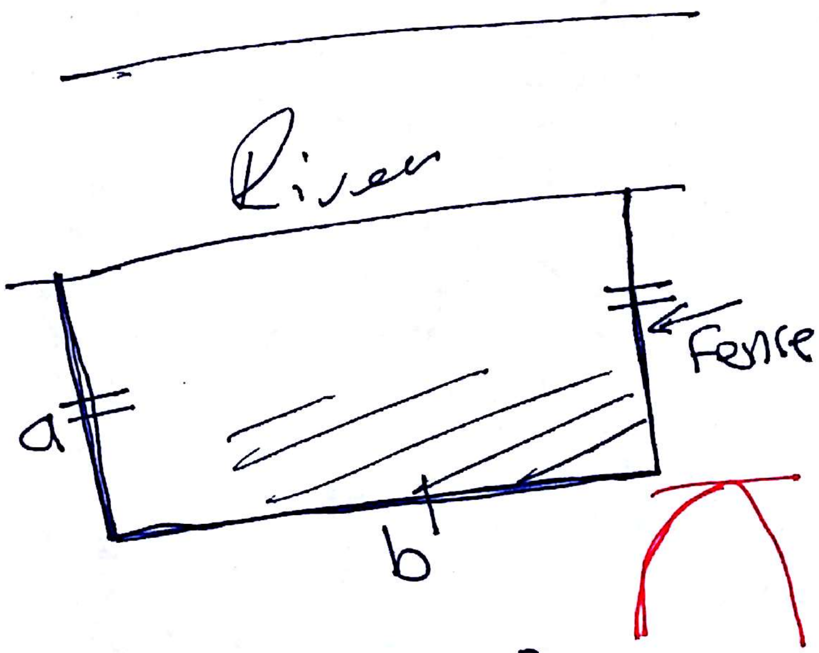
NOV 17, 2016

A Farmer wants to fence off his farming land. ONE side of the land is Connected to a rive and hence does not Need fencing. design the largest fied when he has 100 m of fencing.



Area = $A = a \cdot b$

$a + b + a = 100$ m.

max A where

$2a + b = 100$ m.

$b = 100 - 2a$

$A(a) = a(100 - 2a) = 100a - 2a^2$

$0 < a < 50$

A=0

$\frac{dA}{da} = 0 \Rightarrow 100 - 4a = 0$

$\Downarrow$

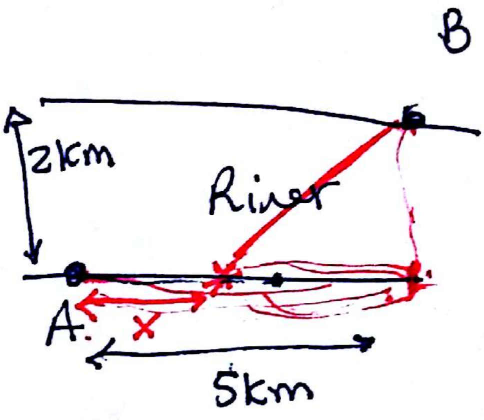
$a = 25$

$\frac{d^2A}{da^2} = -4 \rightarrow$ it is Neg. at $a = 25 \Rightarrow$ this is a maximum.

in a sport competition, each athlete starts from Point A and finish at point B.

while runing in the shore speed of JACK is 15 Km/hr

and when swimming it is 5 km/hr.



Design the best path for him (minimise time)

x is the distance of running before swimming.

$$t_1 = t_{running} = \frac{\text{distance of running}}{\text{speed}} = \frac{x}{15}$$

$$t_2 = t_{swimming} = \frac{\text{swimming distance}}{\text{speed}} = \frac{\sqrt{2^2+(5-x)^2}}{5}$$

$$t_2 = \frac{1}{5}\sqrt{4+25-10x+x^2} = \frac{1}{5}\sqrt{x^2-10x+29}$$

$$t_{tot} = t_1 + t_2 = \frac{x}{15} + \frac{\sqrt{x^2-10x+29}}{5}$$

$$\frac{d\,tot}{dx} = 0 \rightarrow \frac{1}{15} + \frac{2x-10}{10\sqrt{x^2-10x+29}} = 0 \qquad 0 < x < 5$$

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$$\frac{1}{15} + \frac{2x-10}{10\sqrt{x^2-10x+29}} = 0 \qquad x$$

$$10\sqrt{x^2-10x+29} + 15(2x-10) = 0$$

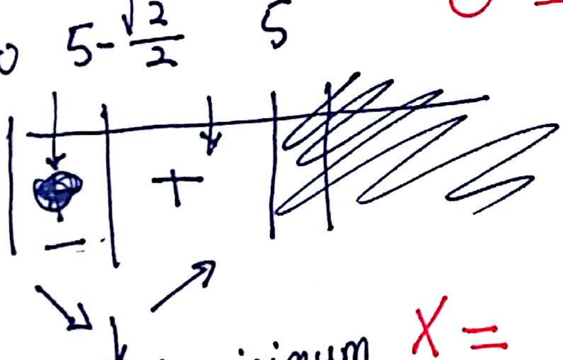
$$\sqrt{x^2-10x+29} = -\frac{15}{10}(2x-10) =$$

$$-\frac{3}{2}(2x-10) = -3x+15$$

$$\Rightarrow \quad x^2-10x+29 = (15-3x)^2 = 225-90x+9x^2$$

$$0 = 8x^2-80x+196 = 0$$

$$2x^2-20x+49 = 0$$



local minimum

$$x = \frac{20 \pm \sqrt{400-4(2)(49)}}{4} =$$

$$5 \pm \frac{\sqrt{400-8(49)}}{4}$$

$8(49) = 392$

$$x = 5 - \frac{\sqrt{400-8(49)}}{4} = 5-\frac{\sqrt{8}}{4} = 5-\frac{\sqrt{2}}{2}$$

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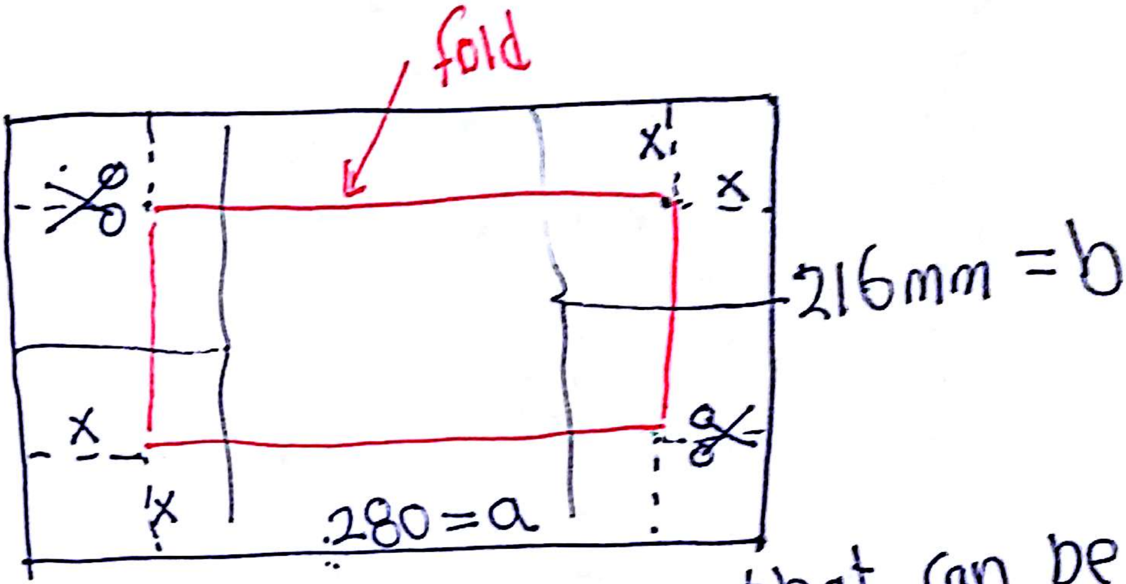


Find the largest open box that can be made by cutting square from the corners of a rectangle and folding it.

$$V = (a-2x)(b-2x)(x)$$

$$\Downarrow$$

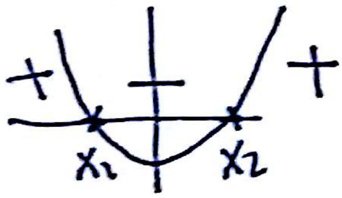
$$0 < x < \frac{b}{2}$$

$$V = 0$$

$$V = (ab - 2(a+b)x + 4x^2)x$$

$$= 4x^3 - 2(a+b)x^2 + abx$$

$$\frac{dV}{dx} = 12x^2 - 4(a+b)x + ab = 0$$



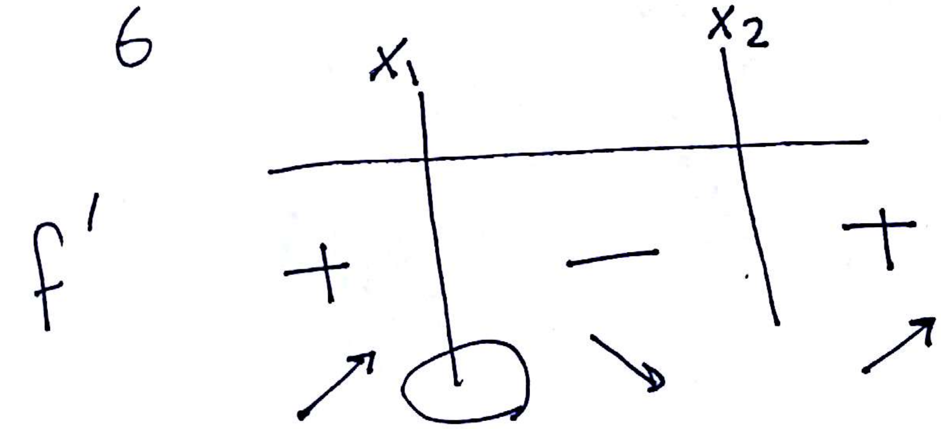
$$12x^2 - 4(a+b)x + ab = 0$$

$$x = \frac{4(a+b) \mp \sqrt{(4(a+b))^2 - 4(12)ab}}{24}$$

$$X = \frac{4(a+b) \pm \sqrt{16a^2 + 16b^2 + 32ab - 48ab}}{24}$$

$$= \frac{4(a+b) \pm \sqrt{16a^2 + 16b^2 - 16ab}}{24}$$

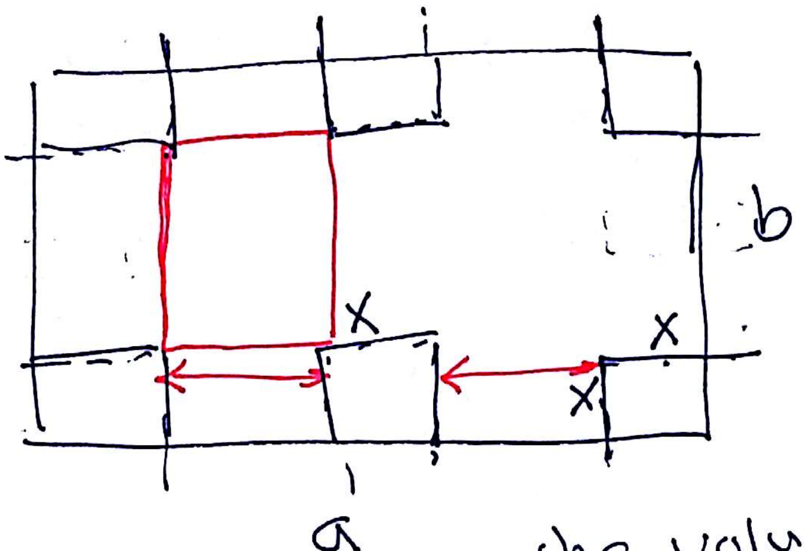
$$= \frac{a+b \pm \sqrt{a^2 + b^2 - ab}}{6}$$



$$X_{max} = \frac{a+b - \sqrt{a^2+b^2-ab}}{6}$$

$$X_2 > \frac{b}{2}$$

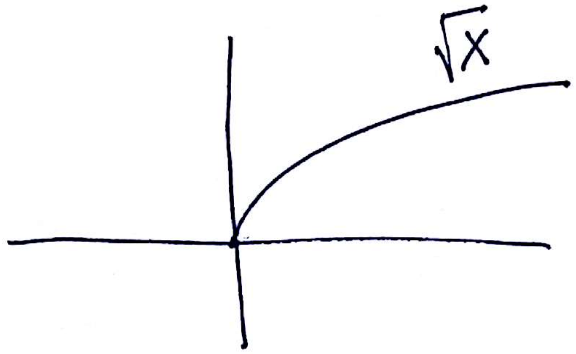
$a = 280$ mm
$b = 216$ mm
$\longrightarrow X = 40$ mm

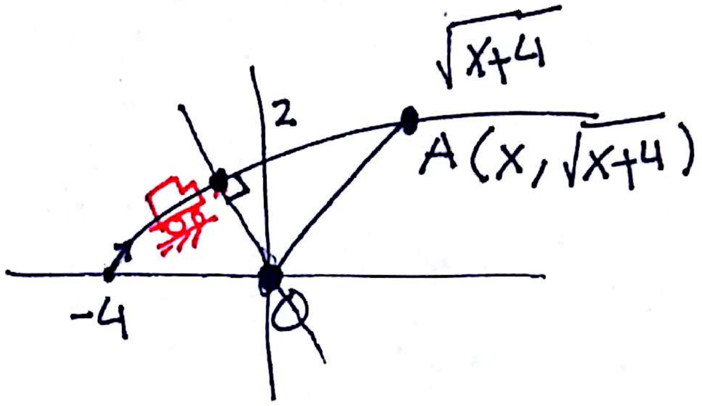


find x that maximize the volume.

$$V = \left(\frac{a-3x}{2}\right)(b-2x)\cdot x$$

You're left behind in a deserted island.
the search team ~~can't~~ couldnot find you past few days
You observed that the path that search team follows is like $y=\sqrt{x+4}$ where your island is at $(0,0)$.
find the point on the path of the search team that is in minimum distance from you so that hopefully they can see or hear you if you make fire or noises.





|OA| = distance from search team and island when search team is at point A.

$$D = \sqrt{(x_A - x_O)^2 + (y_A - y_O)^2} \Rightarrow 1$$

$$D = \sqrt{(x-0)^2 + (\sqrt{x+4} - 0)^2} = \sqrt{x^2 + x + 4}$$

$$\frac{dD}{dx} = 0 \rightarrow \frac{2x+1}{2\sqrt{x^2+x+4}} = 0 \rightarrow x = \frac{-1}{2}$$

$$\begin{array}{c|c|c} & -\frac{1}{2} & \\ \hline \frac{dD}{dx} & - & + \end{array}$$

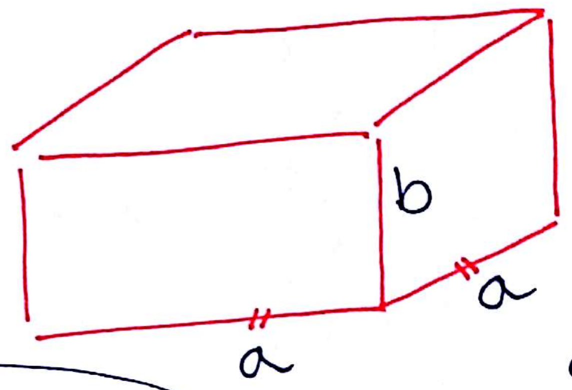
↘ local min ↗

if D is minimise, for same D^2 is also minimised and vice v.

$$D^2 = 1x^2 + x + 4 \rightarrow \frac{d(D^2)}{dx} = 2x + 1 \rightarrow x = \frac{-1}{2}$$

7 marks 9. A business wants to manufacture a huge closed box with a square base a square top and rectangular sides. The material used for the four upright sides of the box costs \$2 per square metre, and the material used for the base and top of the box costs \$8 per square metre.

Find the dimensions of the box with lowest possible cost that has volume 32m^3.



$a = 100$
$b = 32/10000$

$V = a \cdot a \cdot b = a^2 b$

Cost = (Area of side walls) 2 + (Area base) 8 + (Area top) 8

$= (4ab)\,2 + 2(8)(a^2) \Rightarrow$

$C = \cancel{16}\ 8ab + 16a^2$

$V = 32\,m^3 \rightarrow b = \dfrac{32}{a^2}$

$C = 8a\left(\dfrac{32}{a^2}\right) + 16a^2 = \dfrac{256}{a} + 16a^2$

$\dfrac{dC}{da} = 0 \rightarrow \dfrac{-256}{a^2} + 32a = 0 \rightarrow a$

$0 < a < \infty$

$-256 + 32a^3 = 0 \longrightarrow$

$$a^3 = \frac{256}{32} = \frac{2^8}{2^5} = 8$$

$$a = 2.$$

$$\frac{d^2C}{da^2} = \frac{(256)2}{a^3} + 32$$

$$\therefore \left.\frac{d^2C}{da^2}\right|_{a=2} = \frac{(256)2}{8} + 32 > 0$$

$\Downarrow$

local minimum.