a profit flow $\pi + d(q)$ while it remains the industry leader.

The first order condition for the followers is given by equation (3), whereas the first order conditions for a leader investing in R&D are

$$c_x (x_2^*, q^*) = V_2 (q^*) - V_1 \quad \text{and} \quad c_q (x_2^*, q^*) = x_2^* V_2' (q^*) . \tag{17}$$

The first condition tells us that the leader will invest in speed as a function of the incremental rent from innovating, $V_2(q) - V_1$. The second condition tells us that quality and speed are complementary at the moment of choosing the optimal quality. It is not hard to verify that $V_2' > 0$, as higher quality increases the leader's profit flow after improving its product.

Similarly, the post-merger equilibrium is characterized by the followers' value equation (7), the merged firm equation (8), and by replacing the leader condition (6) with its analog of equations (15) and (16).

Proposition 8 (Innovating leader)**.** *The industry has a unique equilibrium before and after the merger. With a merger, the leader's innovation rate, the leader's choice of quality, and the followers' pace of innovation remain constant. A merger, however, reduces overall R&D expenditure. As a consequence, mergers are welfare* *improving.*

As before, the solution of the followers problem (2) in conjunction with the entry and exit conditions determine the equilibrium value of V_1 and the investment rate of the followers, x_1^*. The key difference with the baseline model arises in determining the value of being an innovating leader, V_2, and the number of followers in the industry. In equilibrium, V_2 depends on the leader's own quality investment decision and the total number of followers, n. Figure 2.b illustrates the intuition of how the number of followers and the value of being a leader that has innovated V_2 are jointly determined. Fixing an arbitrary quality $q > 0$, the equilibrium equation (16) can be written as

$$V_2(n) = \frac{\pi + d(q) + nx_1^* W}{r + nx_1^*},$$

a decreasing function of n, as an increase in the number of followers shortens the life span of the leader, decreasing its value. On the other hand, given the value of V_1, it is not hard to verify that equation (15) implies an increasing relation between n and V_2. That is, a larger number of competitors has to be compensated with larger R&D returns, $V_2(q)$, to keep the value of the innovating leader fixed at V_1. The equilibrium is, thus, the unique intersection of those curves.

When a merger occurs, the entry and exit conditions guarantee $W = F = K$. This implies that the equilibrium condition for the followers before and after the merger are identical, which determine that both the value of being a leader that has innovated once and the followers' investments remain constant (i.e., $L_1 = V_1$ and $y_1^* = x_1^*$, respectively). Consequently, the number of followers m adjusts so that both the pace of innovation, $(m-1)y_1^* + \phi y_M = nx_1^*$, and the quality choice of the leader remain unchanged in equilibrium.

3.6 Price Competition in a Quality Ladder

For simplicity, previous sections have analyzed the impact of a merger ignoring that firms may compete in the downstream product market. Here we show that the results carry through to environments in which innovations lead to a lower marginal cost of production and firms compete in prices in the product market. We do this by simply showing that such a scenario maps directly into the setting in Section 3.5.

Suppose the product market is represented by a hyperbolic demand $q = A/p$, where q is quantity, p the price, and A is any positive constant. The industry leader invest in R&D in order to reduce the current lowest marginal cost in the market by an endogenously-determined factor $1 - \alpha \in [0, 1]$. Followers instead invest to reduce the marginal cost by a predetermined factor $\beta \in (0, 1)$.[13] That is, if the current marginal cost is τ, breakthroughs of size $(1 - \alpha)$ and β would reduce marginal cost to $(1 - \alpha)\tau$ and $\beta\tau$, respectively. We assume that the cost of investing in technologies that drives the marginal cost to zero is prohibitively costly, which makes $c(x, \alpha)$ consistent with the assumptions in Section 3.5.

It is not hard to check that in this model the equilibrium price in the product market is equal to the second lowest marginal cost, κ. At this price, a succeeding follower would earn a profit flow of $\pi = (\kappa - \beta\kappa)A/\kappa = (1 - \beta)A$, which is independent of the number of innovations that have taken place. That is, first time innovators earn a constant profit flow as in Section 3.5.

Finally, note that the profit flow of a leader that has achieved two consecutive innovations is $d(\alpha) = (1 - \beta(1 - \alpha))A$, which is increasing and (trivially) concave in α.[14] Note also that $d(0) = 0$. Consequently, the price competition environment described here maps directly into the setting in Section 3.5, which implies that all the results above apply to this case as well.

3.7 Irreversible Investments

We now investigate the industry equilibrium when start-up investments, K, are irreversible. The irreversibility of K introduces an exit friction under which firms may choose not to exit the industry when facing more competition due to a merger. In concrete, we assume that by exiting, a firm does not recover any if of its initial investment.[15]

Since prior to allowing the merger no firm was faced with incentives to exit the market, the pre-merger analysis corresponds to that in Section 2.2. As a result, we focus on analyzing the post-merger period. Since the exit condition is non-binding for the market leader and merged firm, the only equation that changes is that of

[13] The assumption that β is exogenously given can be relaxed. If followers were to choose how much to improve the current technology, their choice of β would be the same regardless of the marginal cost in the industry. Thus, the model would still map to that of Section 3.5.

[14] Note also that the incremental flow of a t-th innovation is $A\beta\alpha_t \prod_{j=2}^{t-1}(1 - \alpha_j)$, where $1 - \alpha_k$ is the endogenously-determined cost reduction factor of the k-th innovation. Note that the incremental flow is decreasing in t, which justifies that leaders will pursue only so many improvements when the cost function, c, includes a fixed cost of performing R&D.

[15] More generally, qualitatively identical results are obtained by assuming that firms recover a fraction $\delta \in [0, 1)$ of K. For ease in exposition, we present the case $\delta = 0$.

the followers, which becomes

$$rF = \max\left\{0, \max_{y_i} y_i(L-F) + \phi y_M(W-F) - c(y_i)\right\}, \tag{18}$$

where W is the pre-merger value of being a follower defined by (2). The interpretation of this value function is analogous to that in the baseline model, with the difference that by exiting a follower does not recover its start-up investment.[16] The exit friction reduces the opportunity cost of firms, which will create cases where the market will accommodate "too many" firms relative to the frictionless case.

Using equation (12) define $\tilde{\phi}$ as the value of ϕ under which $m = n-1$ in the baseline model, i.e.,

$$\tilde{\phi} f(\tilde{\phi}(V-M)) = 2f(V-K). \tag{19}$$

In words, $\tilde{\phi}$ is the value of ϕ such that no firm chooses to enter or exit after a merger. When the efficiency gains of a merger are small ($\phi \in [1, \tilde{\phi}]$) firms will enter (rather than exit) the market.[17] Thus, the number of firms under exit friction m^f equals the baseline number of firm $m > n-1$. Since no firm exercises its option of exiting the market, the exit friction plays no role for these values of ϕ. Consequently, all the results in Proposition 4 apply to the case of small efficiency gains.

On the other hand, when the efficiency gains from a merger are large ($\phi > \tilde{\phi}$), there would be exit in the frictionless case. With exit frictions, however, firms choose not to exit because their opportunity cost is less attractive than remaining in the race: $0 \leq F \leq K$. This implies that there are "too many" firms relative to the frictionless case (i.e., $m^f = n-1 > m$). Figure 3.a illustrates this point.

Proposition 9 (Irreversible Investments)**.** *An R&D enhancing merger under irreversible investments,*

1. *Increases market concentration ($m \leq m^f < n$).*
2. *Increases the speed of innovation when $\phi > \tilde{\phi}$, while it does not affect the speed of innovation when $\phi \in [1, \tilde{\phi}]$.*

Figure 3.b depicts aggregate investments $\hat{y}$ after a merger takes place. When a merger leads to small efficiency gains, entry ensures that the pace of innovation

[16] Note that in Section 2, firms had incentives to merge for all values of $\phi > 1$. With exit frictions, incentives to merge arise only when ϕ is high enough to compensate for the losses that the merged firm suffers when liberating capital. That is, ϕ must be such that $M - 2K > 0$. Examples that satisfy this condition are available upon request.

[17] Existence of $\tilde{\phi}$ follows from similar arguments to those given in Lemma 3.

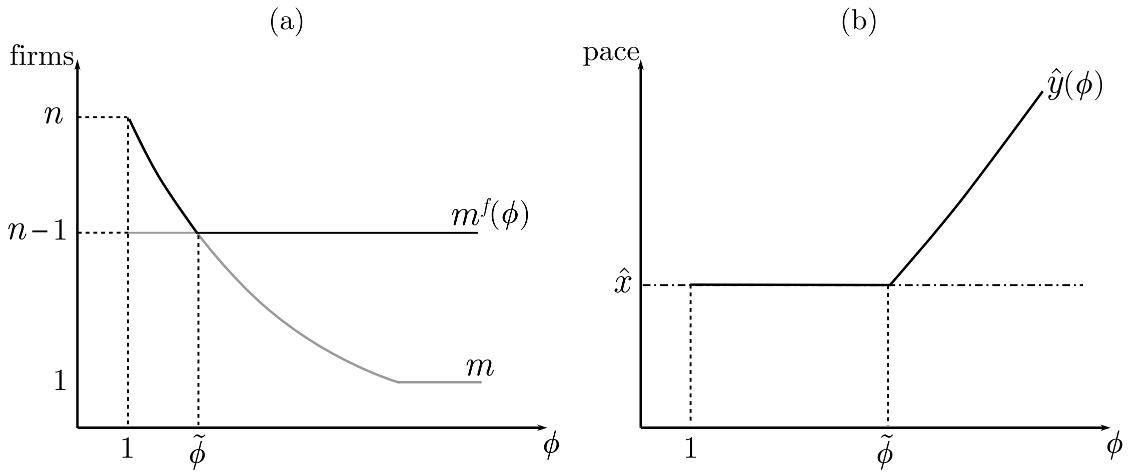

Figure 3: Exit frictions.

corresponds to that in the pre-merger scenario. Once the exit friction starts to bind, there are "too many" firms investing with respect the frictionless case, which speeds up the innovation process. Thus, irreversible investments simply reinforce the desirability of a merger from an innovation standpoint.

4 Discussion and Concluding Remarks

The tension between productive efficiency and market concentration is at heart of traditional merger analysis. Several authors argue that analyzing a merger in innovative industries based on this tradeoff alone is inappropriate. For instance, the merging firms may not even compete in the product market. To this end, we propose a dynamic framework for merger analysis in innovative industries. The framework follows a "creative destruction" approach where new products replace old ones and firms compete to invent a sequence of products. Our approach captures many of the issues that make mergers in innovative industries special: market structure changes rapidly by the introduction of new products, market leaders face the threat of being replaced by new products, merging firms may not even have products in the product market, and R&D may be towards products that are yet to reach the market. We use our framework to analyze how mergers with R&D efficiencies affect market outcomes over time.

We find that timely (but costly) entry is a sufficient condition to guarantee the desirability of a merger from an innovation standpoint. In particular, a merger either decreases the waiting time between innovations or keeps the waiting time constant while reducing the industry expenditure in R&D. We show that this finding is robust to many variations of the model, which makes it a useful result for competition policy.

We do wish to point out that our welfare results are from an innovation standpoint only. To see this, consider the setting in Section 3.6 where firms compete in prices in the product market and to achieve innovations that lower the marginal cost of production. A merger between the market leader and the follower with second lowest marginal cost increases the price in the product market and, at the same time, is welfare improving along the innovation dimension. This example shows that there may be a tension between the classic Williamson tradeoff in the product market and the dynamic benefits of R&D efficiencies.

One may also argue that in some industries entry costs are irreversible and have evolved to become prohibitive, implying that market structure is essentially fixed. In Marshall and Parra (2015) we consider such case and note that some new issues arise. We find that mergers create a tradeoff between less competition in the patent race and R&D efficiency gains, which is created by the absence of the stabilizing effects of entry and exit. We also argue that in this case, in particular, a leader and a follower may have extra incentives to merge as a merger would extend the expected lifespan of the leader by decreasing the pace of innovation in the industry. As argued in Section 3.2, this preemptive motive does not exist when timely (but costly) entry is feasible.

We believe that this framework and set of results contribute to the development of a comprehensive theory of mergers. In particular, it is a first step towards understanding the key trade-offs generated by mergers in innovative industries.

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Appendix

A Preliminary Results

Lemma 1. *The function $f(z)$ implicitly defined by $c'(f(z)) = z$ satisfies:*

1. *$f(z) > 0$ for all $z > 0$ and $f(0) \geq 0$.*
2. *$f'(z) > 0$ for all $z > 0$.*
3. *Let $h(z) = zf(z) - c(f(z))$ for $z \geq 0$. Then $h'(z) = f(z) > 0$ for all $z \geq 0$.*

Proof. 1. From $c(x)$ being increasing and differentiable, $c'(x) > 0$ for all $x > 0$. From $c(x)$ being convex, $c''(x) > 0$ for all $x > 0$. Thus, $c'(x)$ is unbounded above and for each z there exists a unique value of $x = f(z) > 0$ such that $c'(x) = z$.
2. The result follows from the derivative of the inverse function being equal to $f'(z) = 1/c'(f(z)) > 0$.
3. Differentiating h and using $c'(f(z)) = z$ delivers $h'(z) = f(z)$. The result then follows from claim 1. □

Lemma 2. *The following are increasing in ϕ:*

1. *The value of being a merged firm, M.*
2. *The R&D of the merged firm, y_M^*.*
3. *The arrival rate of the merged firm, ϕy_M^*, which is also unbounded above.*

Proof. 1. Implicit differentiation of (11) delivers

$$\frac{dM}{d\phi} = (V - M)\frac{y_M^*}{r + \phi y_M^*} > 0.$$

2. Using the expression above and Lemma 1.2, the derivative of $y_M^* = f(\phi(V-M))$ is given by

$$\frac{dy_M^*}{d\phi} = f'(\phi(V - M))(V - M)\frac{r}{r + \phi y_M^*} > 0.$$

3. The result is a consequence of the previous claim and ϕ being unbounded above. □

Lemma 3. *There exist a unique value of ϕ_1 implicity defined by*

$$\frac{\pi - rV}{V - K} - \phi_1 f(\phi_1(V - M)) = 0.$$

Proof. Define $\psi(\phi) = (\pi - rV)/(V - K) - \phi f(\phi(V - M))$. From equation (5) if follows that $\psi(1) = (n - 1)f(V - K) > 0$, since prior to the merger it is true that $n > 1$ and $V > K$ (see Proposition 1 and Lemma 1.1). From Lemma 2.3 we know that $\phi f(\phi(V-M))$ is increasing and unbounded above in ϕ. Thus, ψ single-crosses zero from above and uniqueness of ϕ_1 follows. □

B Proofs

Proof of Proposition 1 Equation (4) follows from the entry and exit conditions and replacing x^* into equation (2). It has a unique solution such that $V > K$ as the right hand side of (4): (i) satisfies $-c(f(0)) < rK$ if $V = K$; (ii) is strictly increasing in V (from the the envelope theorem we know that its derivative is $f(V-K)$, which we found is positive in Lemma 1.1), and; (iii) is unbounded above as its second derivative is $f'(V-K)$, which we found is positive in Lemma 1.2. The number of firms follows from replacing $\hat{x} = nx^*$ in equation (1) and solving for n. In equilibrium $n > 0$, otherwise $V = \pi/r$ and $c(0) < V - K$, which implies that entry would occur.

Proof of Proposition 2 Using Lemma 1.3 and implicit differentiation of equation (4) we obtain

$$\frac{dV}{dK} = \frac{x^* + r}{x^*} \qquad \frac{dV}{dr} = \frac{K}{x^*} \qquad \frac{dx^*}{dK} = \frac{rf'}{x^*} \qquad \frac{dx^*}{dr} = \frac{Kf'}{x^*}$$

By Lemma 1.1 and 1.2 all derivatives are positive. Differentiating equation (5) we obtain:

$$\frac{d\hat{x}}{d\pi} = \frac{1}{V-K} > 0 \qquad \frac{dn}{d\pi} = \frac{1}{x^*}\frac{d\hat{x}}{d\pi} > 0$$

$$\frac{d\hat{x}}{dK} = -r\left(\frac{\pi - rK}{x^*(V-K)} + \frac{d\hat{x}}{d\pi}\right) < 0 \qquad \frac{dn}{dK} = \frac{1}{x^*}\left(\frac{d\hat{x}}{dK} - \frac{nrf'}{x^*}\right) < 0$$

$$\frac{d\hat{x}}{dr} = -\frac{d\hat{x}}{d\pi}\left(V + \frac{K(\pi - rK)}{x^*(V-K)}\right) < 0 \qquad \frac{dx^*}{dr} = \frac{d\hat{x}}{dr} - \frac{nKf'}{(x^*)^2} < 0.$$

Proof of Proposition 3 The entry and exit conditions imply that equation (7) coincides with equation (4), implying that the equilibrium value for L is the same as that for V in Proposition 1, i.e., $L = V$. The entry and exit conditions, the followers first order condition in (9), and $L = V$ imply that $y^* = x^*$. The value of M is determined by replacing the merged firm first order condition (9) in (8) and solving. To see that M has a unique solution observe that the left hand side of (11) is increasing in M, and the right hand side is decreasing in M by Lemma 1.3. Hence, if there is a solution, it must be unique. Imposing $M = V$ delivers $rV > -c(f(0))$. Imposing $M = K$ delivers

$$\begin{aligned} rK &= (V-K)f(V-K) - c(f(V-K)) \\ &< \phi(V-K)f(\phi(V-K)) - c(f(\phi(V-K))), \end{aligned}$$

where the equality follows from (4) and the inequality follows from $\phi > 1$ and Lemma 1.3. The result then follows from the intermediate value theorem and it immediately implies that $M \in (K, V)$ when $\phi > 1$, and $M = K$ when $\phi = 1$. Finally, an expression for m is obtained by replacing $y_{-M} = (m-1)y^*$ and y_M^* into equation (6).

Proof of Proposition 4 1. From (12) we know that $n > m$ if and only if $\phi y_M > y^*$. When $\phi = 1$, condition (11) implies $\phi y_M = y^*$. From Lemma 2.3, we know that ϕy_M increases with ϕ. Combined, these deliver the result.
2. In equilibrium, we have $V = L$. Then, using equations (1) and (6) in equilibrium, we obtain that

$$nx_i^* = \frac{rV - \pi}{K - V} = \frac{rL - \pi}{K - L} = (m-1)y_i^* + \phi y_m^*,$$

which proves the result.
3. Using $x^* = y^*$ and equation (12), we write the change in expenditure as

$$\Delta \text{Exp} = nc(x^*) - ((m-1)c(x^*) + c(y_M^*)) = \frac{\phi y_M^*}{x^*} c(x^*) - c(y_M^*).$$

Using the equilibrium values of (2) and (8) we substitute for $c(x^*)$ and $c(y_M^*)$ in the expression above and obtain

$$\Delta \text{Exp} = \phi y_M^* \left(M - \frac{r + x^*}{x^*} K \right) + rM. \tag{20}$$

We note that when $\phi = 1$, $\phi y_M^* = x^*$ and $M = K$ so that $\Delta\text{Exp} = 0$. Claim 1 shows that $dM/d\phi$ and $d(\phi y_M^*)/d\phi$ are both positive, which implies $\Delta\text{Exp} > 0$ for all $\phi > 1$.

Proof of Proposition 5 For $\phi \geq \phi_1$, we have that $m = 1$ and that the pace of innovation is given by $\phi y_M^* = \phi f(\phi(V - M))$. At $\phi = \phi_1$ we know, from Proposition 4, that $\phi_1 y_M^* = nf(V - K)$. Also, from Lemma 2.3 we know that $d(\phi y_M^*)/d\phi > 0$. Combined, these relationships establish that the pace of innovation increases after the merger when $\phi > \phi_1$.

Online Appendix
Mergers in Innovative Industries: A Dynamic Framework

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Supplemental Material – Not for Publication

C Omitted Proofs

Proof of Proposition 6 First, a note on pre- and post-merger equilibria. The entry and exit conditions imply that $W = F = K$, which together with the optimal investment levels, determine $\ell(V) = \ell(L)$. Existence and uniqueness follow from arguments analogous to those in Proof of Proposition 1. The number of firms pre-merger and post-merger are given by

$$n = \frac{\pi - rV}{x^*(\ell(V) - K)} \qquad \text{and} \qquad m = n + 1 - \frac{\phi y_M}{x^*}.$$

The arguments to establish the effects of a merger are analogous to those in Proof of Proposition 4. That a merger increases market concentration follows from $m = n$ when $\phi = 1$ and m being decreasing in ϕ. Constant pace of innovation follows from $\ell(V) = \ell(L)$ and the pace of innovation being only a function of the value of the leader. Finally, as in Proof of Proposition 4, the change in expenditure satisfies equation (20), and the same argument applies.

Proof of Proposition 7 See Appendix D.

Proof of Proposition 8 By Proposition 1, the followers have a unique equilibrium which determines V_1. The first order conditions (17) characterize the equilibrium for an innovating leader. The Hessian of problem (15) is

$$(c_{xx}c_{qq} - (c_{xq})^2) + V_2'(q)c_{xq} - xV_2''(q) + V_2'(q)(c_{xq} - V_2'(q))$$

The first parenthesis is positive because convexity of costs. The second and third terms are positive because concavity of $d(q)$ implies $V_2'(q) > 0$ and $V_2''(q) < 0$. Using condition (17) the last parenthesis can be written as $c_{xq} - c_q/x \geq 0$. Where positive sign follows concavity in x of c_q. Thus the Hessian is positive, the solution is strict local maximum, and existence of equilibrium is guaranteed.

Uniqueness follows from observing that corner solutions (i.e., no R&D of infinite R&D) are not optimal, and that every point satisfying (17) is a locally unique maximum. Thus, since there are no minimum or saddle points, at most one maximum exists.

After a merger the frictionless assumption implies $W = F = K$. Thus, the followers value equation in a merger scenario becomes:

$$rK = \max_{y_1} y_1(L_1 - K) - c(y_1)$$

which is the same value equations of the followers pre-merger. In equilibrium this implies $L_1 = V_1$ and $y_1^* = x_1^*$. Let $\hat{q}$ represent the quality choice after a merger, the equilibrium value equations for a leader in the different states are

$$\begin{aligned} rV_1 &= \max_{y_2,\hat{q}} \pi + y_2(L_2(\hat{q}) - V_1) - c(y_2,\hat{q}) + ((m-1)x_1 + \phi y_M)(K - V_1) \\ rL_2(\hat{q}) &= \pi + d(\hat{q}) + ((m-1)x_1 + \phi y_M)(K - L_2(\hat{q})) \end{aligned}$$

For any value of ϕy_M, if m adjust so that $(m-1)y_1^* + \phi y_M^* = nx_1^*$. Then, the leader faces the same problem as in the pre-merger scenario. Therefore, $\hat{q} = q$ and since the problem has a unique equilibrium, the result follows.

To establish the results in Proposition 4 observe that a merger increases market concentration because $m = n$ when $\phi = 1$ and m decreases in ϕ. Constant pace of innovation follows from $V_1 = L_1$, $V_2 = L_2$, and the same choice of quality improvement after the merger. Finally, the change in expenditure can be written as in equation (20) which is increasing in ϕ and takes the value of zero when $\phi = 1$, proving the result that the expenditure falls after the merger.

Proof of Proposition 9 1. Follows directly from the definition of m^f.
2. We separately analyze the cases when $\phi \in [1, \tilde{\phi}]$, $\phi > \tilde{\phi}$. When $\phi \in [1, \tilde{\phi}]$, there is entry of firms after the merger as $m > n - 1$, implying that $F = K$ and that $L = V$. Consequently, the arguments in the proof of Proposition 4 apply, and the result follows.

When $\phi > \tilde{\phi}$. The arrival rate of innovations is given by $\hat{y}(\phi) = (n-2)f(L - F) + \phi f(\phi(V - M))$. The derivative of $\hat{y}(\phi)$ with respect to ϕ is given by

$$\begin{aligned} \frac{d\hat{y}}{d\phi} &= \frac{d\phi y_M^*}{d\phi}\left[1 - \frac{(n-2)(L-F)f'(L-F)}{r + (n-1)y^* + n(L-F)f'(L-F) + \phi y_M^*}\right] \\ &= \frac{d\phi y_M^*}{d\phi}\frac{r + (n-1)y^* + 2(L-F)f'(L-F) + \phi y_M^*}{r + (n-1)y^* + n(L-F)f'(L-F) + \phi y_M^*} > 0, \end{aligned}$$

where we make use of $d\phi y_M/d\phi > 0$ (see Lemma 2.3) and

$$\frac{d(L-F)}{d\phi} = -\frac{d\phi y_M^*}{d\phi}\frac{(L-F)}{r + (n-1)y^* + n(L-F)f'(L-F) + \phi y_M^*}.$$

Since $\hat{y}(\phi_1) = \hat{x}$ and $d\hat{y}/d\phi > 0$, the result follows.

D Equilibrium with Finite Patent Length

Suppose patents have a maximal duration of T years. Denote by $x_{l,t}$ the investment of the leader at time t and by $x_{i,t}$ the investments of entrant i. Denote the total number of firms participating in the race—including the leader—by n, and define $\mathbf{x}_t = \sum_{k\in n} x_{k,t}$ as the total investment in R&D at time t, and $x_{-j,t} = \sum_{k\in n\setminus j} x_{k,t}$ as the total investment performed by all firms but j.

Let V_t be the value of being the leader with a patent that was issued t years ago and W_t be the value of being a follower that faces a leader with a patent issued t years ago. We can write the payoffs as[18]

$$V_t = \max_{\{x_{i,s}\}_{s=t}^{\infty}} \int_t^T (\pi + x_{i,s}V_0 + x_{-i,s}W_0 - c(x_{i,s}))e^{-\int_t^s r+\mathbf{x}_k dk}ds + e^{-\int_t^T r+\mathbf{x}_k dk}\hat{W}$$
$$W_t = \max\left\{K, \max_{\{x_{j,s}\}_{s=t}^{\infty}} \int_t^{\infty} \left(x_{j,s}V_0 + x_{-j,s}W_0 - c\left(x_{j,s}\right)\right)e^{-\int_t^s (r+\mathbf{x}_k)dk}ds\right\},$$

where $\hat{W}$ is the value of being a follower in a scenario with no active patent. At instant of time t, the value of participating in the race is equal to the expected discounted sum of all future payoffs. For the leader this corresponds to the profit flow, π; the costs of R&D, $c(x_{l,t})$; the value of a new innovation, V_0, arriving at a rate $x_{l,t}$; and the value of becoming a new follower, W_0, arriving at a rate $x_{-i,t}$. The value of the followers is determined in a similar fashion.

Using the principle of optimality, we write the problem above as

$$rV_t = \max_{x_{l,t}} \{\pi + x_{l,t}(V_0 - V_t) + x_{-l,t}(W_0 - V_t) - c(x_{l,t}) + V_t'\}$$
$$rW_t = \left\{rK, \max_{x_{i,t}} \{x_{i,t}(V_0 - W_t) + x_{-i,t}(W_0 - W_t) - c(x_{i,t}) + W_t'\}\right\}$$

with corresponding first order conditions: $c'(x_{l,t}) = V_0 - V_t$ and $c(x_{i,t}) = V_0 - W_t$. Free entry and exit implies $W_t = K$ for all t, thus $c'(x_{i,t}) = V_0 - K$. Note that the followers' investments are constant over time. We denote the followers' investments by $x_i^* = f(V_0 - K)$. Replacing x_i^* into the followers' value, we can see that the equation becomes

$$rK = f(V_0 - K)(V_0 - K) - c(f(V_0 - K))$$

which corresponds to equation (4) in the baseline model. This implies that, in order to make the marginal follower indifferent between entering the race and not we must have $V_0 = V$ in equilibrium. Assuming a quadratic cost function $c\left(x\right) = x^2/2$, using $x_{-l,t} = (n-1)x_i^* \equiv \hat{x}$, and replacing the first order conditions we obtain the following differential equation

$$V_t' = aV_t^2 + bV_t + c$$

[18] For space considerations we use the more compact notation V_t instead of $V(t)$.

where $a = -1/2$, $b = r + \hat{x} + V$, and $c = -(2\hat{x}W + 2\pi + V^2)/2$. By defining $\varphi^2 = b^2 - 4ac$, we can write the general solution to this differential equation as[19]

$$V_t = b + \varphi \frac{1 + \exp\left(\varphi\left(C - t\right)\right)}{1 - \exp\left(\varphi\left(C - t\right)\right)}$$

where C a the constant of integration which defines a particular solution. To obtain the particular solution we use the boundary condition $V_T = \hat{W}$, from where we obtain

$$C = T + \frac{1}{\varphi} \ln\left(\frac{\hat{W} - b - \varphi}{\hat{W} - b + \varphi}\right).$$

By replacing C into the general solution, we obtain equation (14), where $\theta_1 = (2\pi - rK) + V(V-K)$ and $\theta_2 = r + (V-K)$. θ_1 and θ_2 are both positive given the assumptions of the model.

Equilibrium Existence Using the Intermediate Value theorem we prove that for every $V > K$ there is a value of $\hat{x}$ such that $V_0 = V$. To simplify notation, we make use of the equilibrium condition $\hat{W} = W = K$. Define $f(\hat{x}) = V_0 - V$, or

$$f(\hat{x}) = \frac{(\hat{x}K + \theta_1)(e^{\varphi T} - 1) + K\varphi(e^{\varphi T} + 1)}{(\hat{x} + \theta_2)(e^{\varphi T} - 1) + \varphi(e^{\varphi T} + 1)} - V.$$

It is not hard to verify that $\lim_{\hat{x}\to\infty} f(\hat{x}) = K - V < 0$. For the upper limit it is convenient to write $f(\hat{x})$ as

$$f(\hat{x}) = \frac{(\hat{x}(K-V) + \theta_1 - V\theta_2) + (K-V)\varphi\frac{e^{\varphi T}+1}{e^{\varphi T}-1}}{(\hat{x} + \theta_2) + \varphi\frac{e^{\varphi T}+1}{e^{\varphi T}-1}}.$$

Since the denominator is always positive, it is sufficient to find a value of $\hat{x}$ that makes the numerator positive. Take the value of $\hat{x} = x_o$ that makes $\varphi^2 = 0$.[20] Then, it suffices to show that $x_o(K-V) + \theta_1 - V\theta_2 > 0$. Replacing, we obtain

$$x_o(K-V) + \theta_1 - V\theta_2 = 2(\pi - rK)\left((V-K)^2 + 2(\pi - rK)\right)$$

which is positive under our usual assumptions.

Equilibrium Uniqueness To prove uniqueness we show that V_0 is strictly decreasing in competition, thus $f(\hat{x})$ single crosses zero from above. To this end, we show that the derivative of $f(\hat{x})$ is globally negative. To determine the sign of

[19]The expression φ^2 is increasing in $\hat{x}$. Depending on $\hat{x}$, the value φ could be real or imaginary. We will assume throughout the proof that φ is real, which will be the case in equilibrium.

[20]In order to guarantee a positive speed of innovation in equilibrium we need $x_o = -\theta_2 + \sqrt{(V-K)^2 + 2(\pi - rK)} > 0$. This is guaranteed by $\pi > (r + \sqrt{2rK})^2/2$ which is a slightly stronger condition than $\pi > rK$.

$df/d\hat{x}$ we decompose the derivative as follows

$$\frac{df(\hat{x})}{d\hat{x}} = \frac{\partial V_0}{\partial \hat{x}} + \frac{\partial V_0}{\partial \varphi}\frac{d\varphi}{d\hat{x}}.$$

We know that $d\varphi/d\hat{x} > 0$ and

$$\frac{\partial V_0}{\partial \hat{x}} = -\frac{((V-K)^2 + 2(\pi - rK))\left(e^{T\varphi}-1\right)^2}{((V-K+r+\hat{x})(e^{\varphi T}-1)+\varphi(1+e^{\varphi T}))^2} < 0$$

under our usual assumptions. Finally,

$$\frac{\partial V_0}{\partial \varphi} = -\frac{((V-K)^2 + 2(\pi - rK))\left(e^{2(T\varphi)} - 2T\varphi e^{T\varphi} - 1\right)}{((V-K+r+\hat{x})\left(e^{\varphi T}-1\right)+\varphi\left(1+e^{\varphi T}\right))^2} < 0.$$

To establish this last inequality, we need to show that $e^{2(T\varphi)} - 2T\varphi e^{T\varphi} - 1 > 0$. Define the function $h(y) \equiv e^{2y} - 2ye^{y} - 1$ for $y \geq 0$. We show that $h(y) > 0$ for all $y > 0$, which implies the result. Observe that $h(0) = 0$ and $h'(y) = 2e^{y}(e^{y} - y - 1)$ which is positive for all $y > 0$ as the term in parenthesis is zero at $y = 0$ and has a positive slope. Therefore, $\partial V_0/\partial\varphi < 0$ which together with $d\varphi/d\hat{x} > 0$ and $\partial V_0/\partial\hat{x}$ proves that $df/d\hat{x} < 0$ for all $\hat{x}$.

Proof of Proposition 7 The arguments to establish the effects of a merger are analogous to those in Proof of Proposition 4. Market concentration increases in ϕ and the pace of innovation is constant due to the free entry and exit conditions not altering $V(0)$ nor V_t as a consequence of a merger. That expenditure decreases with the merger follows from the merger not affecting the investment dynamics of the leader and by the followers' change of expenditure satisfying equation (20), which is increasing in ϕ.