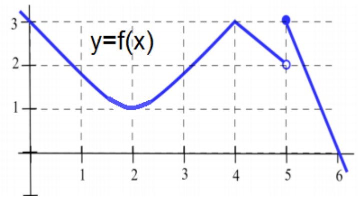
M104 Quiz 2-V1 Thursday Oct 6, 2016 Grade: ______

First Name: ______ Last Name: ______

Student-No: ______ Section: ______

Short answer questions

1. [2 marks] In reference to the graph provided, determine if the statements below are true (T) or false (F). Each part is worth 0.5 mark.



(a) $f(x)$ is continuous on $[1, 5]$.

Solution: False, the function is not continuous from left at $x = 5$.

(b) At $x = 4$, the function is continuous but does not have derivative.

Solution: True

(c) Graph of $f'(x)$ has only one root between $[0, 6]$

Solution: True, it is point $x = 2$, because the tangent line at that point is flat.

(d) $f'(3) > 0$ and $f'(6) < 0$.

Solution: True

Long answer questions — you must show your work

2. [3 marks] $f(x) = \left(\sqrt{x-1}\right)$. Using the limit definition for derivative, show that $f'(5) = 1/4$. No marks will be given to solutions that involve rules of differentiation.

Solution: Limit definition for derivative states

$$
\begin{aligned}
f'(5) = \lim_{h\to 0}\frac{f(5+h)-f(5)}{h} &= \lim_{h\to 0}\frac{\sqrt{5+h-1}-\sqrt{5-1}}{h} \\
&= \lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}\times\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2} \\
&= \lim_{h\to 0}\frac{4+h-4}{h\left(\sqrt{4+h}+2\right)} \\
&= \lim_{h\to 0}\frac{\not{h}}{\not{h}\left(\sqrt{4+h}+2\right)} \\
&= \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}.
\end{aligned}
$$

Marking scheme: 1pt for writing the limit correctly, 1pt for using the correct multiplier to simplify the function and 1pt for complete solution.

3. Consider the piecewise function $f(x)$ defined below:

$$f(x) = \begin{cases} ax\cos(x) & x \le 0 \\ \frac{x-1}{x+1} + be^x & x > 0 \end{cases}$$

(a) [2 marks] find b such that $f(x)$ is continuous everywhere.

Answer: $b = 1$

Solution: Function $f(x)$ is continuous everywhere because it is composed of "nice" functions. The only place that we need to check is $x = 0$ where the two branches come together. For continuity we must have

$$\begin{aligned}
&\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0) \Rightarrow \\
&\lim_{x\to 0^-} ax\cos(x) = \lim_{x\to 0^+} \frac{x-1}{x+1} + be^x \Rightarrow \\
&0 = -1 + b \Rightarrow b = 1
\end{aligned}$$

Marking scheme: 1pt for writing conditions of continuity and 1pt for finding b correctly

(b) [2 marks] Find a such that the function $f(x)$ is differentiable everywhere.

Answer: $a = 3$

Solution: Again we only need to check the function at $x = 0$.

$$\begin{aligned}
&\lim_{x\to 0^-} f'(x) = \lim_{x\to 0^+} f'(x) \Rightarrow \\
&\lim_{x\to 0^-} a\cos x - ax\sin x = \lim_{x\to 0^+} \frac{2}{(x+1)^2} + be^x \Rightarrow \\
&a = 2 + b = 2 + 1 = 3.
\end{aligned}$$

Marking scheme: 1pt for taking derivative correctly and 1pt for finding a correctly

(c) [1 mark] Find the equation of tangent line to the curve of $f(x)$ at $x = 0$ on the curve.

Answer: $y = 3x$

Solution: Slope of tangent line is $f'(0) = a\cos(0) - a(0)\sin(0) = 3$. Tangent line also passes through $(0, 0)$, so equation of tangent line is $y = 3x$.

Marking scheme: 1pt for finding the correct equation of tangent line.