$(\ln t)' = \frac{1}{t}$ Sep 29, 2016

$$y = \frac{1+\ln t}{t}$$

$$y' = \frac{(1+\ln t)'\,t - (1+\ln t)(t)'}{t^2}$$

$$= \frac{\frac{1}{t}\cdot t - (1+\ln t)1}{t^2} =$$

$$\frac{1-1-\ln t}{t^2} = -\frac{\ln t}{t^2}$$

$$y = \frac{1-x}{e^x}$$

$$y' = \frac{(1-x)'\,e^x - (1-x)(e^x)'}{(e^x)^2} =$$

$$\frac{-e^x - (1-x)e^x}{e^{2x}} = \frac{-e^x - e^x + xe^x}{e^{2x}}$$

$$= \frac{e^x(x-2)}{e^{2x}} = \frac{x-2}{e^x}$$

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find value of a and b such that function $f(x)$ is differentiable everywhere

$$f(x) = \begin{cases} ax+b & x \geq 0 \\ 2\sin x + 3\cos x & x < 0 \end{cases}$$

* to make it differentiable, we need to first make it continuity

each branch is a nice function and therfore continuous. the only point that needs further investigation is $\boxed{x=0}$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

$$\lim_{x\to 0^-} 2\sin x + 3\cos x = 2\sin(0) + 3\cos(0) = 3$$

$$\lim_{x\to 0^+} ax+b = b$$

$f(0) = b$ $\quad \boxed{b=3} \Rightarrow$ to be continuous.

$f'(0) = \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h}$

from right

$$f' = \begin{cases} a & x \geq 0 \\ 2\cos x - 3\sin x & x < 0 \end{cases}$$

$\lim_{x \to 0^+} f' = \lim_{x \to 0^-} f' \to a = 2\cos(0) - 3\sin(0)$

$\boxed{a = 2}$

700 terms

$\lim_{x \to 1} \frac{x^{700} - 1}{x - 1} = (x-1)(. \quad\quad)$

$x \Rightarrow h = x - x_0$

$f'(x_0) = \lim_{h \to 0} \frac{f(\underbrace{x_0 + h}_{x}) - f(x_0)}{h} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$

$f(x) = x^{700} \longrightarrow f'(x) = 700x^{699}$

$x_0 = 1 \quad f'(1) = 700$

$f'(1) = \lim_{x \to 1} \frac{x^{700} - 1^{700}}{x - 1} = 700$

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$$\lim_{x\to\frac{\pi}{6}} \frac{\sin x - \frac{1}{2}}{x - \frac{\pi}{6}} = \frac{0}{0}$$

$$f'(x_0) = \lim_{x\to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$= \lim_{x\to\frac{\pi}{6}} \frac{\sin x - \sin(\frac{\pi}{6})}{x - \frac{\pi}{6}} =$$

$$f(x) = \sin x$$

$$x_0 = \frac{\pi}{6}$$

$$= \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$$

$$f'(x) = \cos x$$

$$\lim_{x\to\frac{\pi}{6}} \frac{2\sin x - 1}{x - \frac{\pi}{6}} = 2 \lim_{x\to\frac{\pi}{6}} \frac{\sin x - \frac{1}{2}}{x - \frac{\pi}{6}}$$

$$= 2\cos(\frac{\pi}{6}) = \sqrt{3}$$

$f'(x)$ is itself a function and ~~&~~ it can have derivative it self

$$f'' = (f')' = \lim_{h\to 0} \frac{f'(x+h) - f'(x)}{h}$$

$$f(x) = x\sqrt{x} = x^{\frac{3}{2}}$$

$$f'(x) = \frac{3}{2} x^{\frac{1}{2}}$$

$$f''(x) = \frac{3}{2}(\frac{1}{2}) x^{\frac{1}{2}-1} = \frac{3}{4} x^{-\frac{1}{2}}$$

$$f''' = \frac{-3}{8} x^{-\frac{3}{2}}$$

$f(x) = xe^x$, find $f^{(25)}(x)$

$f'(x) = (x)'e^x + x(e^x)' = e^x + xe^x = \underline{e^x(x+1)}$

$f''(x) = (e^x)'(x+1) + e^x(x+1)' = e^x(x+1) + e^x = e^x(x+2)$

$$f^{(25)} = e^x(x+25)$$

$f(x) = e^x \sin x$, $f^{(100)}$

take derivative 4 times and then try to realise what pattern is repeating.

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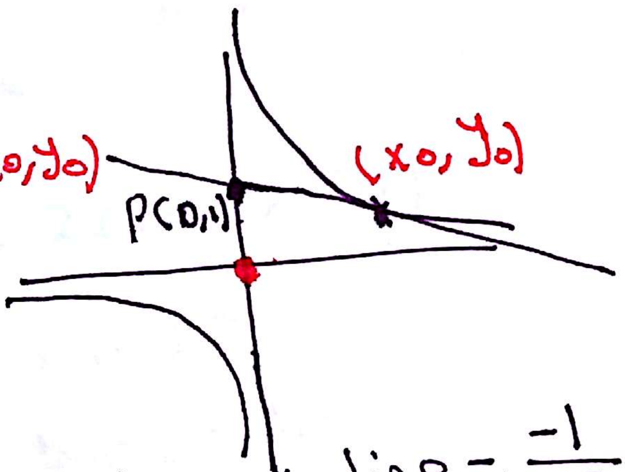
find tangent line to the curve $y = \frac{1}{x}$

Passing through $P(0,1)$

first. assume tangent point is (x_0, y_0)

$y_0 = \frac{1}{x_0} \Leftarrow$ tangent point is on the curve.



$f'(x) = \frac{-1}{x^2} \Rightarrow$ slope of tangent line $= \frac{-1}{x_0^2}$

$y - y_0 = \frac{-1}{x_0^2}(x - x_0) \leftarrow$ equation of tangent line

$y - \frac{1}{x_0} = \frac{-1}{x_0^2}x + \frac{1}{x_0} \Rightarrow \boxed{y = \frac{-1}{x_0^2}x + \frac{2}{x_0}}$

Plug coordinate of Point P $\Rightarrow 1 = \frac{-1}{x_0^2}(0) + \frac{2}{x_0} \rightarrow \boxed{x_0 = 2}$

$$y = \frac{-1}{4}x + 1$$

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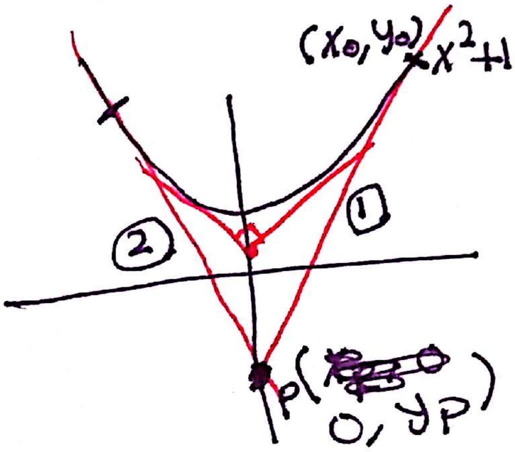
$$f(x) = x^2 + 1$$

find a point on y axis from which we can draw two perpendicular tangent line to the curve.



(x_0, y_0) is the tangent point
it is on the curve $\Rightarrow$

$$y_0 = \underbrace{x_0^2 + 1}$$

$f'(x) = 2x \Rightarrow$ slope of tangent line $= 2x_0$

$y - y_0 = m(x - x_0)$ $\leftarrow$ equation of tangent line.

$$y - (x_0^2 + 1) = 2x_0(x - x_0)$$

$$y = 2x_0 x - 2x_0^2 + x_0^2 + 1 = 2x_0 x - x_0^2 + 1$$

$$y_p = 2x_0 (0) - x_0^2 + 1 \rightarrow y_p = 1 - x_0^2$$

demand that point P is on the tangent line.

$$x_0^2 = 1 - y_p \rightarrow x_0 = \pm\sqrt{1 - y_p}$$

$\rightarrow m_1 = 2\sqrt{1-y_p}$
$m_2 = -2\sqrt{1-y_p}$

$$m_1 = \frac{-1}{m_2} \Rightarrow m_1 \cdot m_2 = -1$$

$$2(\sqrt{1-y_p})(2)(-\sqrt{1-y_p}) = -1$$

$$-4(1 - y_p) = -1 \Rightarrow 1 - y_p = \frac{1}{4} \rightarrow \boxed{y_p = \frac{3}{4}}$$