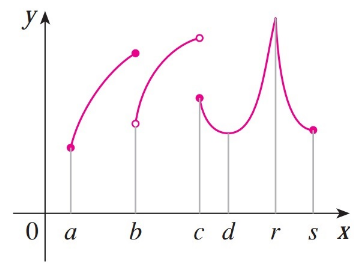
First Name: ____________ Last Name: ____________

Student-No: ____________ Section: ____________

Short answer questions

1. 3 marks In reference to the graph of function y provided, determine if the statements below are true or false on $[a, s]$. Each part is worth 0.5 mark.



(a) The function has three points of discontinuity.

Solution: False, there are two discontinuity points at $x = b$, $x = c$.

(b) The function is not differentiable at three points.

Solution: True, the function is not differentiable at $x = b$, $x = c$ and $x = r$.

(c) The function has an absolute minimum and an absolute maximum, therefore Extreme Value theorem can be applied to it.

Solution: False, the function is not continuous, so EVT does not apply.

(d) The function has two local minima.

Solution: False, only $x = d$ is the local minima.

(e) $x = c$ is a local maximum.

Solution: False, $f(c)$ is smaller than points on its left

(f) The function has four critical points.

Solution: True, The critical points are $x = b, c, d, r$

Long answer questions — you must show your work

2. 3 marks Show that for any $-2 \leq t \leq 2$, the inequality $-2 \leq t\sqrt{4 - t^2} \leq 2$ holds.
Hint: How does this question translate in terms of absolute maximum and absolute minimum?

Solution: We basically want to show that the absolute minimum and absolute maximum of the function $f(t) = t\sqrt{4-t^2}$ on the closed interval of $[-2, 2]$ are -2 and 2 respectively. First of all, the function is continuous on the closed interval. so

- evaluate end points

  $$f(-2) = -2\sqrt{4-(-2)^2} = -2\sqrt{4-4} = 0, \quad f(2) = -2\sqrt{4-(-2)^2} = -2\sqrt{4-4} = 0.$$

- find the critical points.

  $$f'(t) = \sqrt{4-t^2} + t\frac{-2t}{2\sqrt{4-t^2}} = \sqrt{4-t^2} - \frac{t^2}{\sqrt{4-t^2}} = \frac{4-t^2-t^2}{\sqrt{4-t^2}} = \frac{4-2t^2}{\sqrt{4-t^2}}$$

  Now let's solve

  $$f'(t) = 0 \Rightarrow \frac{4-2t^2}{\sqrt{4-t^2}} = 0 \Rightarrow 4-2t^2 = 0 \Rightarrow t = \pm\sqrt{2}$$

  You can try that solving $f'(t)$ *not defined* leads to finding the end points which we do not count as critical points.

- Evaluate the end points:

  $$f(\sqrt{2} = \sqrt{2}\sqrt{4-2} = 2, \quad f(-\sqrt{2}) = -\sqrt{2}\sqrt{4-2} = -2.$$

- Therefore the absolute maximum is $max(0, 2, -2) = 2$ and absolute minimum is $min(0, 2, -2) = -2$

Marking scheme: 1 pt for evaluating end points, 1 pt for finding critical points, 1 pt for complete answer.

3. (a) 1 mark The cost function of a bicycle manufacturing company is given by $C = 10 + 0.1q^2 - 0.001q^3$ in thousand dollars where q is the number of bicycles manufactured. Using marginal cost, approximate the cost of manufacturing the 11th bicycle.

Solution:

$$MC = \frac{\mathrm{d}C}{\mathrm{d}q} = 0.2q - 0.003q^2 \Rightarrow$$

$$MC(10) = 0.2(10) - 0.003(10)^2 = 2 - 0.03 = 1970 \text{ thousand dollars} = 1970 \text{ dollars}$$

Marking scheme: 1pt for a correct answer

(b) 3 marks The relation between the demand (q) and price (p) of a product can by estimated by $p^2 + 2q^2 = 900$. Find the price elasticity of when $p = 10$ and recommend if the price has to be lowered or raised.

Solution: we take the derivative

$$2p + 4q\frac{\mathrm{d}q}{\mathrm{d}p} = 0 \Rightarrow \frac{\mathrm{d}q}{\mathrm{d}p} = -\frac{p}{2q}$$

therefore,

$$E_D = \frac{p}{q}\frac{\mathrm{d}q}{\mathrm{d}p} = -\frac{p}{q}\frac{p}{2q} = -\frac{p^2}{2q^2}$$

We substitute $p = 10$ to find

$$100 + 2q^2 = 900 \Rightarrow 2q^2 = 800 \Rightarrow q^2 = 400 \Rightarrow q = 20$$

We find

$$E_D = -\frac{10^2}{2(20)^2} = -\frac{1}{8}.$$

We see $|E_D| < 1$, so system is inelastic, increase the price.

Marking scheme: 1pt for correct implicit differentiation, 1pt for finding the Elasticity, 1pt for correct recommendation based on their value of Elasticity.