let $f(x) = x^5 + x^3$. Find equation of the tangent line $f^{-1}(x)$ at $x=2$

$$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$$

$$\boxed{y = f'(a)(x-a) + f(a)}$$

→ tangent line equation for any general function $f(x)$

$g(x) = f^{-1}(x)$

find tangent line to g at $x=2$

$g(2)$, $g'(2)$

$$\boxed{\begin{aligned} f(x) &= y \\ f^{-1}(y) &= x \end{aligned}}$$

$g(2) = f^{-1}(2) = 1$

$x^5 + x^3 = 2 \rightarrow x = 1$

$$\boxed{f^{-1}(f(x)) = x}$$

$f(1) = 2 \rightarrow \cancel{f^{-1}}(\cancel{f}(1)) = f^{-1}(2)$

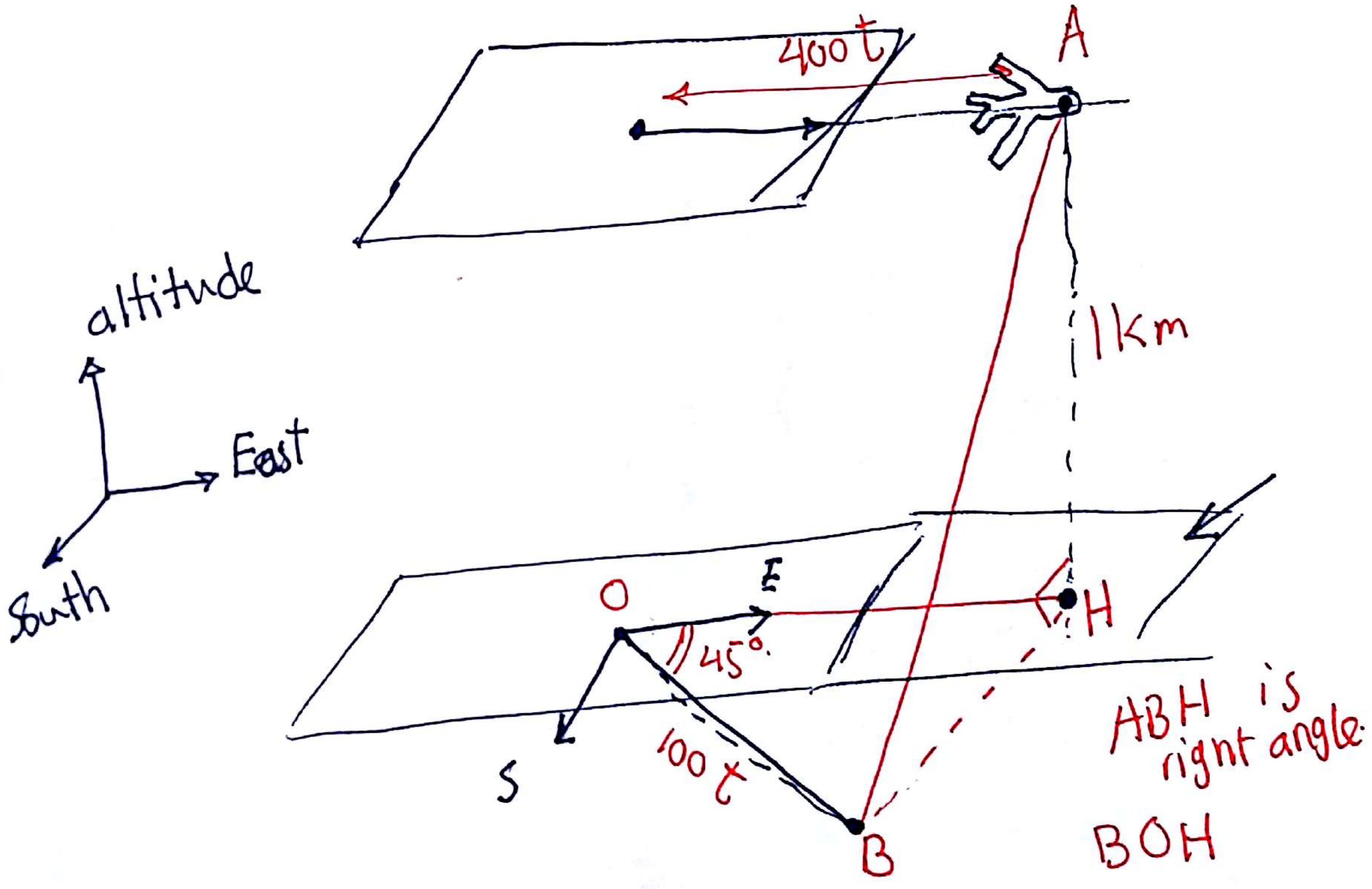
$\Rightarrow 1 = f^{-1}(2)$

$$\frac{d}{dx} f^{-1}(x) = g'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(1)}$$

$$f'(x) = 5x^4 + 3x^2 \rightarrow f'(1) = 8$$

$$g'(2) = \frac{1}{f'(1)} = \frac{1}{8}$$

$$y = \frac{1}{8}(x-2) + 1 = \frac{1}{8}x + \frac{3}{4}$$



$\underset{OBH}{\Delta}$: use Cosine law :

$$BH^2 = OB^2 + OH^2 - 2OB \cdot OH \cdot \cos\frac{\pi}{4}$$

$\underset{ABH}{\Delta}$: $AH^2 + BH^2 = AB^2$

$$AB^2 = AH^2 + OB^2 + OH^2 - 2OB \cdot OH \frac{\sqrt{2}}{2}$$

$$1 + (100t)^2 + (400t)^2 - \cancel{2}(100t)(400t)\frac{\sqrt{2}}{\cancel{2}}$$

$$AB^2 = 1 + 10000t^2 + 160000t^2 - 40000\sqrt{2}\,t^2$$

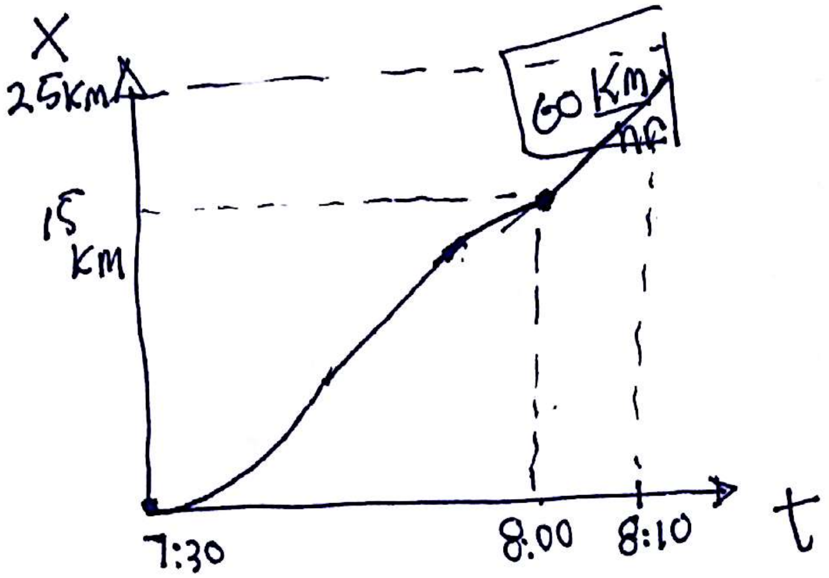
$$2AB \cdot \frac{dAB}{dt} = \cancel{2t(180000 - 40000\sqrt{2})}$$

$$2(170000 - 40000\sqrt{2})t$$

$$t = 36\,s = \frac{36}{3600}\,hr = 0.01\,hr$$

✓

Linear approximation.



$$X(8{:}00 + 0{:}10) = x(8{:}00) + v \cdot \Delta t$$

$$\phantom{X(8{:}00 + 0{:}10)} \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$25 = 15 \qquad 60 \cdot \frac{1}{6}$$

8:00   10 minutes

$$x(\underset{\uparrow}{t} + \underset{\uparrow}{\Delta t}) = \underline{x(t)} + \frac{dx}{dt} \cdot \Delta t$$

for a function $f(x)$, $\mathcal{L}(x)$ is linear approximation of $f(x)$ Near point $x = a$ if

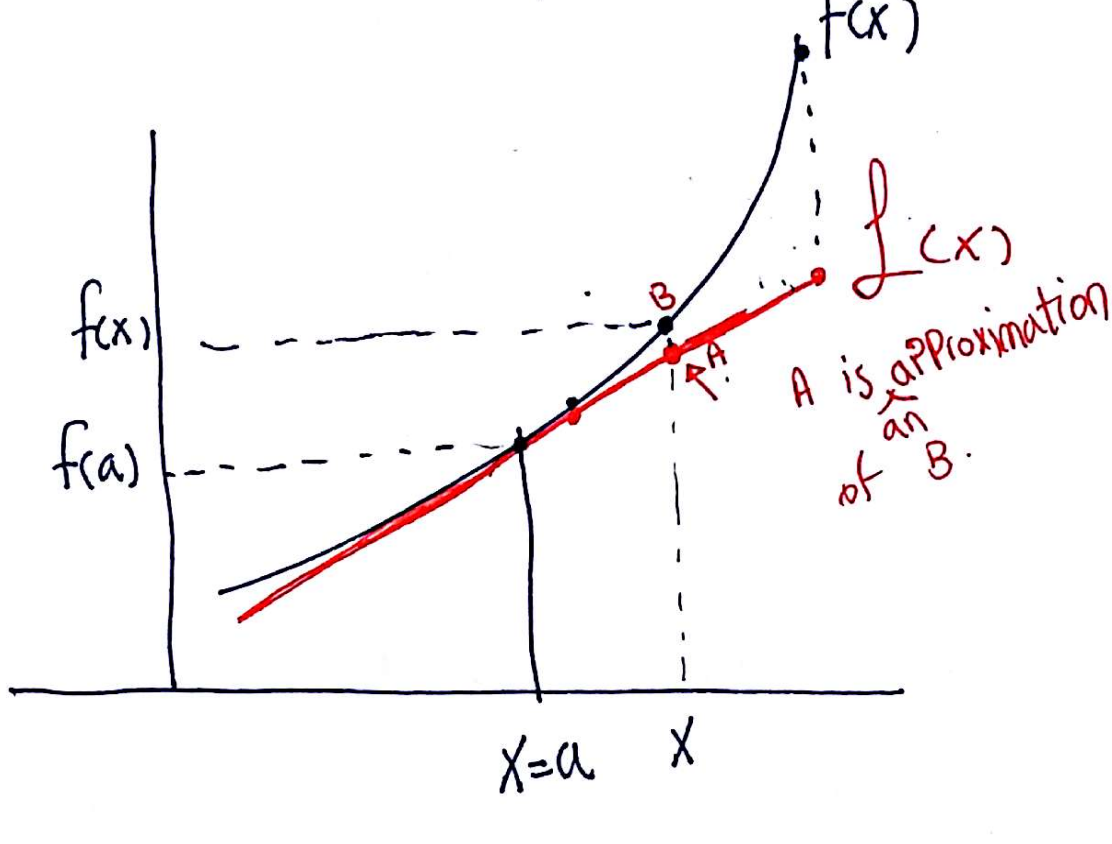
$$f(x) \approx \mathcal{L}(x) = f(a) + f'(a)(x-a)$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$x \approx a$

$$f'(a) \sim \frac{f(x) - f(a)}{x - a}$$

$$f(x) \simeq \underbrace{f(a) + f'(a)\,(x - a)}$$



linearize $f(x) = \sin(x)$ Near $\underbrace{x = 0}_{a}$

$a=0$

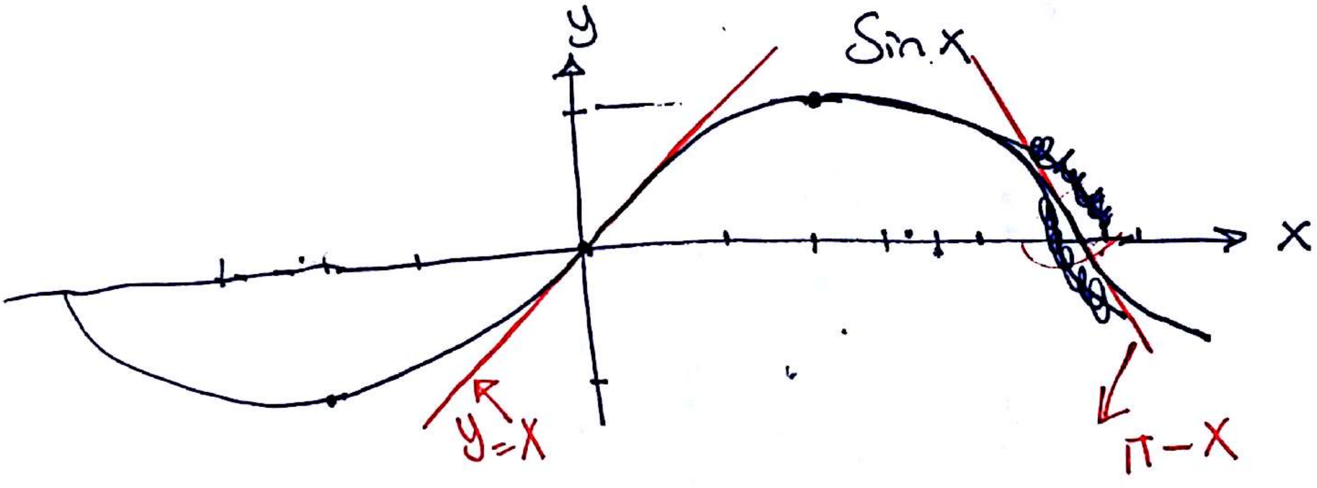
$f(a) = \sin(a) \Rightarrow f(0) = \sin(0) = 0$

$f'(a) = \cos(a) \Rightarrow f'(0) = \cos(0) = 1$

$L(x) = f(a) + f'(a)(x-a) \Rightarrow$

$L(x) = f(0) + f'(0)(x-0)$

$0 + 1(x) = x$



$\sin(0.05) \simeq 0.05$

Near $x=0$, $\sin(x) \simeq x$

linearize $f(x) = \sin(x)$ near $\underline{x = \pi}$

$a = \pi$

$f(a) = f(\pi) = \sin(\pi) = 0$

$f'(a) = \cos(\pi) = -1$

$$\mathcal{L}(x) = f(a) + f'(a)(x-a)$$

$$= 0 \quad -1(x-\pi)$$

$$=$$

$$= \pi - x$$

$\sin(0.99\pi) \simeq \pi - 0.99\pi = 0.01\pi$

Approximate $\sqrt[3]{9}$ using linear approximation.

$f(x) = \sqrt[3]{x}$

linearize function near $a = 8$

$f(a) = \sqrt[3]{8} = 2$

$f'(x) = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3x^{2/3}} = \frac{1}{3\sqrt[3]{x^2}}$

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$$f'(a) = f'(8) = \frac{1}{3}\frac{1}{\sqrt[3]{8^2}} = \frac{1}{3}\frac{1}{\sqrt[3]{64}}$$

$$= \frac{1}{3}\frac{1}{4} = \frac{1}{12}$$

$$\mathcal{L}(x) = f(a) + f'(a)(x-a)$$

$$= 2 + \frac{1}{12}(x-8)$$

$$f(9) \simeq \mathcal{L}(9) = 2 + \frac{1}{12}(9-8)$$

$$= \frac{25}{12}$$

Approximate $\sqrt{26}$ using linear approximation.

$f(x) = \sqrt{x}$

$\downarrow$

linearize at $a = 25$

$$\boxed{\begin{array}{c} f(x) = \sqrt{x^2+1} \\ \downarrow \\ \text{linearize at} \\ a = 5 \end{array}}$$

$f(a) = \sqrt{25} = 5$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(a) = \frac{1}{2\sqrt{25}} = \frac{1}{10}$

$$f(x) \simeq \mathcal{L}(x) = 5 + \frac{1}{10}(x-25)$$

$$= 5 + \frac{1}{10}x - 2.5$$

$$= 2.5 + \frac{1}{10}x$$

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x	$\sqrt{x}$	$\frac{x}{10}+2.5$
24	4.8998	4.9
24.9	4.989989	4.99
25		
25.1	5.00999	5.01
26	5.099	5.1

$f'(x) = \frac{+1}{2\sqrt{x}}$

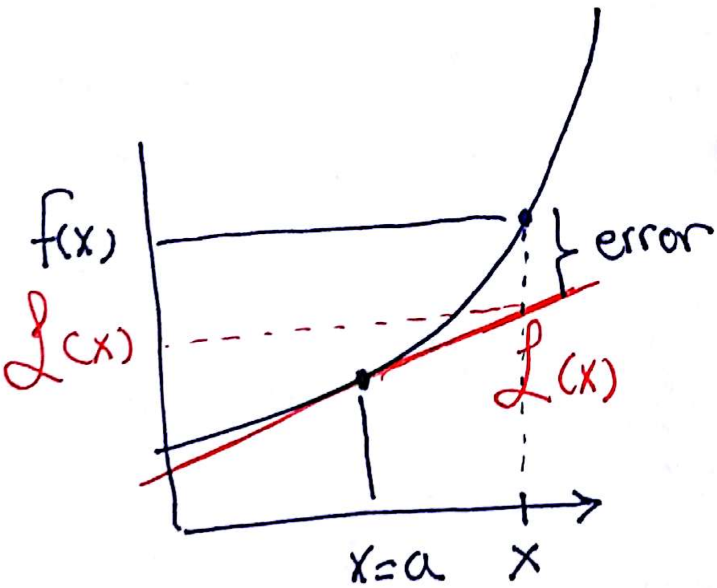
$\downarrow$

$f''(x) = \frac{-1}{4x\sqrt{x}}$

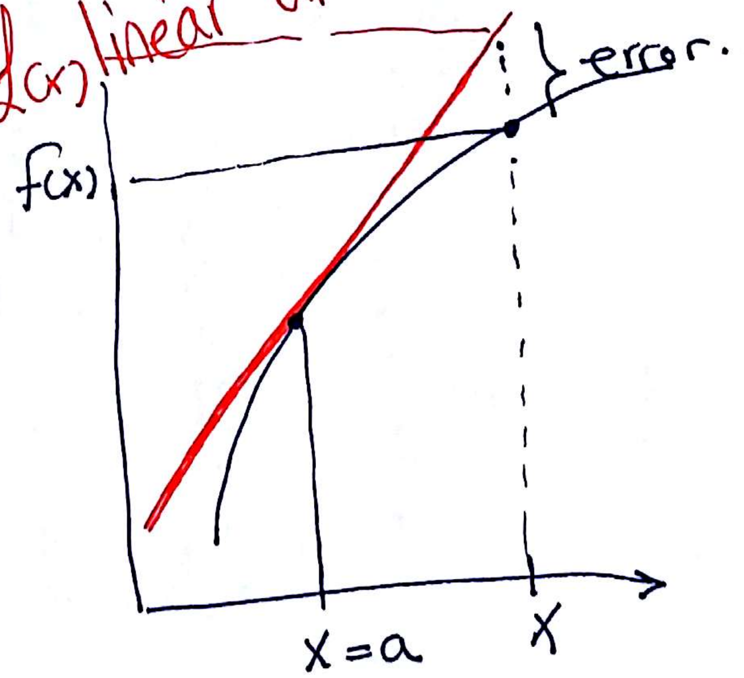
$f'' < 0 \rightarrow$ over estimate

exact value

$\mathcal{L}(x)$ linear approximation



under estimate



overestimate

$f'' > 0 \longrightarrow$ linear approximation is under estimate

$f'' < 0 \longrightarrow$ linear approximation is overestimate

$$\text{Error} = |f(x) - L(x)| < \frac{M}{2}|x-a|^2 .$$

M is absolute value of f'' between x and a

$f(x) = x^2$, linearize the function near

$x = 3$.

$f(3) = 9$

$f'(x) = 2x \rightarrow f'(3) = 6$

$\} \rightarrow L(x) = 6(x-3)+9$

$= 6x - 9$

$f'' = 2 \longrightarrow M = 2$

$\rightarrow$ Error of Approximation $< \frac{2}{2}|x-3|^2$

$f'' > 0 \rightarrow$ underestimate

$(3.1)^2 =$

$f(3.1) \cong L(3.1) = 6(3.1) - 9 = 18.6 - 9 = 9.6$

Error of this approximation $< |3.1-3|^2 = 0.01$

$(3.1)^2 \approx 9.6 + 0.01$