Oct 6, 2016

find derivative of $\sin(y) + y^3 = 6 - x^3$

W.R.T x

$$\frac{d}{dx}\left(\sin(y) + y^3\right) = \frac{d}{dx}\left(6 - x^3\right)$$

$$\cos y \cdot \frac{dy}{dx} + 3y^2 \cdot \frac{dy}{dx} = -3x^2$$

$$\frac{dy}{dx}\left(3y^2 + \cos y\right) = -3x^2$$

derivative of $y(x)$ W.R.T x ←

$$\frac{dy}{dx} = \frac{-3x^2}{3y^2 + \cos y}$$

W.R.T y

$$\frac{d}{dy}\left(\sin(y) + y^3\right) = \frac{d}{dy}\left(6 - x^3\right)$$

$$\cos y + 3y^2 = -3x^2 \cdot \frac{dx}{dy}$$

derivative of $x(y)$ W.RT y

$$= \frac{dx}{dy} = \frac{\cos y + 3y^2}{-3x^2}$$

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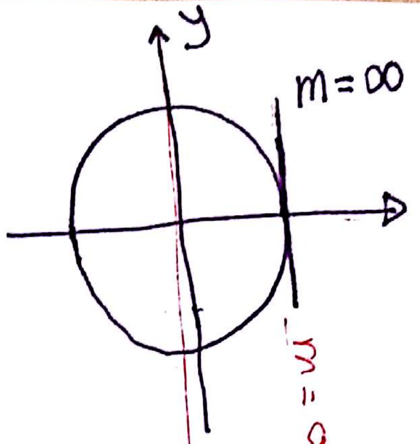
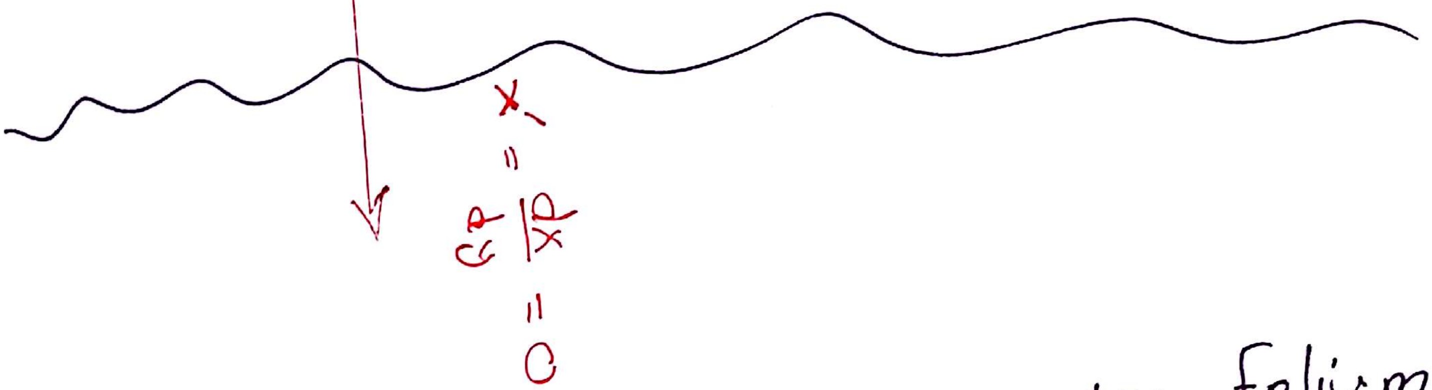
$m = \infty \rightarrow y' = \infty \rightarrow y'$ does not exist





$x^3 + y^3 - 3xy = 0$ Descarter folium.

$\Downarrow$

Fermat

find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left(x^3 + y^3 - 3xy\right) = 0$$

$$3x^2 + 3y^2 y' - 3xy' - 3y = 0 \checkmark$$

$$y'(3y^2 - 3x) = 3y - 3x^2$$

$$\boxed{y' = \frac{3y - 3x^2}{3y^2 - 3x}}$$

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Find $\dfrac{d^2y}{dx^2} = y''$

$$\frac{d}{dx}\left(3x^{2'} + 3y^2 y' - 3xy' - 3y\right) = 0$$

$$6x + 6yy'y' + 3y^2 y'' - 3y' - 3xy'' - 3y' = 0$$

$$y''(3y^2 - 3x) = 6y' - 6yy'^2 - 6x$$

$$y'' = \frac{6y' - 6yy'^2 - 6x}{3y^2 - 3x}$$

$$\frac{d}{dx}(3y^2y')' = (3y^2)'y' + 3y^2(y')'$$

$$6yy'y' + 3y^2y''$$

$$= 6yy'^2 + 3y^2y''$$

$x^2 - y^2 - xy = 1$ find tangent lines at $x = 1$ on the curve.

$$x = 1 \Rightarrow 1 - y^2 - y = 1 \rightarrow -y^2 - y = 0 \rightarrow -y(y+1)$$

$$y = 0, -1$$

$$\frac{d}{dx}(x^2 - y^2 - xy) = \frac{d}{dx}(1) = 1$$

$$2x - 2yy' - y - xy' = 0$$

$(1,0)$ $2 - 0 - 0 - y' =$

$y' = 2$

$$y - 0 = 2(x-1)$$

$$y = 2x - 2$$

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$2x - 2yy' - y - xy' = 0 \longrightarrow (1, -1)$

$2 - 2(-1)y' - (-1) - y' = 0$

$2 + 2y' + 1 - y' = 0 \longrightarrow y' + 3 = 0 \longrightarrow y' = -3$

$y - (-1) = -3(x - 1)$

$y + 1 = -3x + 3 \longrightarrow y = -3x + 2$

$y - y_0 = m(x - x_0)$

Logarithmic differentiation.

$x^3 \quad a^x$

$y = x^x$ $\quad \cancel{x \cdot x^{x-1} = x^x}$

$\ln y = \cancel{x}\ln(x^x) = x \ln x$

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x \ln x) \Rightarrow$

$\frac{1}{y} \cdot y' = \ln x + x\frac{1}{x} = \ln x + 1$

$y' = y(\ln x + 1) = x^x(\ln x + 1)$

$g(t) = (\cos t)^{e^t}$, find $\frac{dg}{dt}$

$\ln(g(t)) = \ln\left((\cos t)^{e^t}\right) = e^t \ln \cos t$

$\frac{d}{dt}(\ln(g(t))) = \frac{d}{dt}\left(e^t \ln(\cos t)\right)$

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$$\frac{g'(t)}{g(t)} = e^t \ln(\cos t) + e^t \frac{-\sin t}{\cos t}$$

$$g'(t) = g(t)\left(e^t \ln(\cos t) - e^t \tan t\right).$$

$$= (\cos t)^{e^t} e^t\left(\ln(\cos t) - \tan t\right)$$

$$y = \frac{\sin^2 x \; (x-1)^2}{(2x-1)^2}$$

$\ln(a+b) = ?$

$\ln(a/b) = \ln a - \ln b$

$\ln(a \cdot b) = \ln a + \ln b$

$$\ln y = \ln\left(\frac{\sin^2 x \cdot (x-1)^2}{(2x-1)^2}\right) =$$

$$= \ln\left(\sin^2 x \; (x-1)^2\right) - \ln\left((2x-1)^2\right)$$

$$= \ln\left((\sin^2 x)\right) + \ln\left((x-1)^2\right) - \ln\left((2x-1)^2\right)$$

$$\ln y = \underline{2\ln(\sin x)} + 2\ln(x-1) - 2\ln(\underline{2x-1})$$

$$\frac{y'}{y} = \frac{2}{\sin x}\cos x + \frac{2}{x-1} - 2\,\frac{2}{2x-1}$$

$$y' = \frac{\sin^2 x \; (x-1)^2}{(2x-1)^2}\left(2\cot x + \frac{2}{x-1} - \frac{4}{2x-1}\right)$$

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Find derivative of $y = \log_a x$

$a^{\log_a x} = x$

$\frac{d}{dx}(a^x) = a^x \ln a$

$a^y = \cancel{a}^{\log_a x} = x$

$a^y \cdot \ln a \cdot y' = 1$

$y' = \frac{1}{\ln a} \frac{1}{a^y} = \frac{1}{\ln a} \frac{1}{x}$

find derivative of $y = a^x$

$\ln y = x \boxed{\ln a}$

$\frac{y'}{y} = \ln a \to y' = \ln a \cdot y = a^x \ln a$

$\frac{d}{dx}(a^y) = \frac{d}{dy}(a^y) \cdot \boxed{\frac{dy}{dx}}$

$\frac{d}{dy}(a^y) \cdot y'$

$= a^y \ln a \cdot y'$

$\frac{d(\,)}{dx} = \frac{d(\,)}{\cancel{dy}} \cdot \frac{\cancel{dy}}{dx}$

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