2. We want to sketch the curve of function $f(x) = x\sqrt{4-x^2}$ $\Leftarrow$

$$4-x^2 \geq 0 \Downarrow -x^2 \geq -4 \Downarrow x^2 \leq 4 \Downarrow -2 \leq x \leq 2$$

(a) find the domain of the function

$D = [-2, 2]$ we have a square root, therefore ______

(b) find the x and y intercept.

$(0,0) \leftarrow$ $y_{intercept}$: $x=0 \rightarrow y=0$

$(\pm 2, 0) \leftarrow$ $x_{intercept}$: $y=0 \rightarrow x\sqrt{4-x^2}=0$

$\rightarrow x=0$, $\sqrt{4-x^2}=0 \rightarrow 4-x^2=0 \Downarrow x=\pm 2$

(c) is function odd, even or periodic?

$f(-x) = -x\sqrt{4-(-x)^2} = -x\sqrt{4-x^2} \rightarrow -f(x)$

the function is odd.

(d) find the asymptotes of the function
for vertical asymptotes the candidate is where ______ Therefore

No vertical asymptote

For the horizontal asymptotes we find the following limits:

$\lim_{x\to\pm\infty} x\sqrt{4-x^2}$ = don't exist

But

NO Horizantal asymptote

(e) increasing or decreasing, local max and local min.
we need to find ______

$$f'(x) = \sqrt{4-x^2} + x\frac{-2x}{2\sqrt{4-x^2}} = \sqrt{4-x^2} - \frac{x^2}{\sqrt{4-x^2}} =$$

so the critical points are :

$$\frac{4-x^2-x^2}{\sqrt{4-x^2}} = \frac{4-2x^2}{\sqrt{4-x^2}}$$

use second derivative test to classify critical points.

$f'=0 \rightarrow 4-2x^2=0 \rightarrow x^2=2 \rightarrow x=\pm\sqrt{2}$ ✓

f' Notdefined $\rightarrow \sqrt{4-x^2}=0 \rightarrow 4-x^2=0 \rightarrow \boxed{x=\pm 2}$

they are endpoints therefore Not critical Point

	-2		$-\sqrt{2}$		$\sqrt{2}$		2
f'		$-$ ↘		$+$ ↗		$-$ ↘	

$x=-\sqrt{2}$ local min

$x=\sqrt{2}$ local

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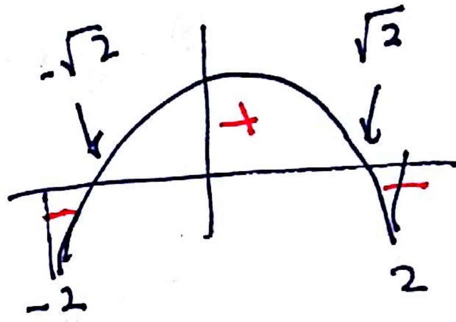
$$4-2x^2 = 2(2-x^2) = 2(\sqrt{2}-x)(\sqrt{2}+x)$$

	-2 to $-\sqrt{2}$	$-\sqrt{2}$ to $\sqrt{2}$	$\sqrt{2}$ to 2
$(\sqrt{2}-x)$	+	+	−
$(\sqrt{2}+x)$	−	+	+
$(4-2x^2)$	−	+	−



$$f'' = \frac{-4x\sqrt{4-x^2} + \dfrac{+\cancel{2}x}{\cancel{2}\sqrt{4-x^2}}(4-2x^2)}{(\sqrt{4-x^2})^2}$$

$$= \frac{-4x(4-x^2) + x(4-2x^2)}{(\sqrt{4-x^2})^3} =$$

$$= \frac{-16x+4x^3+4x-2x^3}{(\sqrt{4-x^2})^3} = \frac{2x^3-12x}{(\sqrt{4-x^2})^3}$$

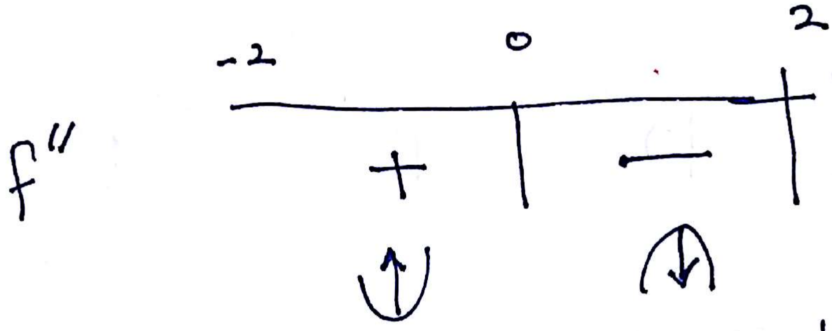
$f''=0 \rightarrow 2x^3-12x=0 \rightarrow 2x(x^2-6)=0$

$\underline{x=0}$ or $\underline{x=\pm\sqrt{6}}$

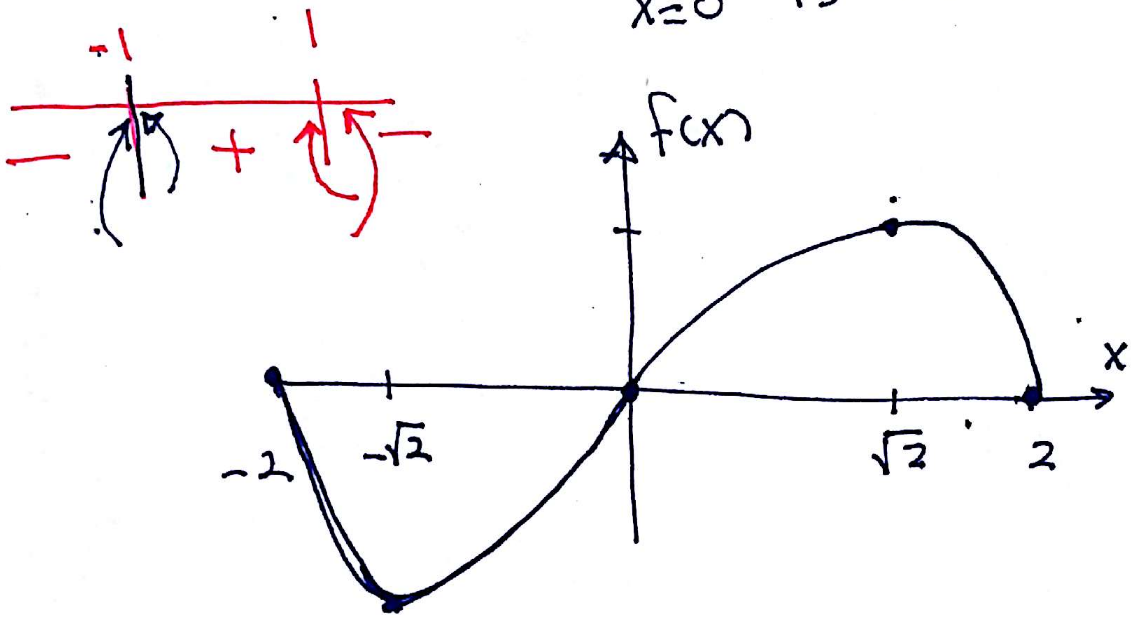
Not in domain

f'' not defined → Nothing here

↳ $(\sqrt{4-x^2})^3=0 \rightarrow x=\pm 2$



$x=0$ is inflection point.



$f(\sqrt{2}) =$
$\sqrt{2}\sqrt{4-2} = 2$

$f(-\sqrt{2}) =$
$-\sqrt{2}\sqrt{4-2} = -2$

$f(x) = \dfrac{x^3}{1-x^2}$ Domain: $1-x^2 = 0$
$\Downarrow$
$x = \pm 1$

$\mathbb{R} - \{\pm 1\}$

y-int : $x = 0 \rightarrow y = 0$

x-int : $y = 0 \rightarrow x^3 = 0 \rightarrow x = 0$

Symetry: $f(-x) = \dfrac{(-x)^3}{1-(-x)^2} = \dfrac{-x^3}{1-x^2} = -f(x)$
$\Downarrow$
odd.

Asymptotes.

vertical: $\lim\limits_{x\to 1^-} \dfrac{x^3}{1-x^2} = \dfrac{1}{0^+} = \infty$

$\lim\limits_{x\to 1^+} \dfrac{x^3}{1-x^2} = \dfrac{1}{0^-} = -\infty$

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$$\lim_{x \to -1^-} \frac{x^3}{1-x^2} = \frac{-1}{0^-} = +\infty$$

$x < -1$
$\Downarrow$
$x^2 > 1$
$\Downarrow$
$1 - x^2 < 0$

$$\lim_{x \to -1^+} \frac{x^3}{1-x^2} = \frac{-1}{0^+} = -\infty$$

$x = 1$ and $x = -1$ vertical asymptote.

Horizontal Asymptote.

$$\lim_{x \to +\infty} \frac{x^3}{1-x^2} = \lim_{x \to \infty} \frac{x^3}{-x^2} = \lim_{x \to \infty} -x = -\infty$$

$$\lim_{x \to -\infty} \frac{x^3}{1-x^2} = +\infty$$

NO Hori. asymp.

$$f'(x) = \frac{3x^2(1-x^2) - x^3(-2x)}{(1-x^2)^2} =$$

$$\frac{3x^2 - 3x^4 + 2x^4}{(1-x^2)^2} = \frac{3x^2 - x^4}{(1-x^2)^2}$$

$f' = 0 \longrightarrow 3x^2 - x^4 = 0 \rightarrow x^2(3 - x^2) = 0 \rightarrow x = 0$
$\rightarrow x = \sqrt{3}$
$\rightarrow x = -\sqrt{3}$

$f' = \text{Not def} \rightarrow (1-x^2)^2 = 0 \rightarrow \boxed{\cancel{x = \pm 1}}$ Not domain

	$-\infty$	$-\sqrt{3}$	0	$\sqrt{3}$	∞
f'	$-$	$+$	$+$	$-$	
	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$	

$x = -\sqrt{3}$ is local min

$x = \sqrt{3}$ is " max

$$f'' = \frac{6x + 2x^3}{(1-x^2)^3}$$

$f'' = 0 \Longrightarrow 6x + 2x^3 = 0 \rightarrow 2x(x^2+3) = 0$

$\rightarrow 2x = 0 \longrightarrow x = 0$

$\rightarrow x^2 + 3 = 0 \longrightarrow$ does not have solution

f'' Not defined

$\rightarrow (1-x^2)^3 = 0 \rightarrow \boxed{\cancel{x = \pm 1}}$ Not in domain.

	$-\infty$	-1	0	$+1$	∞
$2x(x^2+3)$	$-$	$-$	$+$	$+$	
$(1-x^2)^3$	$-$	$+$	$+$	$-$	
f''	$+ \cup$	$- \cap$	$+ \cup$	$- \cap$	

$x = \sqrt{3}$ local max

$x = -\sqrt{3}$ local min

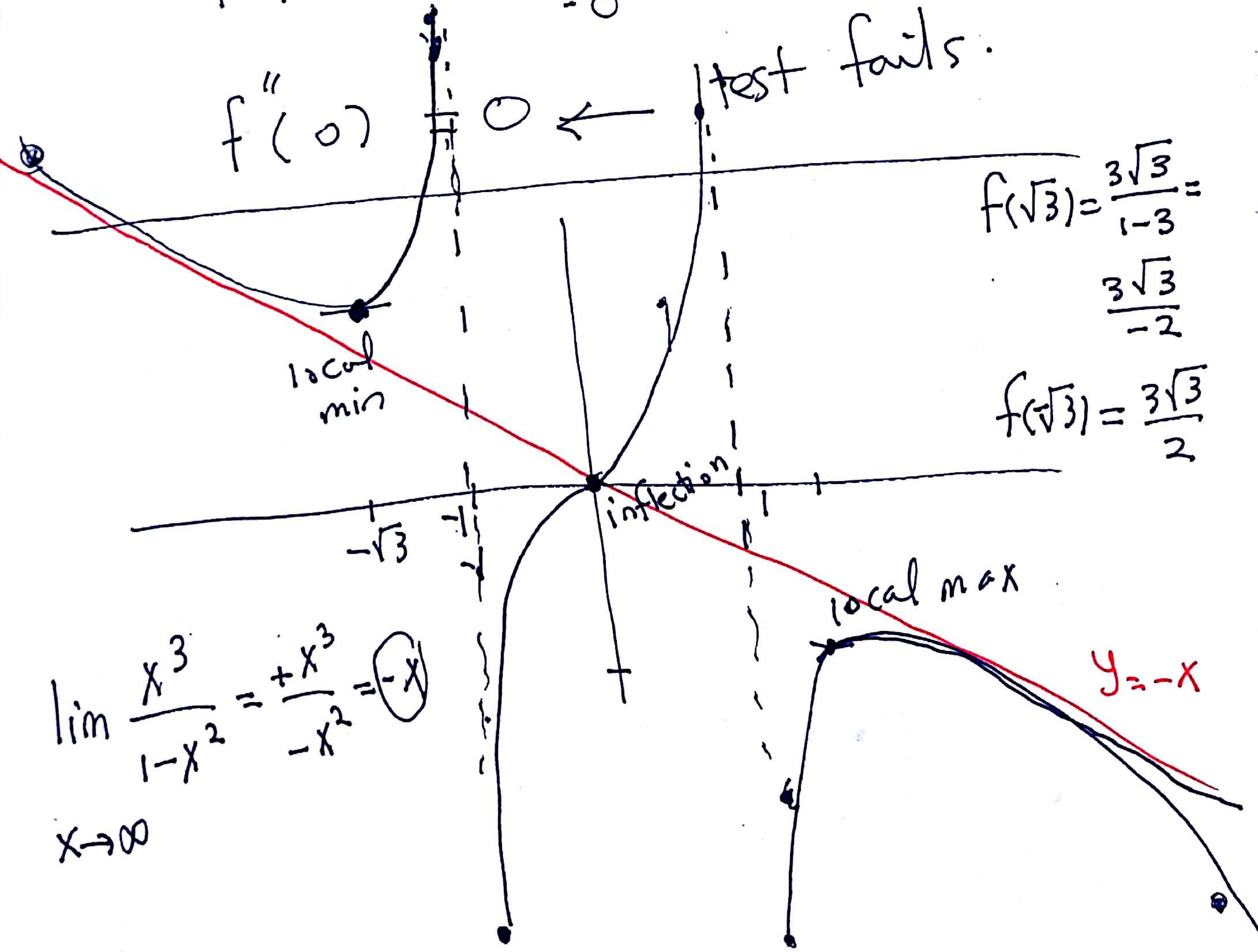
$x = 0$ Nothing

Second derivative test.

$$f''(\sqrt{3}) = \frac{+}{(1-3)^3} = \frac{+}{-8} < 0 \Rightarrow \text{local max}$$

$$f''(-\sqrt{3}) = \frac{-}{-8} > 0 \rightarrow \text{local min.}$$

$$f''(0) = 0 \leftarrow \text{test fails.}$$



$$f(\sqrt{3}) = \frac{3\sqrt{3}}{1-3} = \frac{3\sqrt{3}}{-2}$$

$$f(-\sqrt{3}) = \frac{3\sqrt{3}}{2}$$

$$\lim_{x\to\infty} \frac{x^3}{1-x^2} = \frac{+x^3}{-x^2} = -x$$

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3. We want to sketch the curve of function $f(x) = \frac{e^x}{\sqrt{x}}$

(a) find the domain of the function

$(0, \infty)$

(b) find the x and y intercept.

x int: $y=0 \to \frac{e^x}{\sqrt{x}} = 0 \to e^x = 0 \to$ None

y. int: $x=0 \to$ No y-intercept

(c) is function odd, even or periodic?

Domain is only + nu.

it can't be symmetric.

(d) find the asymptotes of the function

vertical asymptote: $\boxed{x=0}$

$$\lim_{x \to 0^+} \frac{e^x}{\sqrt{x}} = \frac{1}{0^+} = \infty$$

~~(e) increasing or decreasing, local max and local min.~~

Horizontal asymp

$$\lim_{x \to \infty} \frac{e^x}{\sqrt{x}} = \infty$$ No Hori. Asymptote

$$f'(x) = \frac{e^x\sqrt{x} - e^x \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2} = \frac{\frac{2e^x x}{2\sqrt{x}} - \frac{e^x}{2\sqrt{x}}}{x}$$

$$= \frac{2e^x x - e^x}{2x\sqrt{x}} = \frac{e^x(2x-1)}{2x\sqrt{x}}$$

$f'(x)=0 \rightarrow 2e^x x - e^x = 0$

$e^x(2x-1)=0 \rightarrow$

$e^x = 0 \rightarrow$ D.S.

$2x-1=0 \rightarrow x = \frac{1}{2}$

f' Not defined

$\hookrightarrow x\sqrt{x} = 0 \rightarrow \boxed{\cancel{x=0}}$

Not in domain

	0		$\frac{1}{2}$		∞
f'		$-$		$+$	
		$\searrow$		$\nearrow$	

$x=\frac{1}{2}$ local ~~max.~~ min.