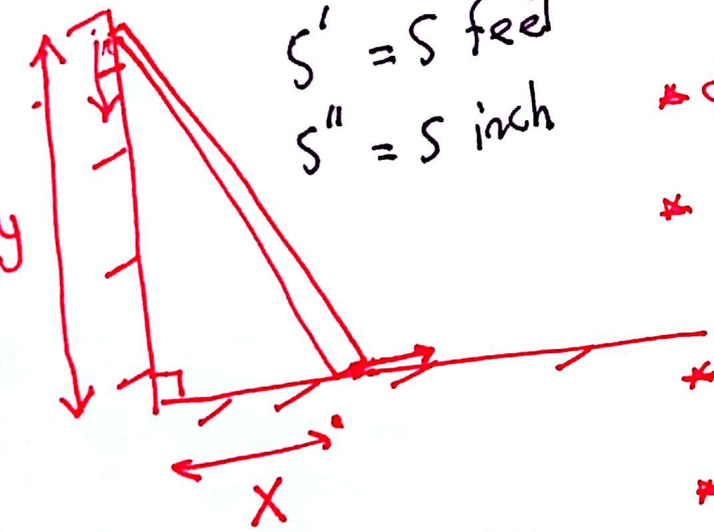
One end of a 13′ ladder is on the ground and the end rests on the wall. If the ~~area~~ bottom end slides at a rate of 3 ft/s away from the wall, $\boxed{\text{how fast}}$ is top end slides down when the bottom of the ladder is 5′ away from the wall?

length of ladder = 13 feet



5′ = 5 feet

5″ = 5 inch

* distance of top end from ground $= y$
* rate of change of $= \dfrac{dy}{dt}$
* distance of bottom end from wall $= x = 5'$
* rate of change of $= \dfrac{dx}{dt} = \dfrac{3\,ft}{s}$

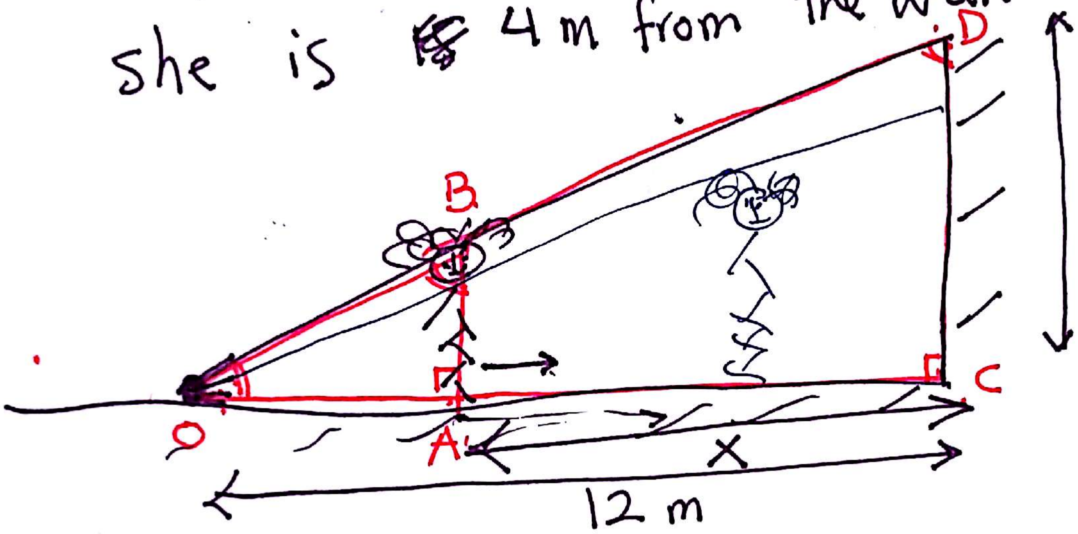
Pythagoras $\to x^2 + y^2 = 13^2 \Rightarrow (5)^2 + y^2 = 13^2 \to y = \sqrt{169-25} = \sqrt{144} = 12$

$$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(13^2) = 0$$

$$2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow 2\cdot(5)(3) + 2(12)\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{30}{24} = -\frac{5}{4}\frac{ft}{s}$$

a spotlight on the ground shines on a wall 12 m away. if a two meter tall women walk from the spotline to the wall at a speed of $\boxed{1.6\ m/s}$, then how fast is the length of her shadow on the wall decreases when she is 4 m from the wall.



Height = 2 m

distace of spotline from the wall = 12 m

distance of the woman from the wall = X

length of her shadow = Y

Rate of change of distance of woman from the wall

Rate of change of her shadow.

$\triangle OAB \sim \triangle OCD \Rightarrow \frac{OA}{OC} = \cancel{\frac{OB}{OD}} = \frac{AB}{CD}$

$\frac{12-x}{12} = \frac{2}{y} \rightarrow 12y - xy = 24$

$x=4 \Rightarrow 8y = 24 \Rightarrow y=3$

$$\frac{d}{dt}\left(12y - xy\right) = \frac{d}{dt}(24) = 0$$

$$12\frac{dy}{dt} - \frac{dx}{dt}y - x\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -1.6\ \frac{m}{s}$$

$$12\frac{dy}{dt} - (-1.6)\cdot(3) - 4\left(\frac{dy}{dt}\right) = 0$$

$$8\frac{dy}{dt} = -4.8 \Rightarrow \frac{dy}{dt} = \frac{-4.8}{8} = -0.6\frac{m}{s}$$

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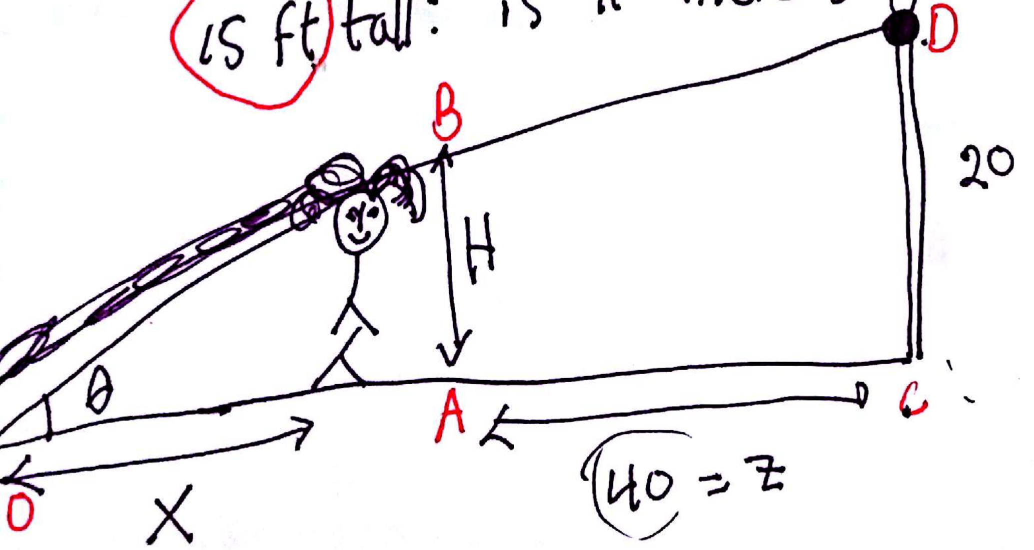
while in Winderland, Alice eats a cookie that makes her height grow taller at the rate of $0.5\ \frac{ft}{min}$. If she is standing 40 ft away from a light which is 20 ft tall. how fast is the length of her shadow on ground changing when she is 15 ft tall? is it increasing or decreasing?



Constant → height of light = 20 ft

→ distance of Alice from the light = 40ft

Her hight = H

rate of change of her height = $\frac{dH}{dt}$

shadow length = X

rate of chang of shadow length = $\frac{dX}{dt}$

You can use Similar triangles

$\triangle OAB \sim \triangle OCD$

$\triangle OAB$. $\tan\theta = \frac{H}{X}$ $\Rightarrow$ $\frac{H}{X} = \frac{20}{X+40}$

$\triangle OCD$ $\tan\theta = \frac{20}{X+40}$

$$\Downarrow$$

$$HX + 40H = 20X$$

$$\Downarrow$$

$$15X + 40(15) = 20X$$

$$\Downarrow$$

$$600 = 5X$$

$$\Downarrow$$

$$X = 120$$

$$\frac{d}{dt}(HX + 40H) = \frac{d}{dt}(20X)$$

$$\frac{dH}{dt}X + H\frac{dX}{dt} + 40\frac{dH}{dt} = 20\frac{dX}{dt}$$

$$(0.5)(120) + 15\frac{dX}{dt} + 40(0.5) = 20\frac{dX}{dt} \Rightarrow 80 = 5\frac{dX}{dt}$$

$$\frac{dX}{dt} = 16 \text{ ft/min}$$

$$\lim_{x \to 1} \frac{x^{100} - 1}{2x - 2} = \frac{0}{0} \quad \text{Requires simplification}$$

$$\boxed{f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}}$$

$$\lim_{x \to 1} \frac{x^{100} - 1}{2x - 2} = \frac{1}{2} \boxed{\lim_{x \to 1} \frac{x^{100} - 1}{x - 1}} = \frac{1}{2} \underbrace{(x^{100})'}_{\text{when } x = 1}$$

$$f(x) = x^{100}$$

$$x_0 = 1$$

$$\frac{1}{2} \underbrace{(100 x^{99})}_{x = 1} = \frac{1}{2} 100 = 50$$

$$\lim_{x \to 1} \frac{\sqrt{2-x} - 1}{\sqrt{17-x} - 4} = \frac{1-1}{4-4} = \frac{0}{0} \quad \text{Requires simp.}$$

$$\lim_{x \to 1} \frac{\sqrt{2-x} - 1}{\sqrt{17-x} - 4} \times \frac{\sqrt{2-x} + 1}{\sqrt{2-x} + 1} \times \frac{\sqrt{17-x} + 4}{\sqrt{17-x} + 4} =$$

$$\lim_{x \to 1} \frac{((2-x) - 1)(\sqrt{17-x} + 4)}{(17 - x - 16)(\sqrt{2-x} + 1)} = \lim_{x \to 1} \frac{\cancel{(1-x)}(\sqrt{17-x} + 4)}{\cancel{(1-x)}(\sqrt{2-x} + 1)} = \frac{4+4}{1+1}$$

$$= 4$$

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$3y^2 - 2x^3 - 4x^2y + 5y = 7$

Find ~~to~~ points where tangent line is <u>horizontal</u>.

$$\frac{d}{dx}\left(3y^2 - 2x^3 - 4x^2y + 5y\right) = \frac{d}{dx}(7) = 0$$

$$6y\frac{dy}{dx} - 6x^2 - 8xy - 4x^2\frac{dy}{dx} + 5\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 0 \Rightarrow -6x^2 - 8xy = 0$$

$$-6x^2 - 8xy = 0 \rightarrow x(-6x-8y) = 0$$

$$x = 0 \Rightarrow 3y^2 + 5y = 7 \rightarrow 3y^2 + 5y - 7 = 0$$

find y by quadratic form

$$-6x - 8y = 0 \Rightarrow x = \frac{-8}{6}y$$

$$3y^2 - 2\left(\frac{-8}{6}y\right)^3 - 4\left(\frac{-8}{6}y\right)^2 y + 5y = 7$$

$$3y^2 - 2\left(-\frac{64}{27}\right)y^3 - \left(\frac{16}{9}\right)4y + 5y = 0$$

$$\frac{128}{27}y^3 + 3y^2 - \frac{19}{9}y = 0 \Rightarrow y\left(\frac{128}{27}y^2 + 3y - \frac{19}{9}\right) =$$

$y = 0$

$$\frac{128}{27}y^2 + 3y - \frac{19}{9} = 0 \rightarrow$$

$f(x) = x^2 e^x$

Prove there is Point where tangent line is parallel to $y = 2016x - 30$

$y = 2016x - 3 \longrightarrow$ slop $= 2016$

$f' = 2016$ $\qquad f'(x) = 2xe^x + x^2 e^x$

$$2xe^x + x^2 e^x = 2016$$

$$\Downarrow$$

$$g(x) = 2xe^x + x^2 e^x - 2016 = 0$$

IVT $\Rightarrow$ $g(x)$ is continuous everywhere.

$x = 0 \Rightarrow g(0) = -2016 < 0$

$x = 2016 \quad g(2016) = 2(2016)e^{2016} + 2016^2 e^{2016} - 2016 > 0$

$g(0) < 0 < g(2016)$

$\Downarrow$

By IVT there must exist a c

$c \in [0, 2016]$, for which

$g(c) = 0 \Rightarrow$

$2c\,e^c + c^2 e^c = 2016$