office hour tmrw 3:00 LSK 300C.

let f(x) be defined on an interval I containing x=c. If there is an open interval containing for which f(c) in the minimum value, then f(c) is called a relative minimum (local minimin)

we say f(x) has a relative minimum at (c, f(c)).

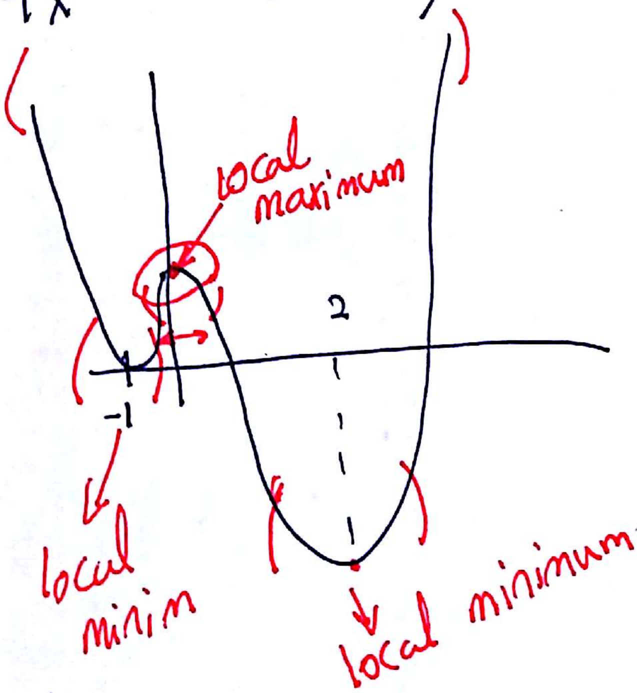
Similarly defined for ~~maximinimum~~ local maximinimum

$$f(x) = \frac{1}{5}\left(3x^4 - 4x^3 - 12x^2 + 5\right). \text{ on } [-2,3].$$

$X = -1$ local minimum

$x = 0$ local maximum

$x = 2$ local minimum.



3 local Extrema.

local Extrema are points that are either local max or local min.

Fermat theory: if $f(x)$ has a local minimum or local maximum at $f(c)$, and if $f'(c)$ exists, then $f'(c) = 0$

$\Downarrow$

if you want to find local extrema, then look at points where $\begin{cases} f'(x)=0 \\ f'(x) \text{ Not Defined.} \end{cases}$

* critical points are those points for which $f' = 0$ or is not defined.

$\Downarrow$

all local Extrema, they are critical point by Not vice verca.

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find critical points for $f(x) = \frac{1}{5}(3x^4 - 4x^3 - 12x^2 + 5)$

$\to f'(x) = 0$

$\to f'(x)$ Not defined. $\times$

$$f'(x) = \frac{1}{5}(12x^3 - 12x^2 - 24x)$$

$$f'(x) = 0 \to \frac{1}{5}(12x^3 - 12x^2 - 24x) = 0$$

$$\frac{12}{5}x(x^2 - x - 2) = 0 \to \frac{12}{5}x(x-2)(x+1) = 0$$

$x = 0$
$x = 2$
$x = -1$

Find a such that $f(x) = \sin(ax) + 2x + 3 - x^2$ has a critical point at $x = 0$. (2015)

$$f' = a\cos(ax) + 2 - 2x$$

$f' = 0$

~~f' not defined~~ $\leftarrow$ $f(x)$ is a Nice function and it is differentiable everywhere.

$$f'(0) = a\cos(0) + 2 - 2(0) = 0$$

$$a(1) + 2 = 0 \Rightarrow a = -2$$

$$f(x) = \frac{x^2}{x^2 - 1}$$ identify critical Points

$\leftarrow D = \mathbb{R} - \{\pm 1\}$

$x^2 - 1 = 0 \Rightarrow x = \pm 1$

$$f'(x) = \frac{2x(x^2-1) - 2x(x^2)}{(x^2-1)^2}$$

$$= \frac{\cancel{2x^3} - 2x - \cancel{2x^3}}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

$$f'(x) = 0 \longrightarrow \frac{-2x}{(x^2-1)^2} = 0 \rightarrow -2x = 0$$

$$\Downarrow$$

$$\boxed{x=0}$$

$f'(x) =$ Not defined.

$$\Rightarrow \text{denom} = 0 \Rightarrow (x^2-1)^2 = 0$$

$$\Rightarrow x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow \boxed{x = \pm 1}$$

Not critial Point because they are Not in domain.

$f(x) = (x-1)^{2/3} + 2$
identify critical points.

$$f'(x) = \frac{2}{3}(x-1)^{\frac{2}{3}-1}$$

$$= \frac{2}{3}(x-1)^{-1/3} = \frac{2}{3}\,\frac{1}{(x-1)^{1/3}}$$

$$f(x) = \left((x-1)^2\right)^{1/3} + 2 \quad = \frac{2}{3}\,\frac{1}{\sqrt[3]{x-1}}$$

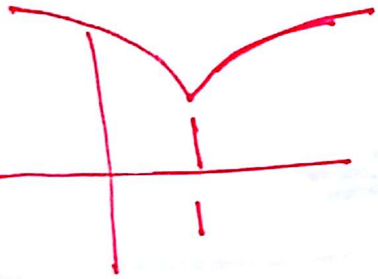
$$\rightarrow \sqrt[3]{(x-1)^2} + 2 \qquad D = \mathbb{R}$$

$f'(x) = 0 \Rightarrow \frac{2}{3} \frac{1}{\sqrt[3]{x-1}} = 0$

No solution.

$f'(x)$ Not defined.

$\frac{2}{3} \frac{1}{\sqrt[3]{x-1}}$ Not defined.



denom $= 0 \to \sqrt[3]{x-1} = 0$

$\to x-1 = 0 \to \boxed{x=1}$

Closed Interval method.

let f be Continuous on closed Interval $[a,b] \Rightarrow$ BY EVT, It has one abs. Min & abs. Max.

1) evaluate end Points.
2) find critical Points & evaluate then
3) lagest one is abs. Max.
smallest one is abs Min.

$f(x) = x^3 - 3x^2 - 9x + 2$

find abs Min & abs Max. on $[-2,2]$

1) evaluate end points

$f(-2) = (-2)^3 - 3(-2)^2 - 9(-2) + 2$
$= -8 - 12 + 18 + 2 = 0$

$f(2) = 2^3 - 3(2)^2 - 9(2) + 2 =$
$8 - 12 - 18 + 2 = -20$

2) Find critical Points.

$f'(x) = 0 \rightarrow 3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0 \Rightarrow 3(x+1)(x-3) = 0$

$x = -1, \ \boxed{\cancel{x=3}}$

$f(-1) = -1^3 - 3(-1)^2 - 9(-1) + 2 = 7$

Abs. Max = $\max(0, -20, 7) = 7$

Abs. Min = $\min(0, -20, 7) = -20$

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Find abs Min. and abs Max of $f(x) = x + \sin x$; on $[0, 2\pi]$

a) evaluate end Points

$f(0) = 0 + \sin(0) = 0$

$f(2\pi) = 2\pi + \sin(2\pi) = 2\pi$

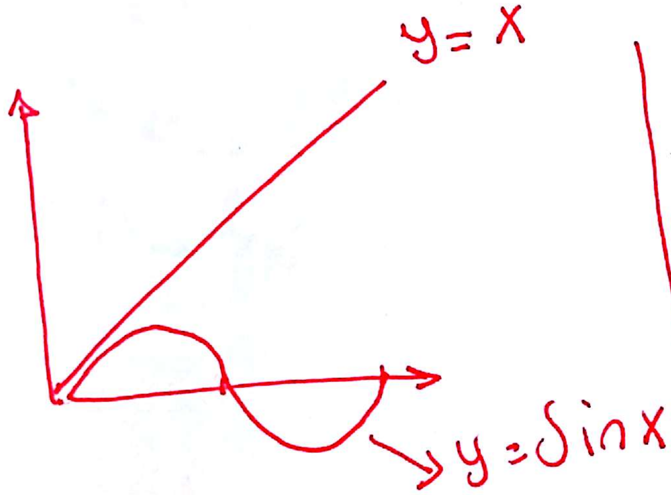
b) $\rightsquigarrow f'(x) = 0 \rightarrow 1 + \cos x = 0$

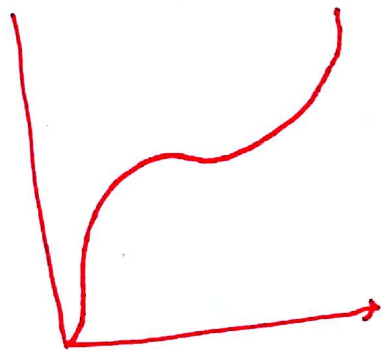
$\rightarrow \cos x = -1 \Longrightarrow \boxed{x = \pi}$

$f(\pi) = \pi + \sin(\pi) = \pi$

Abs Max $= 2\pi$

ABS Min $= 0$





$f(x) = x\sqrt{4-x^2}$.

find Abs Min, abs Max in its domain.

$4-x^2 \geq 0 \Rightarrow -x^2 \geq -4$

$\Downarrow$

$x^2 \leq 4$

$\Downarrow$

$|x| \leq 2$

$\Downarrow$

$\underline{-2 \leq x \leq 2}$

1) $f(2) = 0$

$f(-2) = 0$

2) $f'(x) = \sqrt{4-x^2} + x\dfrac{-2x}{2\sqrt{4-x^2}}$

$= \sqrt{4-x^2} - \dfrac{x^2}{\sqrt{4-x^2}} =$ common denom

$\dfrac{4-x^2 - x^2}{\sqrt{4-x^2}} = \dfrac{4-2x^2}{\sqrt{4-x^2}}$

$f'(x) = 0 \rightarrow 4 - 2x^2 = 0$

$\rightarrow 2x^2 = 4 \Rightarrow x^2 = \frac{4}{2} = 2$

$\Rightarrow x = \pm\sqrt{2}$

$f'(x) = \text{Not defined}$

$\Downarrow$

$\text{denom} = 0 \rightarrow \sqrt{4 - x^2} = 0$

$\rightarrow 4 - x^2 = 0 \rightarrow \boxed{x = \pm 2}$

Not accepte

[illegible]

$f(\sqrt{2}) = \sqrt{2} \cdot \sqrt{4-2} = 2$

$f(-\sqrt{2}) = -\sqrt{2}\sqrt{4-2} = -2$

abs Max = 2

abs Min = -2

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