NOV 3, 2016

vertical and horizontal asymptotes

↓ where function goes to infinity

↓ function blows up. (Singularity)

→ behavior of function at infinity

---

vertical asymptote

$$\lim_{x \to 0^+} \frac{1}{x} = \infty \qquad \frac{1}{0^+} = \infty$$

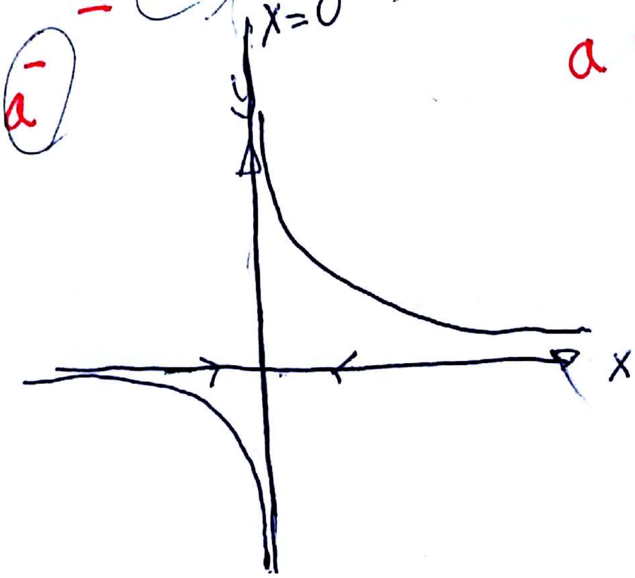
-0.0001

-0.1 -0.01 -0.001 x=0 0.0001 0.001 0.01 0.1

-10 -100 -1000 -10000 | 10000 1000 100 10

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty \qquad \frac{1}{0^-} = -\infty$$

if $\lim_{x \to a^+ \text{ or } a^-} f(x) = +\infty$ or $-\infty$ then $x = a$ is a vertical asymptote.



if $\dfrac{\text{Number} \neq 0}{0} = \pm\infty$

$$\lim_{x \to 0^+} \frac{\cos x}{x(x - \pi/2)} = \frac{1}{0^+\left(\frac{-\pi}{2}\right)} = \frac{1}{0^-} = -\infty$$

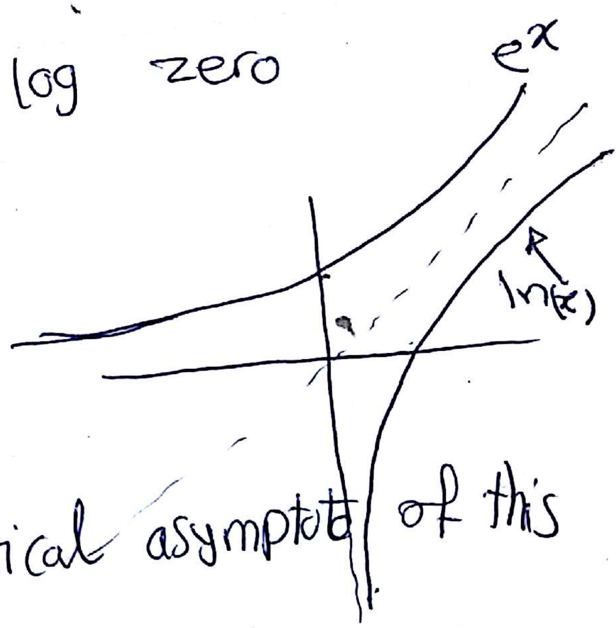
$$\lim_{x \to 0^-} \frac{\cos x}{x\left(x - \frac{\pi}{2}\right)} = \frac{1}{0^-\left(\frac{-\pi}{2}\right)} = \frac{1}{0^+} = +\infty$$

In General, when looking for vertical asymptotes, your candidates are

* points that make denom. zero

→ * Points that a log zero

$$\lim_{x \to 0^+} \ln x = -\infty$$



$\dfrac{x-2}{(x^2-4)}$, find vertical asymptote of this function

Candidates $x^2 - 4 = 0 \to x^2 = 4 \to x = \pm 2$

$$\lim_{x \to -2^+} \frac{x-2}{x^2-4} = \lim_{x \to -2^+} \frac{-4}{(x-2)(x+2)} = \frac{-4}{(-4)(0^+)} = \frac{-4}{0^-} = \infty$$

Scanned by CamScanner

$$\lim_{x\to -2^-} \frac{x-2}{x^2-4} = \lim_{x\to -2^-} \frac{x-2}{(x-2)(x+2)} = \frac{-4}{(-4)(0^-)} = \frac{-4}{0^+} = -\infty$$

$$\lim_{x\to +2^+} \frac{x-2}{x^2-4} = \frac{0}{0}$$

$$= \lim_{x\to +2^+} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \lim_{x\to +2^+} \frac{1}{x+2} = \frac{1}{4}$$

$X=-2$ is vertical asymptote

$X=2$ is NOT vertical asymptote

Horizontal asymptote

$\lim_{x\to +\infty} f(x) = L_1 \rightarrow y=L_1$ is a horizontal asymptote

$\lim_{x\to -\infty} f(x) = L_2 \rightarrow y=L_2$ is a horizontal asymptote.

$$\boxed{\lim_{x\to\infty} \frac{1}{x^r} = 0 \quad \text{for any } r>0}$$

Scanned by CamScanner

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\frac{\text{Number}}{\infty} = 0$$

$$\frac{\infty}{\infty} = \text{simplification}$$

$$\frac{\infty-\infty}{\infty} = \text{Simplification}$$

$$\lim_{x\to\infty} \frac{1x^2-3x+4}{3x^2-8x+1} = \lim_{x\to\infty}\frac{x^2\left(1-\cancel{\frac{3}{x}}+\cancel{\frac{4}{x^2}}\right)}{x^2\left(3-\cancel{\frac{8}{x}}+\cancel{\frac{1}{x^2}}\right)} =$$

$$\lim_{x\to\infty}\frac{x^2}{3x^2} = \frac{1}{3}$$

$y=\frac{1}{3}$ is a horizontal asymptote for this function.

$$\lim_{x\to\infty} \frac{(\text{Polynomail}) \text{ of order } m}{(\text{Polynomial}) \text{ of order } n}$$

- $m < n \leftarrow 0$
- $m = n \leftarrow$ ratio of coefficient of highe Power
- $m > n \leftarrow \infty$

$$\lim_{x\to\infty} \frac{\cancel{x^3}}{\cancel{x^2}} = x \to \infty$$

$$\lim_{x\to\infty} \frac{\cancel{x^2}}{x^3} = \lim_{x\to\infty}\frac{1}{x} = 0$$

Scanned by CamScanner

$$\lim_{x\to\infty} \frac{x^n-1}{x^2+1}$$

determine n, so that the limit exists and has a non-zero answer.

$n=2$

$$\lim_{x\to\infty} \frac{x^2-1}{x^2+1} = \lim_{x\to\infty} \frac{x^2\left(1-\cancel{\frac{1}{x^2}}\right)}{x^2\left(1+\cancel{\frac{1}{x^2}}\right)} = \lim_{x\to\infty} \frac{x^2}{x^2} = 1$$

$$\lim_{x\to+\infty} \frac{\sqrt{4x^2+1}}{5x-1} = \lim_{x\to\infty} \frac{\sqrt{4x^2\left(1+\cancel{\frac{1}{4x^2}}\right)}}{x\left(5-\cancel{\frac{1}{x}}\right)}$$

$$= \lim_{x\to\infty} \frac{\sqrt{4x^2}}{5x} = \lim_{x\to\infty} \frac{2x}{5x} = \frac{2}{5}$$

$$\lim_{x\to-\infty} \frac{\sqrt{4x^2+1}}{5x-1} = \lim_{x\to-\infty} \frac{\sqrt{4x^2}}{5x} = \lim_{x\to\infty} \frac{-2x}{5x} = \frac{-2}{5}$$

Scanned by CamScanner

$$\lim_{x\to\infty} \frac{x^2-\sqrt{x^4-x^3}}{x-1} = \lim_{x\to\infty} \frac{x^2\left(1-\sqrt{1-\frac{1}{x}}\right)}{x-1}$$ $\boxed{\infty \cdot 0}$

$$x^2-\sqrt{x^4-x^3} = x^2-\sqrt{x^4\left(1-\frac{1}{x}\right)} =$$

$$x^2 - x^2\sqrt{1-\frac{1}{x}}$$

$$x^2\left(1-\sqrt{1-\frac{1}{x}}\right)$$

$$\lim_{x\to\infty} \frac{x^2-\sqrt{x^4-x^3}}{x-1} \cdot \frac{x^2+\sqrt{x^4-x^3}}{x^2+\sqrt{x^4-x^3}} =$$

$$\lim_{x\to\infty} \frac{\cancel{x^4}-(\cancel{x^4}-x^3)}{(x-1)\left(x^2+\sqrt{x^4-x^3}\right)} = \lim_{x\to\infty} \frac{x^3}{2x^3}$$

$$x \qquad \sqrt{x^4}$$

$$2x^2$$

$$= \frac{1}{2}$$

Scanned by CamScanner