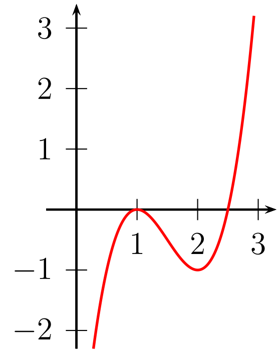
EXAMPLES:

(a) Find the critical numbers of $f(x) = 2x^3 - 9x^2 + 12x - 5$.



Solution: We have

$$f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x-1)(x-2)$$

thus $f'(x) = 0$ at $x = 1$ and $x = 2$. Since $f'(x)$ exists everywhere, $x = 1$ and $x = 2$ are the only critical numbers.

(b) Find the critical numbers of $f(x) = 2x + 3\sqrt[3]{x^2}$.

Solution: We have

$$f'(x) = (2x + 3x^{2/3})' = 2x' + 3(x^{2/3})' = 2 \cdot 1 + 3 \cdot \frac{2}{3}x^{2/3-1}$$

$$= 2 + 2x^{-1/3} = \left\{\frac{2}{1} + \frac{2}{\sqrt[3]{x}} = \frac{2\sqrt[3]{x}}{\sqrt[3]{x}} + \frac{2}{\sqrt[3]{x}} = \frac{2\sqrt[3]{x}+2}{\sqrt[3]{x}}\right\} = \frac{2(\sqrt[3]{x}+1)}{\sqrt[3]{x}}$$

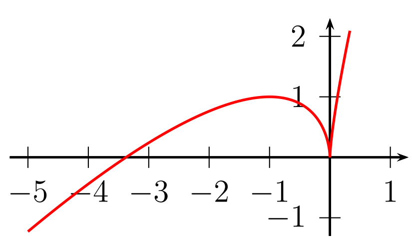
or

$$= 2 + 2x^{-1/3} = \left\{2x^{-1/3} \cdot x^{1/3} + 2x^{-1/3} \cdot 1 = 2x^{-1/3}(x^{1/3}+1)\right\} = \frac{2(\sqrt[3]{x}+1)}{\sqrt[3]{x}}$$

In short,

$$f'(x) = 2 + 2x^{-1/3} = 2x^{-1/3}(x^{1/3}+1) = \frac{2(\sqrt[3]{x}+1)}{\sqrt[3]{x}}$$

It follows that $f'(x)$ equals 0 when $\sqrt[3]{x} + 1 = 0$, which is at $x = -1$; $f'(x)$ does not exist when $\sqrt[3]{x} = 0$, which is at $x = 0$. Thus the critical numbers are -1 and 0.



EXAMPLE: Find the critical numbers of $f(x) = x^{1/3}(2 - x)$.

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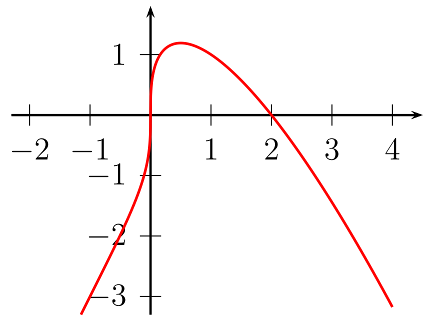
Solution: We have



$$\begin{aligned} f'(x) &= [x^{1/3}(2-x)]' \\ &= (x^{1/3})'(2-x) + x^{1/3}(2-x)' \\ &= \frac{1}{3}x^{-2/3}(2-x) + x^{1/3}\cdot(-1) \\ &= \frac{2-x}{3x^{2/3}} - x^{1/3} \\ &= \left\{\frac{2-x}{3x^{2/3}} - \frac{x^{1/3}\cdot 3x^{2/3}}{3x^{2/3}} = \frac{2-x}{3x^{2/3}} - \frac{3x}{3x^{2/3}} = \frac{2-x-3x}{3x^{2/3}} = \frac{2-4x}{3x^{2/3}}\right\} = \frac{2(1-2x)}{3x^{2/3}} \end{aligned}$$

In short,

$$f'(x) = (x^{1/3})'(2-x) + x^{1/3}(2-x)' = \frac{2-x}{3x^{2/3}} - x^{1/3} = \frac{2(1-2x)}{3x^{2/3}}$$

Here is an other way to get the same result:

$$\begin{aligned} f'(x) &= [x^{1/3}(2-x)]' \\ &= (2x^{1/3} - x^{4/3})' \\ &= 2(x^{1/3})' - (x^{4/3})' \\ &= 2\cdot\frac{1}{3}x^{1/3-1} - \frac{4}{3}x^{4/3-1} \\ &= \frac{2}{3}x^{-2/3} - \frac{4}{3}x^{1/3} = \left\{\frac{2}{3x^{2/3}} - \frac{4x^{1/3}}{3} = \frac{2}{3x^{2/3}} - \frac{4x^{1/3}\cdot x^{2/3}}{3\cdot x^{2/3}} = \frac{2}{3x^{2/3}} - \frac{4x}{3x^{2/3}} = \frac{2-4x}{3x^{2/3}}\right\} \\ &= \frac{2(1-2x)}{3x^{2/3}} \end{aligned}$$

or

$$= \frac{2}{3}x^{-2/3} - \frac{4}{3}x^{1/3} = \left\{\frac{2}{3}x^{-2/3}\cdot 1 - \frac{2}{3}x^{-2/3}\cdot 2x = \frac{2}{3}x^{-2/3}(1-2x)\right\} = \frac{2(1-2x)}{3x^{2/3}}$$

In short,

$$f'(x) = (2x^{1/3} - x^{4/3})' = \frac{2}{3}x^{-2/3} - \frac{4}{3}x^{1/3} = \frac{2}{3}x^{-2/3}(1-2x) = \frac{2(1-2x)}{3x^{2/3}}.$$

It follows that $f'(x)$ equals 0 when $1-2x=0$, which is at $x=\frac{1}{2}$; $f'(x)$ does not exist when $x^{2/3}=0$, which is at $x=0$. Thus the critical numbers are $\frac{1}{2}$ and 0.

THE CLOSED INTERVAL METHOD: To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).

2. Find the values of f at the endpoints of the interval.

3. The largest of the values from Step 1 and 2 is the absolute maximum value; the smallest value of these values is the absolute minimum value.

EXAMPLE: Find the absolute maximum and minimum values of $f(x) = 2x^3 - 15x^2 + 36x$ on the interval $[1, 5]$ and determine where these values occur.

Solution:

Step 1: Since

$$f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6) = 6(x - 2)(x - 3)$$

there are two critical numbers $x = 2$ and $x = 3$.

Step 2: We now evaluate f at these critical numbers and at the endpoints $x = 1$ and $x = 5$. We have

$$f(1) = 23, \quad f(2) = 28, \quad f(3) = 27, \quad f(5) = 55$$

Step 3: The largest value is 55 and the smallest value is 23. Therefore the absolute maximum of f on $[1, 5]$ is 55, occurring at $x = 5$ and the absolute minimum of f on $[1, 5]$ is 23, occurring at $x = 1$.

EXAMPLES:

(a) Find the absolute maximum and minimum values of $f(x) = 2x^3 - 15x^2 + 24x + 2$ on $[0, 2]$ and determine where these values occur.

(b) Find the absolute maximum and minimum values of $f(x) = 6x^{4/3} - 3x^{1/3}$ on the interval $[-1, 1]$ and determine where these values occur.

EXAMPLES:

(a) Find the absolute maximum and minimum values of $f(x) = 2x^3 - 15x^2 + 24x + 2$ on $[0, 2]$ and determine where these values occur.

Solution:

Step 1: Since

$$f'(x) = 6x^2 - 30x + 24 = 6(x^2 - 5x + 4) = 6(x - 4)(x - 1)$$

there are two critical numbers $x = 1$ and $x = 4$.

Step 2: Since $x = 4$ is not from $[0, 2]$, we evaluate f only at $x = 1$ and at the endpoints $x = 0$ and $x = 2$. We have

$$f(0) = 2, \quad f(1) = 13, \quad f(2) = 6$$

Step 3: The largest value is 13 and the smallest value is 2. Therefore the absolute maximum of f on $[0, 2]$ is 13, occurring at $x = 1$ and the absolute minimum of f on $[0, 2]$ is 2, occurring at $x = 0$.

(b) Find the absolute maximum and minimum values of $f(x) = 6x^{4/3} - 3x^{1/3}$ on the interval $[-1, 1]$ and determine where these values occur.

Solution:

Step 1: Since

$$\begin{aligned} f'(x) = (6x^{4/3} - 3x^{1/3})' = 6(x^{4/3})' - 3(x^{1/3})' &= 6 \cdot \frac{4}{3}x^{4/3-1} - 3 \cdot \frac{1}{3}x^{1/3-1} \\ &= 8x^{1/3} - x^{-2/3} \\ &= 8x^{-2/3} \cdot x - 1 \cdot x^{-2/3} \\ &= x^{-2/3}(8x - 1) \\ &= \frac{8x - 1}{x^{2/3}} \end{aligned}$$

there are two critical numbers $x = 0$ and $x = \dfrac{1}{8}$.

Step 2: We now evaluate f at these critical numbers and at the endpoints $x = -1$ and $x = 1$. We have

$$f(-1) = 9, \quad f(0) = 0, \quad f\left(\frac{1}{8}\right) = -\frac{9}{8}, \quad f(1) = 3$$

Step 3: The largest value is 9 and the smallest value is $-\dfrac{9}{8}$. Therefore the absolute maximum of f on $[-1, 1]$ is 9, occurring at $x = -1$ and the absolute minimum of f on $[-1, 1]$ is $-\dfrac{9}{8}$, occurring at $x = \dfrac{1}{8}$.

Appendix

EXAMPLE: Find the absolute maximum and minimum values of $f(x) = 2x + 3$ on $[-1, 4]$ and determine where these values occur.

Solution:
Step 1: Since

$$f'(x) = (2x + 3)' = (2x)' - 3' = 2(x)' - 3' = 2 \cdot 1 - 0 = 2$$

there are no critical numbers.

Step 2: Since there are no critical numbers, we evaluate f at the endpoints $x = -1$ and $x = 4$ only. We have

$$f(-1) = 2(-1) + 3 = -2 + 3 = 1$$

$$f(4) = 2(4) + 3 = 8 + 3 = 11$$

Step 3: The largest value is 11 and the smallest value is 1. Therefore the absolute maximum of f on $[-1, 4]$ is 11, occurring at $x = 4$ and the absolute minimum of f on $[-1, 4]$ is 1, occurring at $x = -1$.

EXAMPLE: Find the absolute maximum and minimum values of $f(x) = 2x^2 - 8x + 1$ on $[-1, 3]$ and determine where these values occur.

Solution:
Step 1: Since

$$f'(x) = (2x^2 - 8x + 1)' = (2x^2)' - (8x)' + 1' = 2(x^2)' - 8(x)' + 1' = 2 \cdot 2x - 8 \cdot 1 + 0 = 4x - 8$$

the number at which f' is zero is 2. Therefore $x = 2$ is the critical number.

Step 2: We evaluate f at the critical number $x = 2$ and at the endpoints $x = -1, 3$. We have

$$f(2) = 2 \cdot 2^2 - 8 \cdot 2 + 1 = 2 \cdot 4 - 8 \cdot 2 + 1 = 8 - 16 + 1 = -7$$

$$f(-1) = 2(-1)^2 - 8(-1) + 1 = 2 + 8 + 1 = 11$$

$$f(3) = 2 \cdot 3^2 - 8 \cdot 3 + 1 = 2 \cdot 9 - 8 \cdot 3 + 1 = 18 - 24 + 1 = -5$$

Step 3: The largest value is 11 and the smallest value is -7. Therefore the absolute maximum of f on $[-1, 3]$ is 11, occurring at $x = -1$ and the absolute minimum of f on $[-1, 3]$ is -7, occurring at $x = 2$.

EXAMPLE: Find the absolute maximum and minimum values of $f(x) = 5 - x^3$ on $[-2, 1]$ and determine where these values occur.

EXAMPLE: Find the absolute maximum and minimum values of $f(x) = 5 - x^3$ on $[-2, 1]$ and determine where these values occur.

Solution:

Step 1: Since

$$f'(x) = (5 - x^3)' = 5' - (x^3)' = 0 - 3x^2 = -3x^2$$

the critical number is $x = 0$.

Step 2: We evaluate f at the critical number $x = 0$ and at the endpoints $x = -2, 1$. We have

$$f(0) = 5 - 0^3 = 5 - 0 = 5$$

$$f(-2) = 5 - (-2)^3 = 5 - (-8) = 5 + 8 = 13$$

$$f(1) = 5 - 1^3 = 5 - 1 = 4$$

Step 3: The largest value is 13 and the smallest value is 4. Therefore the absolute maximum of f on $[-2, 1]$ is 13, occurring at $x = -2$ and the absolute minimum of f on $[-2, 1]$ is 4, occurring at $x = 1$.

EXAMPLE: Find the absolute maximum and minimum values of $f(x) = x^3 + 3x^2 - 1$ on $[-3, 2]$ and determine where these values occur.

Solution:

Step 1: Since

$$f'(x) = (x^3 + 3x^2 - 1)' = (x^3)' + (3x^2)' - 1' = (x^3)' + 3(x^2)' - 1' = 3x^2 + 3 \cdot 2x - 0 = 3x^2 + 6x$$
$$= 3x(x + 2)$$

the critical numbers are $x = 0$ and $x = -2$.

Step 2: We evaluate f at the critical numbers $x = 0, -2$ and at the endpoints $x = -3, 2$. We have

$$f(0) = 0^3 + 3 \cdot 0^2 - 1 = 0 + 0 - 1 = -1$$

$$f(-2) = (-2)^3 + 3 \cdot (-2)^2 - 1 = -8 + 12 - 1 = 3$$

$$f(-3) = (-3)^3 + 3 \cdot (-3)^2 - 1 = -27 + 27 - 1 = -1$$

$$f(2) = 2^3 + 3 \cdot 2^2 - 1 = 8 + 12 - 1 = 19$$

Step 3: The largest value is 19 and the smallest value is -1. Therefore the absolute maximum of f on $[-3, 2]$ is 19, occurring at $x = 2$ and the absolute minimum of f on $[-3, 2]$ is -1, occurring at $x = 0$ and $x = -3$.

EXAMPLE: Find the absolute maximum and minimum values of $f(x) = 3x^4 + 4x^3 - 36x^2$ on $[-2, 3]$ and determine where these values occur.

Solution:

Step 1: Since

$$\begin{aligned} f'(x) = (3x^4 + 4x^3 - 36x^2)' = (3x^4)' + (4x^3)' - (36x^2)' &= 3(x^4)' + 4(x^3)' - 36(x^2)' \\ &= 3 \cdot 4x^3 + 4 \cdot 3x^2 - 36 \cdot 2x \\ &= 12x^3 + 12x^2 - 72x \\ &= 12x(x^2 + x - 6) \\ &= 12x(x - 2)(x + 3) \end{aligned}$$

the critical numbers are $x = 0,\ x = 2$, and $x = -3$.

Step 2: Since $-3 \notin [-2, 3]$, we evaluate f only at the critical numbers $x = 0, 2$ and at the endpoints $x = -2, 3$. We have

$$f(0) = 3 \cdot 0^4 + 4 \cdot 0^3 - 36 \cdot 0^2 = 0 + 0 - 0 = 0$$

$$f(2) = 3 \cdot 2^4 + 4 \cdot 2^3 - 36 \cdot 2^2 = 48 + 32 - 144 = -64$$

$$f(-2) = 3 \cdot (-2)^4 + 4 \cdot (-2)^3 - 36 \cdot (-2)^2 = 48 - 32 - 144 = -128$$

$$f(3) = 3 \cdot 3^4 + 4 \cdot 3^3 - 36 \cdot 3^2 = 243 + 108 - 324 = 27$$

Step 3: The largest value is 27 and the smallest value is -128. Therefore the absolute maximum of f on $[-2, 3]$ is 27, occurring at $x = 3$ and the absolute minimum of f on $[-2, 3]$ is -128, occurring at $x = -2$.

EXAMPLE: Find the absolute maximum and minimum values of $f(x) = -\dfrac{1}{x^2}$ on $[0.4, 5]$ and determine where these values occur.

Solution:

Step 1: We have

$$f'(x) = \left(-\frac{1}{x^2}\right)' = -\left(\frac{1}{x^2}\right)' = -\left(x^{-2}\right)' = -(-2)x^{-2-1} = 2x^{-3} = \frac{2}{x^3}$$

Note that there are no numbers at which f' is zero. The number at which f' does not exist is $x = 0$, but this number is not from the domain of f. Therefore f does not have critical numbers.

Step 2: Since f does not have critical numbers, we evaluate it only at the endpoints $x = 0.4$ and $x = 5$. We have

$$f(0.4) = -\frac{1}{(0.4)^2} = -\frac{1}{0.16} = -6.25 \qquad f(5) = -\frac{1}{5^2} = -\frac{1}{25} = -0.04$$

Step 3: The largest value is -0.04 and the smallest value is -6.25. Therefore the absolute maximum of f on $[0.4, 5]$ is -0.04, occurring at $x = 5$ and the absolute minimum of f on $[0.4, 5]$ is -6.25, occurring at $x = 0.4$.

EXAMPLE: Find the absolute maximum and minimum values of $f(x) = 20 - 3x - \dfrac{12}{x}$ on $[2, 4]$ and determine where these values occur.

Solution:

Step 1: We have

$$f'(x) = \left(20 - 3x - \frac{12}{x}\right)' = \left(20 - 3x - 12x^{-1}\right)' = 20' - (3x)' - \left(12x^{-1}\right)'$$

$$= 20' - 3 \cdot x' - 12\left(x^{-1}\right)'$$

$$= 0 - 3 \cdot 1 - 12(-1)x^{-1-1}$$

$$= -3 + 12x^{-2}$$

Since

$$-3 + 12x^{-2} = -3 + 12 \cdot \frac{1}{x^2} = -3 + \frac{12}{x^2} = \frac{-3x^2}{x^2} + \frac{12}{x^2} = \frac{-3x^2 + 12}{x^2}$$

$$= \frac{-3(x^2 - 4)}{x^2}$$

$$= \frac{-3(x^2 - 2^2)}{x^2}$$

$$= \frac{-3(x - 2)(x + 2)}{x^2}$$

we have

$$f'(x) = -\frac{3(x - 2)(x + 2)}{x^2}$$

The numbers at which f' is zero are $x = 2$ and $x = -2$. The number at which f' does not exist is $x = 0$. But f is not defined at $x = 0$, therefore the critical numbers of f are $x = -2$ and $x = 2$ only.

Step 2: Since $-2 \notin [2, 4]$, we evaluate f only at the critical number $x = 2$ (which is also one of the endpoints) and at the second endpoint $x = 4$. We have

$$f(2) = 20 - 3 \cdot 2 - \frac{12}{2} = 20 - 6 - 6 = 8$$

$$f(4) = 20 - 3 \cdot 4 - \frac{12}{4} = 20 - 12 - 3 = 5$$

Step 3: The largest value is 8 and the smallest value is 5. Therefore the absolute maximum of f on $[2, 4]$ is 8, occurring at $x = 2$ and the absolute minimum of f on $[2, 4]$ is 5, occurring at $x = 4$.

EXAMPLE: Find the absolute maximum and minimum values of $f(x)=2\sqrt[3]{x}$ on $[-8,1]$ and determine where these values occur.

Solution:

Step 1: We have

$$f'(x)=\left(2x^{1/3}\right)'=2\left(x^{1/3}\right)'=2\cdot\frac{1}{3}x^{1/3-1}=\frac{2}{3}x^{-2/3}=\frac{2}{3x^{2/3}}$$

Note that there are no numbers at which f' is zero. The number at which f' does not exist is $x=0$, so the critical number is $x=0$.

Step 2: We evaluate f at the critical number $x=0$ and at the endpoints $x=-8$ and $x=1$. We have

$$f(0)=2\sqrt[3]{0}=2\cdot 0=0$$

$$f(-8)=2\sqrt[3]{-8}=2\cdot(-2)=-4$$

$$f(1)=2\sqrt[3]{1}=2\cdot 1=2$$

Step 3: The largest value is 2 and the smallest value is -4. Therefore the absolute maximum of f on $[-8,1]$ is 2, occurring at $x=1$ and the absolute minimum of f on $[-8,1]$ is -4, occurring at $x=-8$.

EXAMPLE: Find the absolute maximum and minimum values of $f(x)=-5x^{2/3}$ on $[-1,1]$ and determine where these values occur.

Solution:

Step 1: We have

$$f'(x)=\left(-5x^{2/3}\right)'=-5\left(x^{2/3}\right)'=-5\cdot\frac{2}{3}x^{2/3-1}=-\frac{10}{3}x^{-1/3}=-\frac{10}{3x^{1/3}}$$

Note that there are no numbers at which f' is zero. The number at which f' does not exist is $x=0$, so the critical number is $x=0$.

Step 2: We evaluate f at the critical number $x=0$ and at the endpoints $x=-1$ and $x=1$. We have

$$f(0)=-5(0)^{2/3}=-5\cdot 0=0$$

$$f(-1)=-5(-1)^{2/3}=-5\Big((-1)^2\Big)^{1/3}=-5(1)^{1/3}=-5\cdot 1=-5$$

$$f(1)=-5(1)^{2/3}=-5\cdot 1=-5$$

Step 3: The largest value is 0 and the smallest value is -5. Therefore the absolute maximum of f on $[-1,1]$ is 0, occurring at $x=0$ and the absolute minimum of f on $[-1,1]$ is -5, occurring at $x=\pm 1$.

EXAMPLE: Find the absolute maximum and minimum values of $f(x)=x^{4/3}$ on $[-27,27]$ and determine where these values occur.

EXAMPLE: Find the absolute maximum and minimum values of $f(x) = x^{4/3}$ on $[-27, 27]$ and determine where these values occur.

Solution:

Step 1: We have

$$f'(x) = \left(x^{4/3}\right)' = \frac{4}{3}x^{4/3-1} = \frac{4}{3}x^{1/3}$$

Note that there are no numbers at which f' does not exist. The number at which f' is zero is $x = 0$, so the critical number is $x = 0$.

Step 2: We evaluate f at the critical number $x = 0$ and at the endpoints $x = -27$ and $x = 27$. We have

$$f(0) = 0^{4/3} = 0$$

$$f(-27) = (-27)^{4/3} = \left((-27)^{1/3}\right)^4 = (-3)^4 = 81$$

$$f(27) = 27^{4/3} = \left(27^{1/3}\right)^4 = 3^4 = 81$$

Step 3: The largest value is 81 and the smallest value is 0. Therefore the absolute maximum of f on $[-27, 27]$ is 81, occurring at $x = \pm 27$ and the absolute minimum of f on $[-27, 27]$ is 0, occurring at $x = 0$.

EXAMPLE: Find the absolute maximum and minimum values of $f(x) = \sqrt{4 - x^2}$ on $[-2, 1]$ and determine where these values occur.

Solution:

Step 1: We have

$$f'(x) = \left((4 - x^2)^{1/2}\right)' = \frac{1}{2}(4 - x^2)^{1/2-1} \cdot (4 - x^2)' = \frac{1}{2}(4 - x^2)^{-1/2} \cdot (4' - (x^2)')$$

$$= \frac{1}{2}(4 - x^2)^{-1/2} \cdot (0 - 2x) = \frac{1}{2}(4 - x^2)^{-1/2} \cdot (-2x) = -x(4 - x^2)^{-1/2}$$

$$= -\frac{x}{(4 - x^2)^{1/2}} = -\frac{x}{\sqrt{4 - x^2}}$$

Note that the number at which f' is zero is $x = 0$ and the numbers at which f' does not exist are $x = \pm 2$. Therefore the critical numbers are $x = 0, -2, 2$.

Step 2: Since $2 \notin [-2, 1]$, we evaluate f only at the critical numbers $x = 0, -2$ (which is also one of the endpoints) and at the second endpoint $x = 1$. We have

$$f(0) = \sqrt{4 - 0^2} = \sqrt{4 - 0} = \sqrt{4} = 2$$

$$f(-2) = \sqrt{4 - (-2)^2} = \sqrt{4 - 4} = \sqrt{0} = 0$$

$$f(1) = \sqrt{4 - 1^2} = \sqrt{4 - 1} = \sqrt{3}$$

Step 3: The largest value is 2 and the smallest value is 0. Therefore the absolute maximum of f on $[-2, 1]$ is 2, occurring at $x = 0$ and the absolute minimum of f on $[-2, 1]$ is 0, occurring at $x = -2$.

EXAMPLE: Find the absolute maximum and minimum values of $f(x)=\cos x$ on $\left[-\frac{\pi}{2},\frac{5\pi}{2}\right]$ and determine where these values occur.

Solution:
Step 1: We have

$$f'(x)=-\sin x$$

Note that there are no numbers at which f' does not exist. The numbers in $\left[-\frac{\pi}{2},\frac{5\pi}{2}\right]$ at which f' is zero on are $x=0,\pi,2\pi$, so the critical number is $x=0,\pi,2\pi$.

Step 2: We evaluate f at the critical numbers $x=0,\pi,2\pi$ and at the endpoints $x=-\frac{\pi}{2},\frac{5\pi}{2}$. We have

$$f(0)=\cos 0=1$$

$$f(\pi)=\cos\pi=-1$$

$$f(2\pi)=\cos(2\pi)=1$$

$$f\left(-\frac{\pi}{2}\right)=\cos\left(-\frac{\pi}{2}\right)=0$$

$$f\left(\frac{5\pi}{2}\right)=\cos\left(\frac{5\pi}{2}\right)=0$$

Step 3: The largest value is 1 and the smallest value is -1. Therefore the absolute maximum of f on $\left[-\frac{\pi}{2},\frac{5\pi}{2}\right]$ is 1, occurring at $x=0,2\pi$ and the absolute minimum of f on $\left[-\frac{\pi}{2},\frac{5\pi}{2}\right]$ is -1, occurring at $x=\pi$.