NOV 8, 2016

$$\lim_{x\to\infty} \frac{x^3 - 2x^5 - 1}{x^5 + 1} = \lim_{x\to\infty} \frac{-2x^5}{x^5} = -2$$

$$\lim_{x\to\infty} \frac{\ln x}{x} = \frac{\infty}{\infty} = 0$$

$x \to \infty$ $\ln x$ grows much slower than any Polynomial.

any Polynomial grows much slower than any exponential

$$\ln x < x^p < e^x$$

loosly speaking at infinity

$$\lim_{x\to\infty} \frac{x}{\ln x} = \infty$$

$$\lim_{x\to\infty} \frac{x^2+1}{e^x} = 0$$

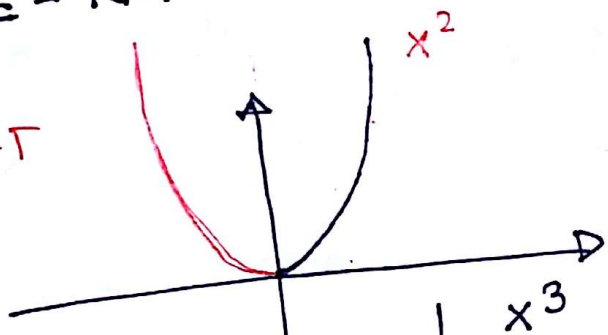
$$\lim_{x\to\infty} \frac{x^{10000}}{e^{0.0001x}} = 0$$

$$y = x\sqrt{\underbrace{4-x^2}_{\geq 0}} \qquad \underbrace{-2 \leq x \leq 2}$$

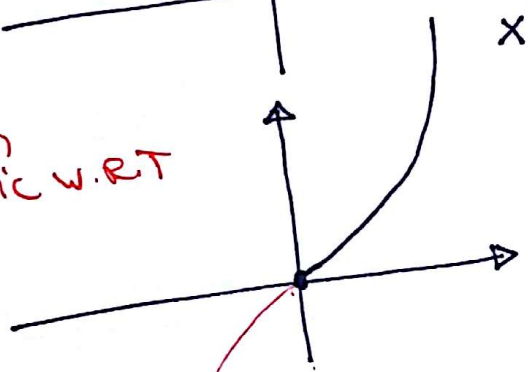
$f(-x) = f(x) \longrightarrow$ even function x^2.

$f(-x) = -f(x) \longrightarrow$ odd function x^3

even function is symetric W.R.T y-axis



odd function is symmetric w.R.T origin.



$\sin(x + 2\pi) = \sin x$

Plot the function $y = x^3 - 3x^2 + 5$

1) Domain: $\mathbb{R}$

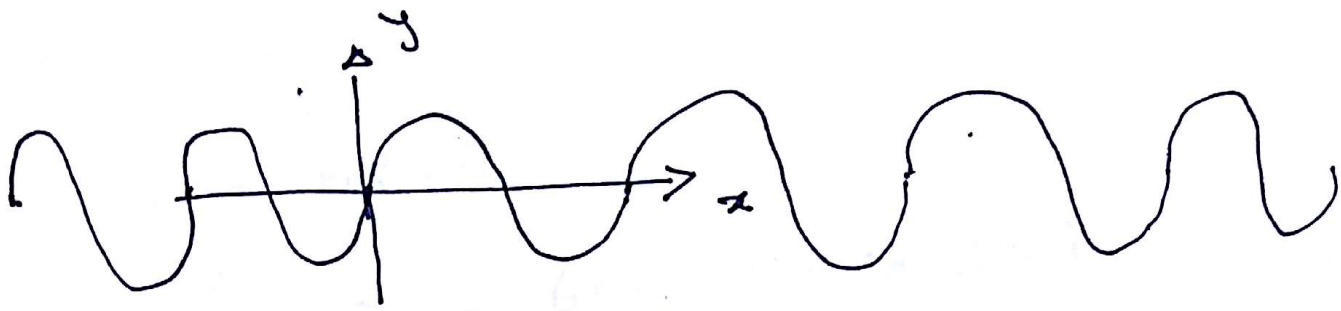
2) x-intercept: $y = 0 \longrightarrow \underline{x^3 - 3x^2 + 5 = 0}$ IVT can't be solved easily ☆ forget about it

y-intercept: $x = 0 \longrightarrow y = 5 \quad (0,5)$

$x^3 + 5 = 3x^2 \longrightarrow \dfrac{x^3+5}{x^2} = 3$

3) $f(-x) = (-x)^3 - 3(-x)^2 + 5 = -x^3 - 3x^2 + 5$ $\neq f(x)$ $\neq -f(+x)$

Not even, Not odd, Not periodic



4) vertical asymptote:

* NO denom
x NO log
} → NO vertical asym.

Horizontal asymptote.

$\lim_{x\to\infty} x^3 - 3x^2 + 5 = \infty$

$\lim_{x\to-\infty} x^3 - 3x^2 + 5 = -\infty$

limits dont exist ↓ no horizontal asymptote.

5) Increasing/decreasing - local max & min

$f'(x) = 3x^2 - 6x = x(3x-6)$

critical → $f'(x)=0 \longrightarrow 3x^2-6x=0 \Rightarrow x(3x-6)=0$; $x=0, x=2$

→ $f'(x)$ not defined → does not have any answer

$(-\infty,0] \cup (2,\infty)$ increasing

$(0,2)$ decreasing.

$x=0$ is local max

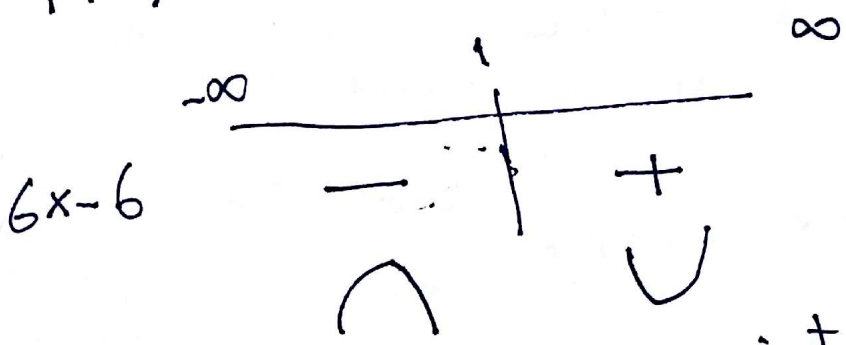
$x=2$ is local min

	$-\infty$		0		2		∞
x		−		+		+	
$3x-6$		−		−		+	
f'		+	↗	−	↘	+	↗

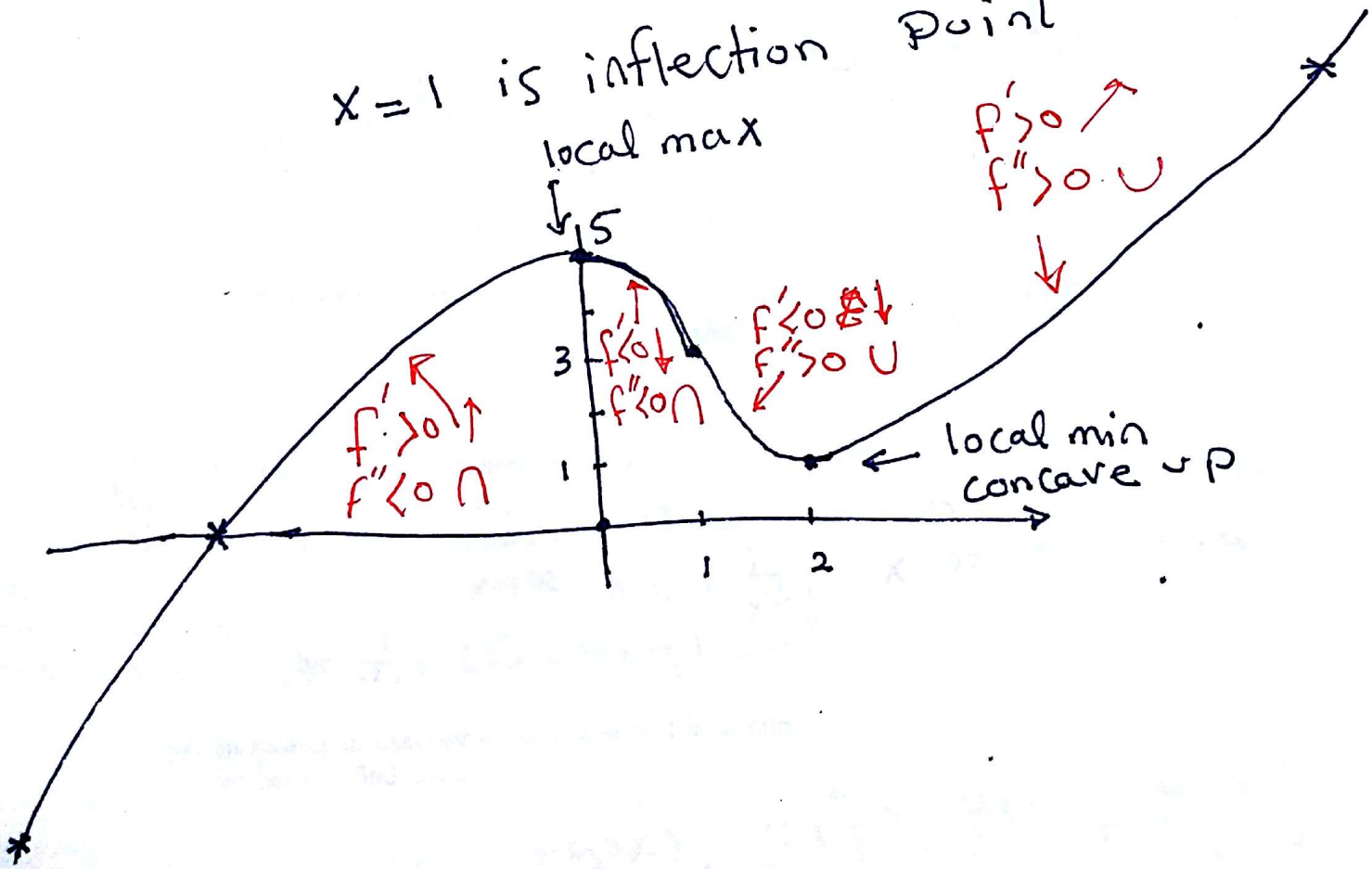
7) Concavity

$f''(x) = 6x - 6$

$f''(x) = 0 \rightarrow 6x - 6 = 0 \rightarrow x = 1$



$x = 1$ is inflection point.



M104 Worksheet | Curve Sketching | Tues-Thur Nov 8, 2016

Name: _____ Student No: _____

1. We want to sketch the curve of function $f(x) = \frac{4x}{x^2+1}$.

(a) find the domain of the function

we have a fraction, therefore we should set it to zero

$\Rightarrow 1 + x^2 = 0 \Rightarrow$ No solution $\rightarrow \mathbb{R}$

(b) find the x and y intercept.

$y_{intercept} : x = 0 \Rightarrow y = \frac{4(0)}{1+0} = 0$

$x_{intercept} : y = 0 \Rightarrow \frac{4x}{1+x^2} = 0 \rightarrow 4x = 0 \rightarrow x = 0$

(c) is function odd, even or periodic?

$$f(-x) = \frac{4(-x)}{(-x)^2+1} = \frac{-4x}{+x^2+1} = -\left(\frac{4x}{x^2+1}\right) = -f(x) \rightarrow \text{odd}$$

(d) find the asymptotes of the function

for vertical asymptotes the candidate is where _____. Therefore

$x^2 + 1 = 0 \Rightarrow$ No solution

Now we need to check ~~_____~~

No vertical asymptote.

For the horizontal asymptotes we find the following limits:

$$\lim_{x\to\infty} \frac{4x}{x^2+1} = \lim_{x\to\infty} \frac{4x}{x^2\left(1+\frac{1}{x^2}\right)} = \lim_{x\to\infty} \frac{4x}{x^2} = \lim_{x\to\infty} \frac{4}{x} = 0$$

$\lim_{x\to-\infty} \frac{4x}{x^2+1} = 0$ (Similarly)

(e) increasing or decreasing, local max and local min.

we need to find _____

$$f'(x) = \frac{4(x^2+1) - 4x(2x)}{(x^2+1)^2} = \frac{4x^2+4-8x^2}{(x^2+1)^2} = \frac{4-4x^2}{(x^2+1)^2}$$

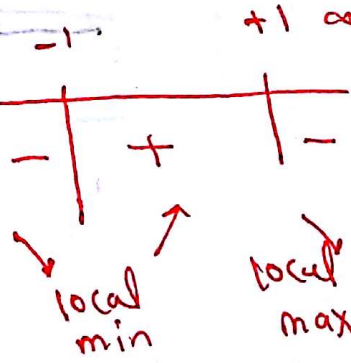
so the critical points are :

$f' = 0 \Rightarrow 4 - 4x^2 = 0 \rightarrow x^2 = 1 \rightarrow x = \pm 1$

$f' =$ Not defined $\Rightarrow (x^2+1)^2 = 0 \rightarrow x^2 = -1 \rightarrow$ No Solution.

$(-\infty, -1) \cup (1, \infty)$ decreasing

$(-1, 1)$ increasing



$$y' = \frac{4-4x^2}{(x^2+1)^2} \longrightarrow y'' = \frac{-8x(x^2+1)^2 - (4-4x^2)(x^2+1)(4x)}{(x^2+1)^4}$$

$$= \frac{(x^2+1)\left(-8x(x^2+1) - 4x(4-4x^2)\right)}{(x^2+1)^4}$$

$$= \frac{-8x^3-8x-16x+16x^3}{(x^2+1)^3} = \frac{8x^3-24x}{(x^2+1)^3}$$

$x=1, \; x=-1$

$f''(1) = \frac{-16}{8} < 0 \Rightarrow$ local max

$f''(-1) > 0 \Rightarrow$ local min.

Inflection Points

$8x^3 - 24x = 0$

$8x(x^2-3) = 0$

$x = 0, \; x = \pm\sqrt{3}$

$$y'' = \frac{8x(x-\sqrt{3})(x+\sqrt{3})}{(x^2+1)^3}$$

	$-\infty$	$-\sqrt{3}$	0	$\sqrt{3}$	∞
$8x$	$-$	$-$	$+$	$+$	
$x-\sqrt{3}$	$-$	$-$	$-$	$+$	
$x+\sqrt{3}$	$-$	$+$	$+$	$+$	
f''	$-$	$+$	$-$	$+$	
		$\cap$	$\cup$	$\cap$	$\cup$

$$f' = \frac{4-4x^2}{(x^2+1)^2} = \frac{4(1-x)(1+x)}{(x^2+1)^2}$$

	$-\infty$		$x=-1$		$x=1$		∞
$1-x$		+		+		−	
$1+x$		−		+		+	
f'		→		+		−	
		↘		↗		↘	
						local max