Example: MaxMin 2

Find All Extrema[1]:

$$f(x) = x^3 - 3x$$

[1] Extrema: local and global maxima and minima

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$$f(x) = x^3 - 3x$$

Since there are no endpoints, we only need to find critical points and singular points. $f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x+1)(x-1)$. So there are no singular points, and the critical points are ± 1.
We know that cubic functions grow hugely positive in one direction, and hugely negative in the other. So, there's no global max or min. We need only decide whether $x = 1$ and $x = -1$ are local extrema.
We can easily graph $f'(x)$, and we see it is an upwards-pointing parabola. It is positive to the left of $x = -1$ and positive to its right, so f is increasing up till $x = -1$, then decreasing after; so $x = 1$ is a local max.
Likewise, $f'(x)$ is negative to the left of $x = 1$ and positive to the right of it; so it's decreasing till $x = 1$ and increasing after. Thus $x = 1$ is a local min.

[1] Extrema: local and global maxima and minima

Example: MaxMin 3

Find All Extrema

$$f(x) = \sqrt[3]{x^2 - 64}, \qquad x \text{ in } [-1, 10]$$

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The endpoints are -1 and 10. We differentiate to identify critical points and singular points:
$f'(x) = \frac{1}{3}(x^2 - 64)^{-2/3}(2x) = \frac{2}{3}x(x^2 - 64)^{-2/3}$. So the critical point is $x = 0$ and the singular points are $x = \pm 8$; but since $x = -8$ is not our domain, we don't have to worry about it.
The global extrema are found by simply comparing the value of the function at the various interesting points.
$f(0) = \sqrt[3]{-64} = -4$; $f(8) = 0$; $f(-1) = -\sqrt[3]{63}$; and $f(10) = \sqrt[3]{100 - 64} = \sqrt[3]{36}$. Of these, -4 is the smallest and $\sqrt[3]{36}$ is the largest, so the global max is $\sqrt[3]{36}$ at $x = 10$, and the global min is -4 at $x = 0$.
Then it's pretty clear that $x = 0$ is a local min. Since -1 and 10 are endpoints, they can't be local mins. So, what of $x = 8$? When x is slightly smaller than 8, or slightly larger than 8, $f'(x)$ is positive; so $f(x)$ is increasing to the left of 8 and also to the right of 8. Then 8 is neither a local max nor a local min.

Example: MaxMin 4

Find the largest and smallest value of $f(x) = x^4 - 18x^2$.

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There are no endpoints given, so we take the domain to be the domain of the function, which is all real numbers. As x goes to infinity or negative infinity, $f(x)$ goes to infinity, so there is no global max, hence no largest value.
To find the global min, we differentiate: $f'(x) = 4x^3 - 36x = 4x(x^2 - 9)$. So the critical points are 0 and ± 3, and there are no singular points.
$f(0) = 0$, and $f(3) = f(-3) = -81$, so the smallest value (and global min) is -81, and it occurs twice (which is fine): at 3 and -3.