$f'(x) = 0 \rightarrow 2e^x x - e^x = 0$

$e^x(2x-1) = 0 \rightarrow$

$e^x = 0 \rightarrow$ D.S.

$2x - 1 = 0 \rightarrow x = \frac{1}{2}$

f' ~~Not~~ Not defined

$\hookrightarrow x\sqrt{x} = 0 \rightarrow \boxed{\cancel{x = 0}}$

Not in domain

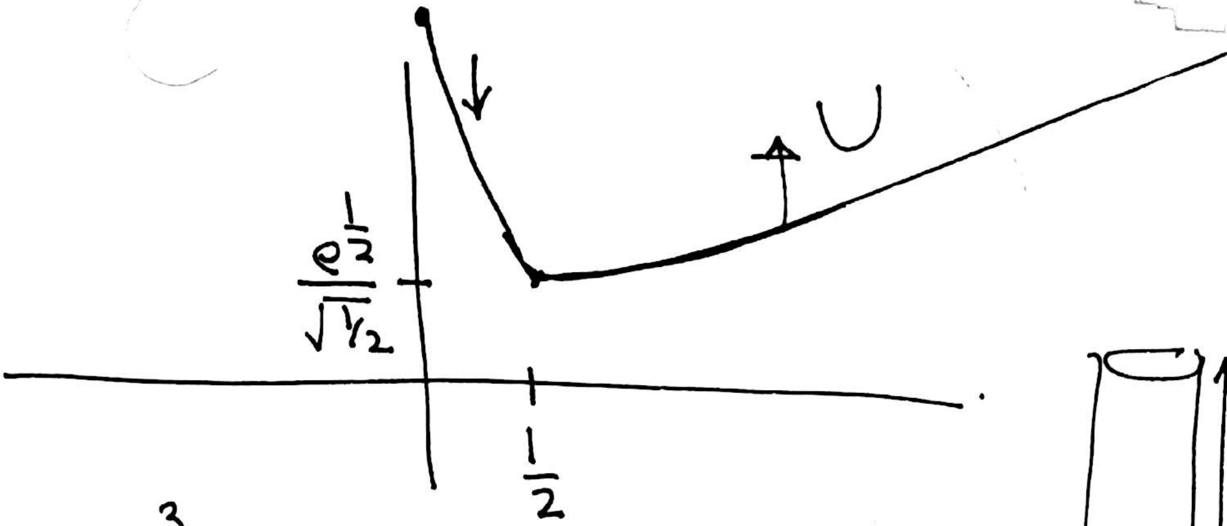
	0	$1/2$	∞
f'	$-$		$+$
	$\searrow$		$\nearrow$

$|_{x=\frac{1}{2}}$ local ~~max~~ min.

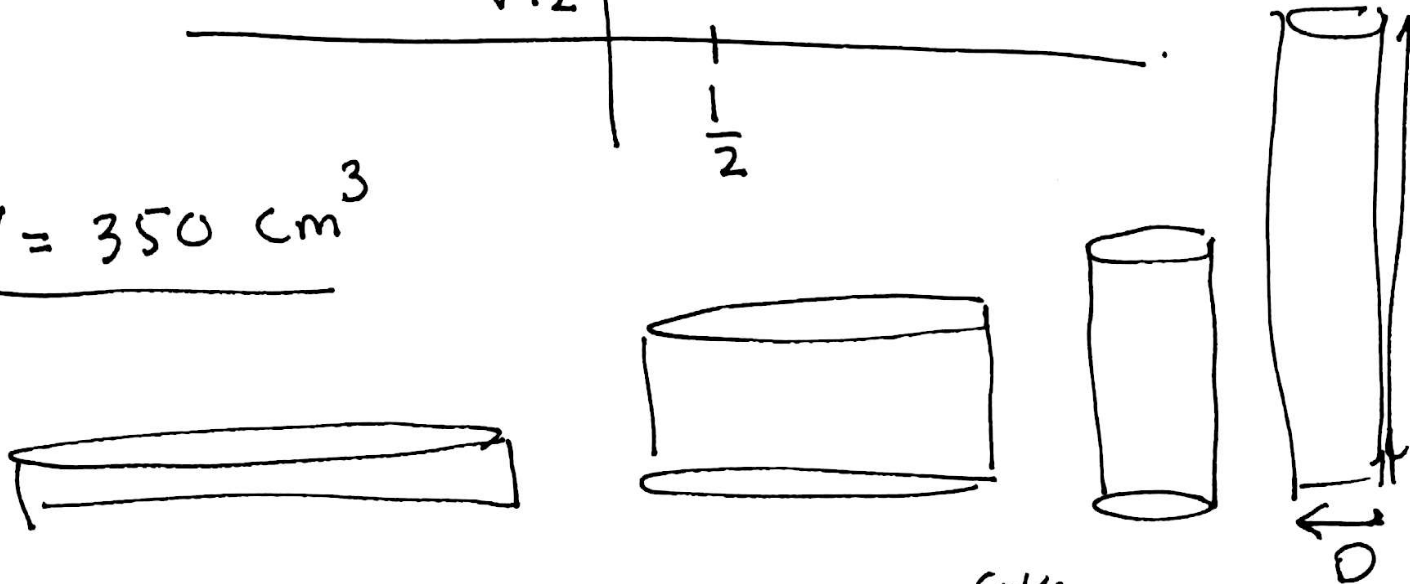
$$f' = \frac{e^x(2x-1)}{2x\sqrt{x}} \rightarrow 2x^{+\frac{3}{2}}$$

$$f'' = \frac{\left(e^x(2x-1) + e^x 2\right)\left(2x\sqrt{x}\right) - e^x(2x-1)\,3x^{1/2}}{4x^3}$$

$$= \frac{e^x\sqrt{x}\left((2x+1)2x - 3(2x-1)\right)}{4x^3}$$



$V = 350\ cm^3$



Coke

given $V = V_0$ ($350\ cm^3$)

design a cylinderical can that has ~~a~~ minim[...] surface area

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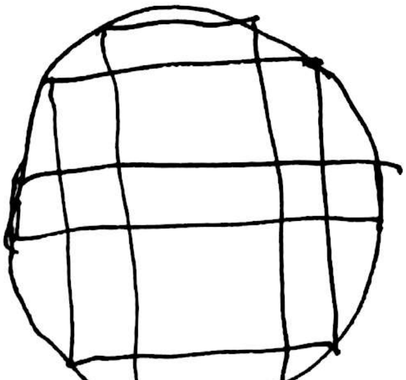
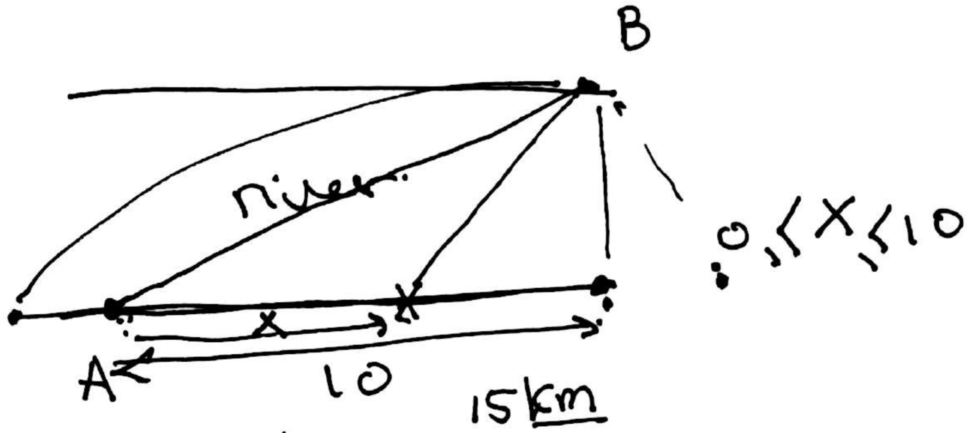


circular cake.

you're allowed to cut

a rectangular piece.

of this cake.



Speed when running $15 \frac{km}{hr}$

" " swimming $3 \frac{km}{hr}$.

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optimisation :

1) understand the problem:

2) Draw a diagram

3) Introduce a notation.

4) reduce to one variable.

5) Solve the problem

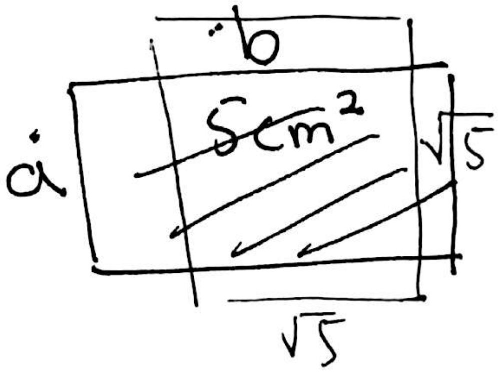
   5-1) finding domain

   5-2) " critical point

   5-3 Classify " " ← second derivative test

6) → check your solution with physical intuition

1) Among all rectangles with Area $= 5\ cm^2$ find the one with smallest perimeter.



$a = 10$
$b = \frac{1}{2}$

$A = a \cdot b$

$P = 2a + 2b$

minimise ~~maximize~~ p when $A = 5\ cm^2$

$A = 5 \rightarrow a \cdot b = 5 \rightarrow b = \frac{5}{a}$

$b = \frac{5}{\sqrt{5}} = \sqrt{5}$

$P = 2a + 2\left(\frac{5}{a}\right) = 2a + \frac{10}{a}$

$a = b$

minimise $P(a)$. $\qquad 0 < a$ [redacted]

* find critical points
* compare end points.

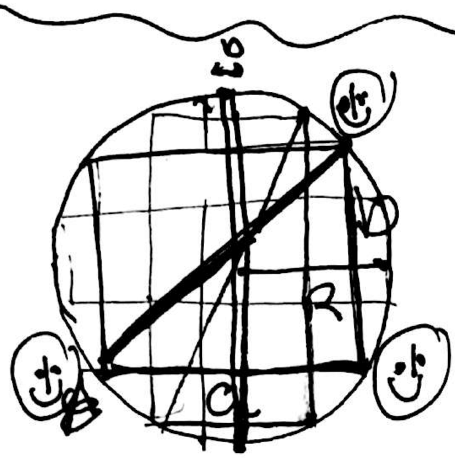
$\frac{dP}{da} = 0 \rightarrow 2 - \frac{10}{a^2} = 0 \rightarrow 2a^2 = 10 \rightarrow a = \pm\sqrt{5}$

$a = \sqrt{5}$

$\frac{d^2P}{da^2} = \frac{+20}{a^3} \qquad \frac{d^2P}{da^2}(\sqrt{5}) > 0 \Rightarrow$ local min

$\cancel{P(0)}$, $P(\sqrt{5})$ $\cancel{(P(\infty))}$

$$\min(P) = P(\sqrt{5}) = 4\sqrt{5}$$



circle. find the largest rectangle that can be inscribed in a circle of ~~[illegible]~~ radius R.

$A = a \cdot b$ & maximise area.

in $(2R)^2 = a^2 + b^2$

$$b^2 = 4R^2 - a^2 \rightarrow b = \sqrt{4R^2 - a^2}$$

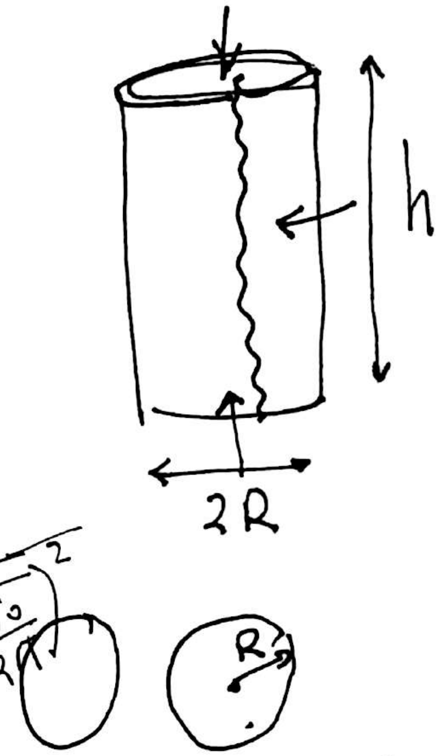
$$A(a) = a \cdot \sqrt{4R^2 - a^2} \qquad 0 < a < 2R$$

$$\frac{dA}{da} = \sqrt{4R^2 - a^2} + a\frac{-\cancel{2}a}{\cancel{2}\sqrt{4R^2 - a^2}} = \sqrt{4R^2 - a^2} - \frac{a^2}{\sqrt{4R^2 - a^2}}$$

$$= \frac{4R^2 - a^2 - a^2}{\sqrt{4R^2 - a^2}} = \frac{4R^2 - 2a^2}{\sqrt{4R^2 - a^2}} \qquad b = \sqrt{2}R$$

$$\frac{dA}{da} = 0 \rightarrow 4R^2 - 2a^2 = 0 \Rightarrow 2a^2 = 4R^2 \rightarrow a = \sqrt{2}R$$

design a cylinderical beverage can with volume V_0 and smallest surface area.



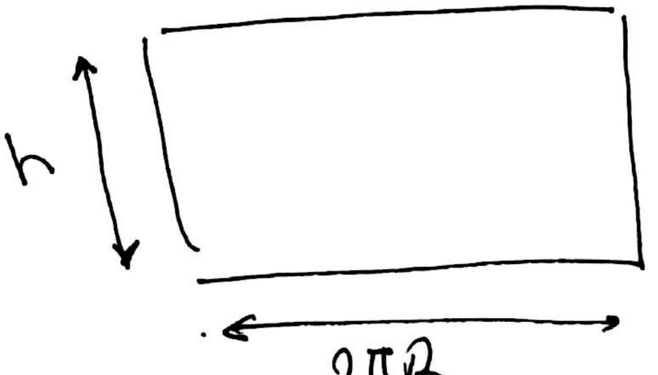
$V = \pi R^2 \cdot h = V_0$

$A = \underbrace{2\pi R^2 + 2\pi R \cdot h}$

minimise A, when $V = V_0$

$V_0 = \pi R^2 h \rightarrow h = \frac{V_0}{\pi R^2} \rightarrow h = \frac{V_0}{\pi\left(\sqrt[3]{\frac{V_0}{2\pi}}\right)^2}$

unroll cylinder



$A = 2\pi R^2 + 2\pi R \frac{V_0}{\pi R^2}$

$A = 2\pi R^2 + \frac{2V_0}{R}$

$\frac{dA}{dR} = 0 \rightarrow 4\pi R - \frac{2V_0}{R^2} = 0$

$4\pi R = \frac{2V_0}{R^2} \rightarrow 4\pi R^3 = 2V_0 \rightarrow R = \sqrt[3]{\frac{V_0}{2\pi}}$

$\frac{d^2A}{dR^2} = 4\pi + \frac{4V_0}{R^3} = 4\pi + \frac{4V_0}{\frac{V_0}{2\pi}} = 4\pi + 8\pi = 12\pi > 0$

$\Downarrow$

minimiser.

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