Oct 20

AVERAGE COST $\bar{C}(x) = \frac{C(q)}{q}$

AVERAGE RATE of change of cost $\frac{\Delta C}{\Delta q} = \frac{C(q_2) - C(q_1)}{q_2 - q_1}$

Marginal cost $MC = \frac{dC}{dq}$

Marginal unit cost $MU C(n) = C(n+1) - C(n)$

A Company produces bycicles with the cost of $C(q) = 10000 + 300\, q^{0.8}$

a) Find Average cost when $q = 1, 10, 100$. Interpret this.

$$\bar{C}(1) = \frac{C(1)}{1} = \frac{10300}{1} = 10300\ \$$$

$$\bar{C}(10) = \frac{C(10)}{10} = \frac{10000 + 300(10)^{0.8}}{10} \cong 1190\ \$$$

$$\bar{C}(100) = \frac{C(100)}{100} = \frac{10000 + 300(100)^{0.8}}{100} = 177\ \$$$

On Average the first 10 bikes cost 1190\$ each to manufacture.

Economies of Scale

b) Find average rate of change when the company is moving from 10 bycikes to 100.

$q_1 = 10$

$q_2 = 100$

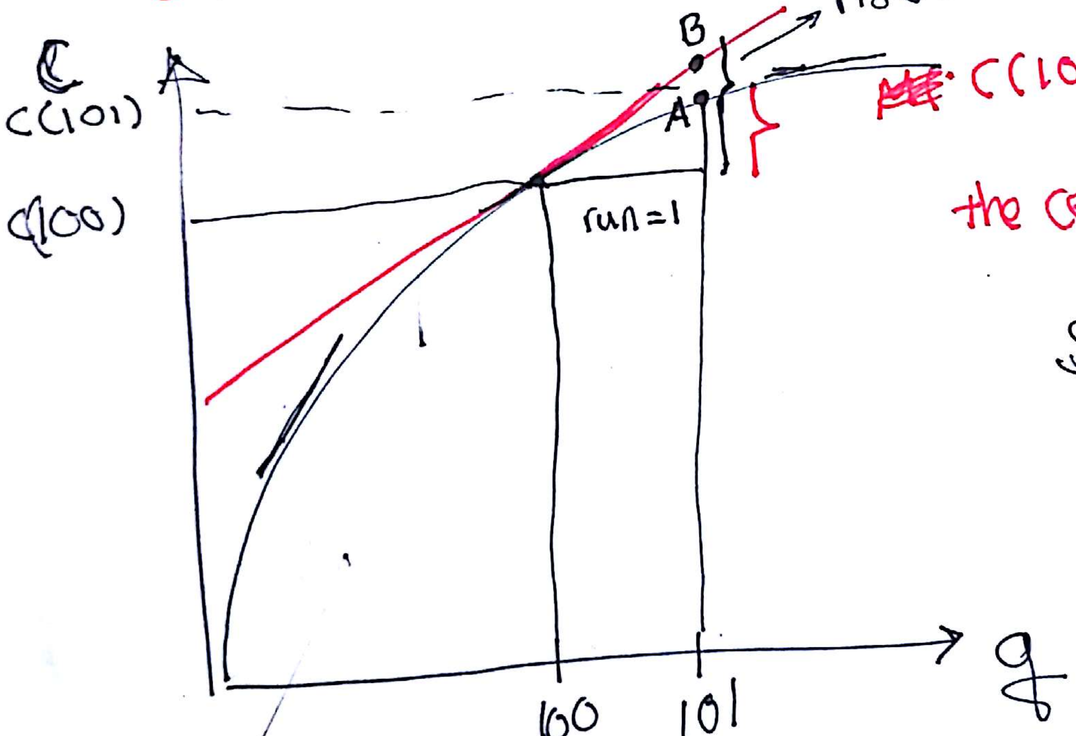
$$\text{AVE. R.C} = \frac{C(100) - C(10)}{100 - 10}$$

$$= \frac{10000 + 300(100)^{0.8} - (10000 + 300(10)}{90}$$

$$= 111.6 \$$$

when moving from $10 \to 100$ on average each bircycle cost you $111.6 \$$

rise = slope = $\frac{dc}{dq}$ = MC



$C(101) - C(100) = MC(10$

the cost of 101^{th} bicyc

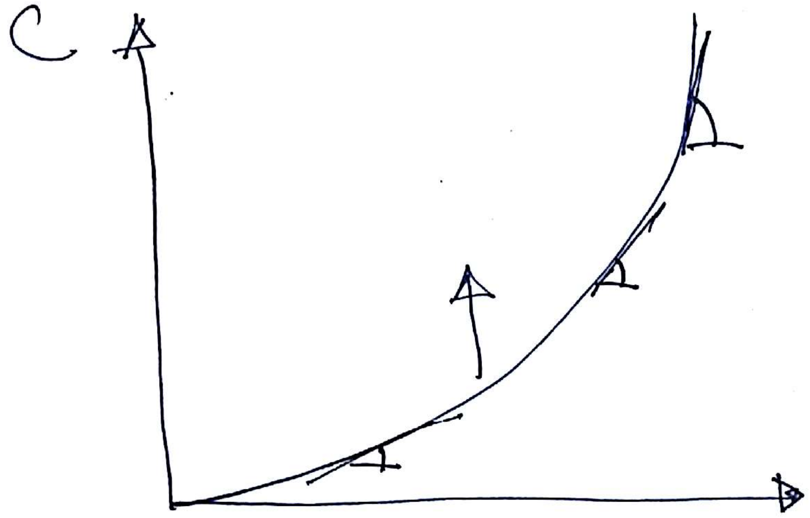
$$\text{Slope} = \frac{\text{rise}}{\text{run}} = .$$

Slope = rise

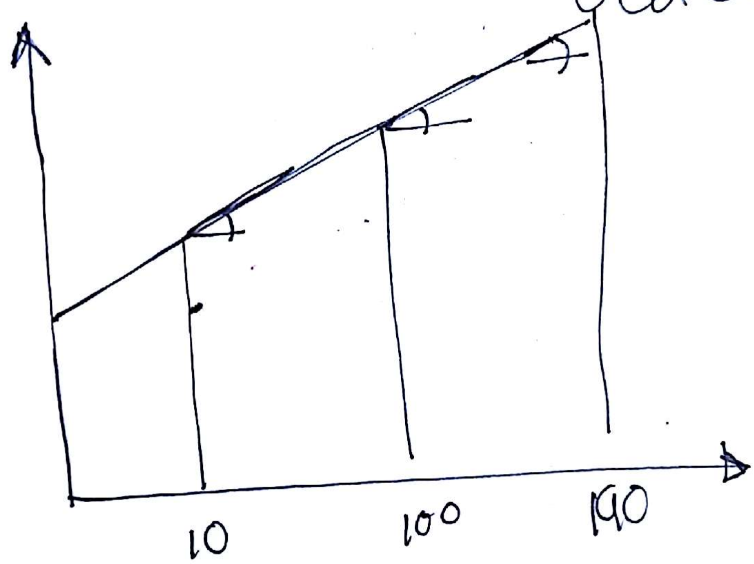
Economy of scales.

MC is approximately MC.

$f'' < 0 \to$ Concave Down $\to$ Economies of Scale



Concave up → Diseconomies of Scale.



for linear cost

MUC = MC always.

Manufacturing of x bicyles/week

costs $C(x) = 500 + 350x - 0.9x^2$

what is the cost of the 301^{th} bike?

a) $MC(300) = C(301) - C(300)$

$= 500 + 350(301) - 0.9(301)^2 - (500 + 350(300) - 0.9(300)^2) = 295.9\$$

b) $MC(300)$ $\quad MC = \frac{dc}{dq} = 350 - 0.18x$

$MC(300) = 350 - 0.18(300) = 296.5\$$

approximatly, the 301^{th} bike cost $296.5\$$

Weekly cost of producing x bicylce is given by $C(q) = 1500 + 100q - 0.01q^2$

and the price function is

$P(q) = 120 - 0.005q$

Determine marginal cost, marginal Revenue & Marginal Profit

$MC = \frac{dC}{dq} = 100 - 0.02q$

$R = P.q = (120 - 0.005q)q = 120q - 0.005q^2$

$MR = 120 - 0.01q$

$P = R - C \rightarrow \frac{dP}{dq} = \frac{dR}{dq} - \frac{dC}{dq}$

$\Downarrow$

$MP = MR - MC$

$MP = \underline{120 - 0.01q - (100 - 0.02q)} =$

$= 20 + 0.01q$

Evalute MP at $q = 250$ and Interpret it.

$MP = 20 + 0.01(250) = 20 + 2.5 = 22.5\$$

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approximately production of 251^{th} bike will have the profit of 22.5 \$.

$$MUP = \bar{P}(251) - \bar{P}(250) = 23 \$$$

Price Elasticity of Demand.

let q be function of P.

$$R = P.q = P.q(P).$$

$$R' = \frac{dR}{dP} = (P.q(P))' = 1.q(P) + P.\frac{dq}{dP}$$

$$= q + P\frac{dq}{dP} = q\Big(1 + \underbrace{\frac{P}{q}\frac{dq}{dP}}_{E_D}\Big)$$

E_D = Price elasticity of demand.

$$\frac{dR}{dP} = q(1+E_D)$$

$E_D < -1 \Rightarrow R' < 0$

$E_D > -1 \Rightarrow R' > 0$

$$E_D = \frac{P}{q} \frac{dq}{dP} < 0$$

$\frac{dq}{dP}$ → law of demand

$E_D < 0$

Find acceleration

$|E_D| < 1 \longrightarrow R' > 0$ Price inelastic

$|E_D| > 1 \longrightarrow R' < 0$ Price elastic

$|E_D| < 1 \rightarrow R' > 0 \rightarrow \frac{dR}{dP} > 0$

If increasing Price $\Longrightarrow$ Increase R.

$|E_D| > 1 \rightarrow R' < 0 \rightarrow \frac{dR}{dP} < 0$

If decreasing Price $\rightarrow$ Increase R.

E_D is always negative therefore we only work with its absolute value

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