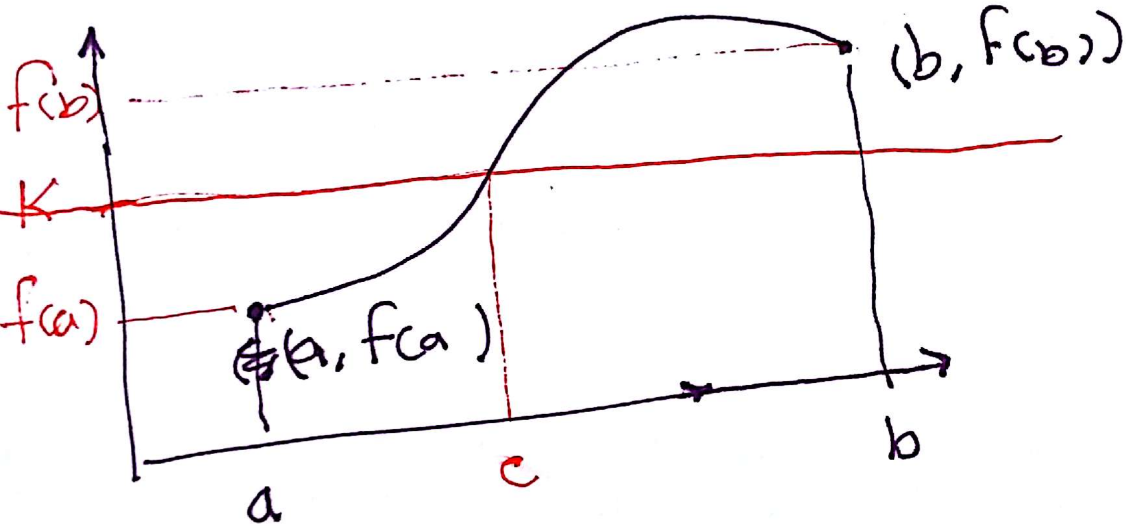
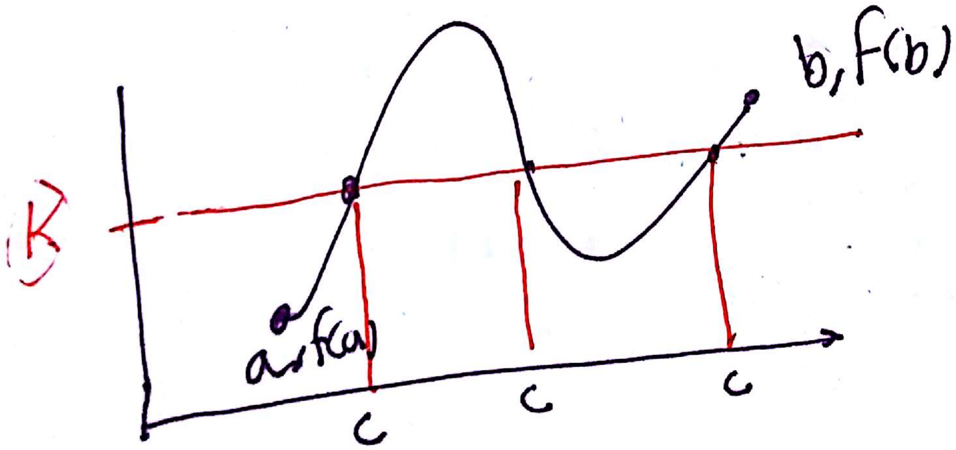
Intermediate value theorem.

Suppose f(x) is Continuous on [a,b] then for any ~~f(a)~~ K between f(a) and f(b), there exists <u>at least</u> on number $a < c < b$ for which $f(c) = K$





there might be more than ONE c

$f(x) = x^3 + x^2 - 1$

Show that $f(x)$ has <u>at least</u> ONE root between $0, 2$

$f(x)=0$

✓ Polynomial $\Rightarrow$ continuous on $[0, 2]$

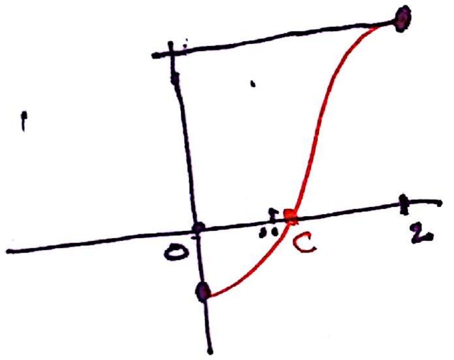
$f(0) = -1$ $\qquad -1 < 0 < 11$

$f(2) = 11$ $\qquad f(a) < 0 < f(b)$

$\Downarrow$ IVT

$0 < c < 2$

$f(c) = 0$



$f(1) = 1 + 1 - 1 = 1$ $\qquad 0 < c < 1$

$-1 < 0 < 1$

$\Downarrow$

$f(a) < 0 < f\left(\frac{a+b}{2}\right)$

$f\left(\frac{1}{2}\right) = \frac{1}{8} + \frac{1}{4} - 1 = \frac{1+2-8}{8} = -\frac{5}{8}$

$\Downarrow$

$\frac{1}{2} < c < 1$

$x + \sin x = 1$

$\boxed{\varepsilon = 10^{-3}}$

$[a, b]$

$b - a = L$

$n=0 \quad L$

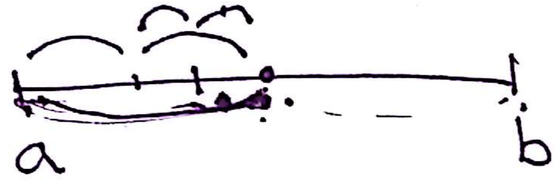
$n=1 \quad L/2$

$n=2 \quad L/4$

$n=3 \quad L/8$

$\vdots$

$n=n \quad L/2^n$



$$L/2^n = \varepsilon \longrightarrow \frac{L}{\varepsilon} = 2^n$$

$$\ln \frac{L}{\varepsilon} = n \ln 2$$

$$\boxed{n = \frac{\ln \frac{L}{\varepsilon}}{\ln 2}}$$

$$\left.\begin{array}{l} L = 2 \\ \varepsilon = 10^{-3} \end{array}\right\} \rightsquigarrow n = \frac{\ln \frac{2}{0.001}}{\ln 2} = 10.96$$

$n = 11$

bisection method.

root finding method.

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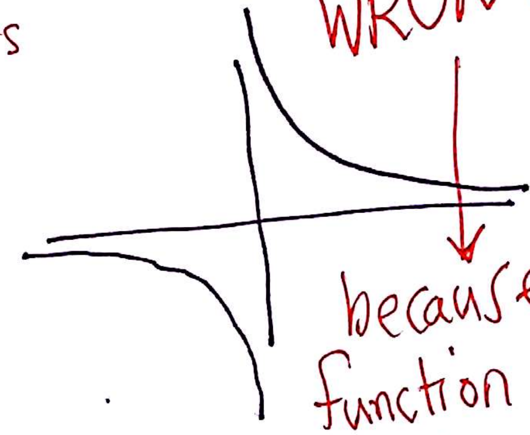
$$f(x) = \frac{1}{x}$$

$f(-1) = -1$ $\qquad$ $f(-1) < 0 < f(1)$

$f(1) = 1$ $\qquad\qquad\qquad \Downarrow$

$\exists c, \ -1 < c < 1, \ f(c) = 0$ $\Leftarrow$ WRONG

there exists



because the function is not Continuous

$x = 0$, undefined.

$\frac{1}{x} = 0$ never has a solution

$$y = \sin\left(\frac{1}{x}\right)$$

does it have limit at $x = 0$

is it Continuous ?

$D = \mathbb{R} - \{0\}$

$\sin\frac{1}{x} = 0 \Rightarrow \sin\frac{1}{x} = \sin(K\pi) \quad K = \pm 1, \pm 2, \dots$

$\frac{1}{x} = K\pi \rightarrow x = \frac{1}{K\pi}, \quad x = \pm\frac{1}{\pi}, \pm\frac{1}{2\pi}, \dots$

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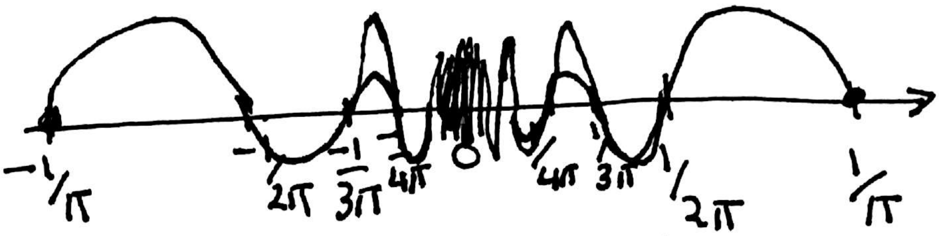
unremovable

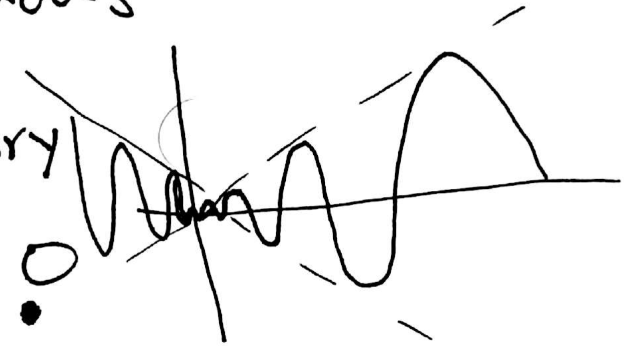


the limit does Not exist

$f(x)\ \cancel{g} = x \cdot \sin\frac{1}{x}$ ←

Not continuous removable

squeeze theory



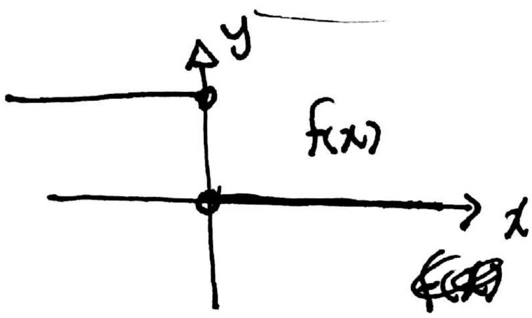
$$\lim_{x\to 0} f(x) = 0$$

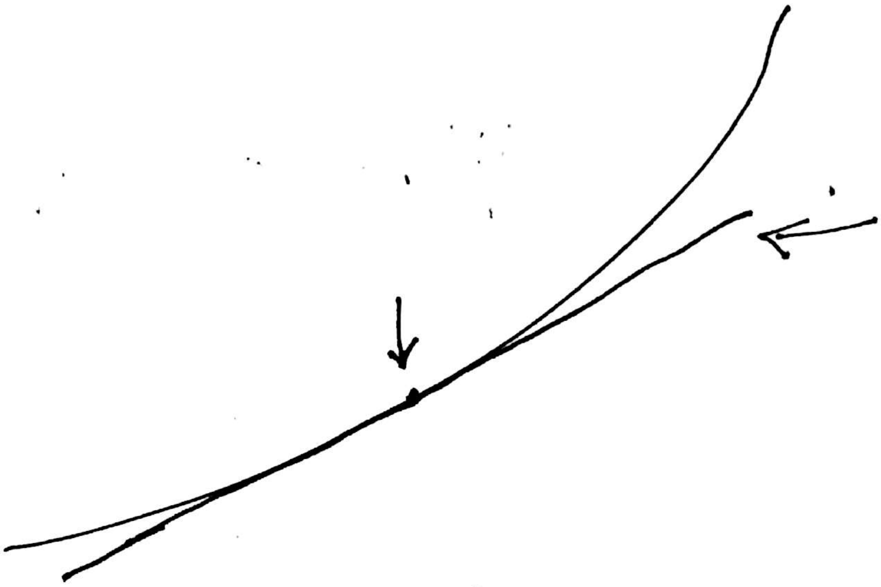
$$f(x) = \begin{cases} x\sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$f(x) = \frac{x - |x|}{x} \qquad |x| \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

$$\lim_{x\to 0^+} f(x) = \frac{x - x}{x} = \frac{0}{x} = 0$$

$$\lim_{x\to 0^-} f(x) = \frac{x - (-x)}{x} = \frac{2x}{x} = 2$$





Rate of change. $f(x)$ $[a,b]$

average rate of change

$$\frac{f(b)-f(a)}{b-a}$$

Instantaneous Rate of change

$$\downarrow = \lim_{b\to a} \frac{f(b)-f(a)}{b-a} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$