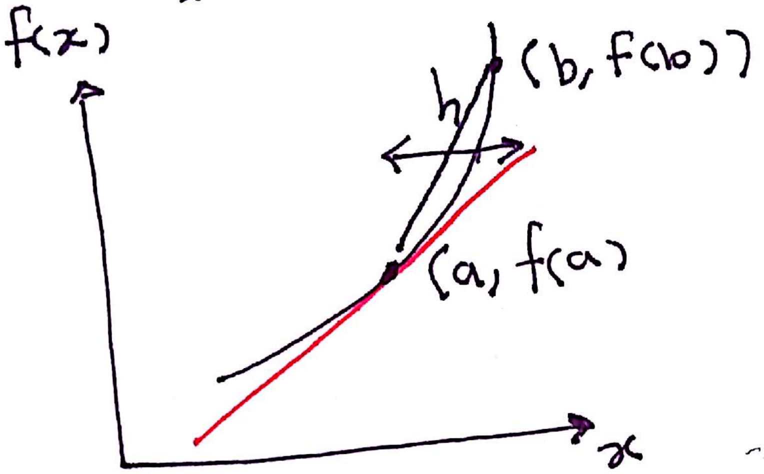
Compounded Interest $\rightarrow \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^{n} = e^{x}$



Secant line = line that connects two points on the curve

slope of secant line $= \dfrac{\text{rise}}{\text{run}} = \dfrac{f(b)-f(a)}{b-a}$

$f(x) = x^2$, slope of secant line $= \dfrac{(1+h)^2 - 1^2}{1+h-1}$

$a = 1$

$b = a+h = 1+h$

$$= \frac{1^2+h^2+2h-1}{h}$$

$$= \frac{h^2+2h}{h} = \frac{\cancel{h}(h+2)}{\cancel{h}}$$

$$= h+2 \rightarrow 2$$

1	0.1	0.01	0.001	0.0001	-0.0001	-0.001	.01
2	2.1	2.01	2.001	2.0001	1.9999	1.999	1.99

2

Slope of tangent line

$$\lim_{b\to a}\frac{f(b)-f(a)}{b-a} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Scanned by CamScanner

RATE of CHANGE $f(x)$

AVERAGE RATE of CHANGE between $x = a, b$ $= \dfrac{f(b) - f(a)}{b - a}$

Instantaneous RATE of change

$$\lim_{b \to a} \frac{f(b) - f(a)}{b - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

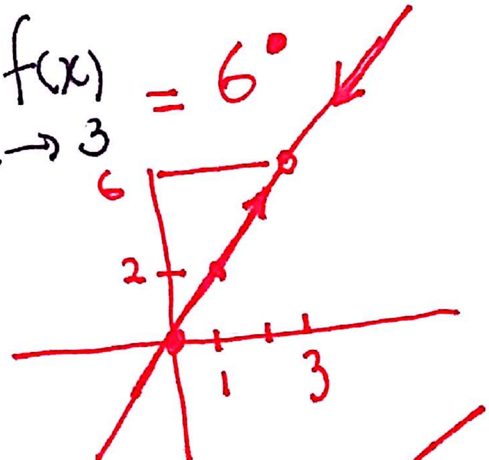
AVE. RATE change $\longleftrightarrow$ slope of secant line $\longleftrightarrow$ AVE. velocity

Inst. RATE change $\longleftrightarrow$ Slope of tangent line $\longleftrightarrow$ Inst. velocity

Evaluating limits.

$$f(x) = \begin{cases} 2x & x \neq 3 \\ 9 & x = 3 \end{cases}$$

$$\lim_{x \to 3} f(x) = 6$$

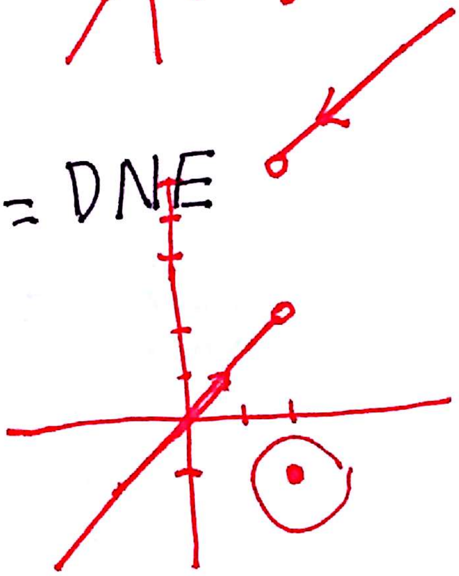


x	2.9	2.99	2.999	3.001	3.01	3.10
$f(x)$	5.8	5.98	5.998	6.002	6.02	6.2

6

$$f(x) = \begin{cases} x & x < 2 \\ -1 & x = 2 \\ x+3 & x > 2 \end{cases}$$

$$\lim_{x \to 2} f(x) = DNE$$



x	1.9	1.99	1.999	2.001	2.01	2.10
$f(x)$	1.9	1.99	1.999	5.001	5.01	5.1

2 5

Scanned by CamScanner

GOOD function

$$\lim_{x \to a} f(x) = f(a)$$

* Polynomial (x^2, x^5, x^{100})
* e^x
* $\sin x$, $\cos x$

Limit laws $\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

* $\lim_{x \to a} f(x)+g(x) = L+M$
* $\lim_{x \to a} f(x)-g(x) = L-M$
* $\lim_{x \to a} c\,f(x) = cL$ $c =$ is a constant
* $\lim_{x \to a} f(x).g(x) = L.M$
* $\lim_{x \to a} (f(x))^n = L^n$ → n is an integer
* $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}$ → except when $M=0$
* $\lim_{x \to a} (f(x))^{\frac{1}{n}} = L^{\frac{1}{n}} = \sqrt[n]{L}$ → n is an integer
  → if n is even, $f(x)$ must be positive around $x=a$

Scanned by CamScanner

$$\lim_{x\to 3} x^2+2x+1 = \lim_{x\to 3} x^2 + \lim_{x\to 3} 2x + \lim_{x\to 3} 1$$

Subst. 16

$$= 9 + 2(3) + 1 = 16$$

$$\lim_{x\to 2} \sqrt[3]{4x^2+3x+5} = \lim_{x\to 2} (4x^2+3x+5)^{1/3}$$

$$x^{1/n} = \sqrt[n]{x}, \quad n \in \mathbb{N}$$

$$= \sqrt[3]{\lim_{x\to 2} (4x^2+3x+5)} =$$

$$= \sqrt[3]{\lim_{x\to 2} 4x^2 + \lim_{x\to 2} 3x + \lim_{x\to 2} 5} =$$

$$= \sqrt[3]{4\lim_{x\to 2} x^2 + 3\lim_{x\to 2} x + \lim_{x\to 2} 5} =$$

$$= \sqrt[3]{4(4) + 3(2) + 5} = \sqrt[3]{16+6+5} = \sqrt[3]{27} = 3$$

$$\sqrt[3]{4(2^2)+3(2)+5} = \sqrt[3]{16+6+5} = \sqrt[3]{27} = 3$$

To find ~~the~~ a limit,
Most of time, you can evaluate the function.
BuT be careful with fractions. and even roots

$$\lim_{x \to 2} e^x + x^2 - \sin x = e^2 + 2^2 - \sin 2 = e^2 + 4 - \sin 2$$

$$\lim_{x \to 2} \frac{x}{x-1} = \frac{2}{1} = 2 \checkmark$$

$$\lim_{x \to 1} \frac{x}{x-1} = \cancel{\frac{\ }{0}}$$

Substitution DOES NOT work.

$$\lim_{x \to 3} \sqrt{x^2+9} = \sqrt{18} \checkmark$$

$$\lim_{x \to 3} \sqrt{x^2-16} =$$

Substitution does not work.

Finding domain

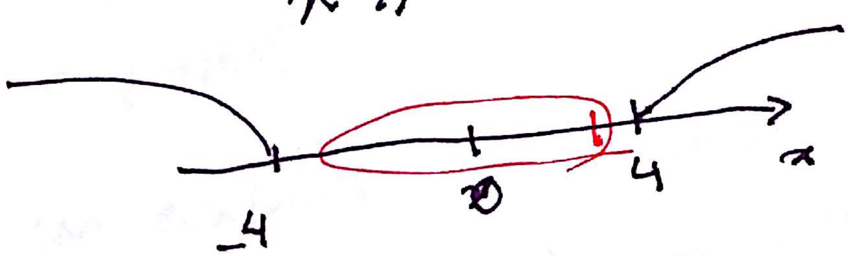
$$f(x) = \sqrt{x^2-16}$$

$$x^2 - 16 \geq 0$$

$$x^2 \geq 16 \rightarrow$$

$$|x| \geq 4 \rightarrow$$

$$x \geq 4 \text{ or } x \leq -4$$



I live put a note on how to determine domain of a function on the course website

Scanned by CamScanner

$$\lim_{x \to 1} \frac{5x^2+6x+1}{8x-4} = \frac{5+6+1}{8-4} = \frac{12}{4} = 3$$

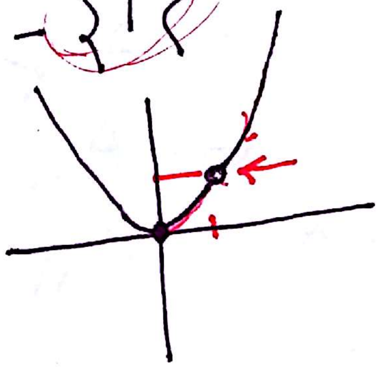
$$\lim_{b \to 2} \frac{3b}{\sqrt{4b+1}-1} = \frac{3(2)}{3-1} = \frac{6}{2} = 3$$

$$\lim_{x \to 1} \frac{x^3-x^2}{x-1} = \frac{0}{0}$$ & $\frac{\text{zero}}{\text{zero}}$ case.

$\frac{\text{zero}}{\text{zero}} \longrightarrow$ Simplify function.

$$\lim_{x \to 1} \frac{x^3-x^2}{x-1} = \lim_{x \to 1} \frac{x^2\cancel{(x-1)}}{\cancel{(x-1)}} = \lim_{x \to 1} x^2 = 1$$

$$f(x) = \frac{x^3-x^2}{x-1} = x^2 \quad D = \mathbb{R} - \{1\}$$



$$= \frac{x^2\cancel{(x-1)}}{\cancel{(x-1)}} \underset{\substack{\uparrow \\ x \neq 1}}{=} x^2$$

$$f(x) = \begin{cases} x^2 & x \neq 1 \\ \text{undefined} & x = 1 \end{cases} \neq x^2$$

$$\lim_{x \to 1} \frac{x^2+x-2}{x^2+4x-5} = \frac{0}{0}$$

$$(x^2+x-2) = (x+2)(x-1)$$

$\downarrow$

add 1

multiply -2

$$(x^2+4x-5) = (x+5)(x-1)$$

$$\lim_{x \to 1} \frac{x^2+x-2}{x^2+4x-5} = \lim_{x \to 1} \frac{(x+2)\cancel{(x-1)}}{(x+5)\cancel{(x-1)}} = \lim_{x \to 1} \frac{x+2}{x+5} = \frac{1}{2}$$

Scanned by CamScanner