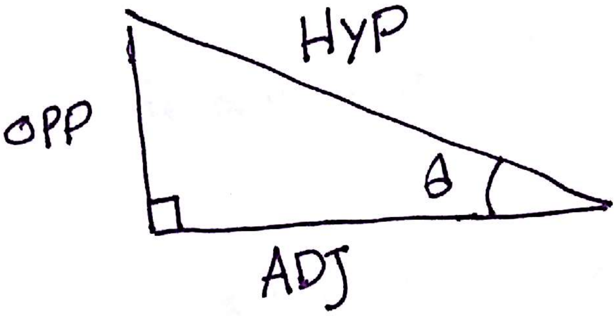
Oct 11, 2016

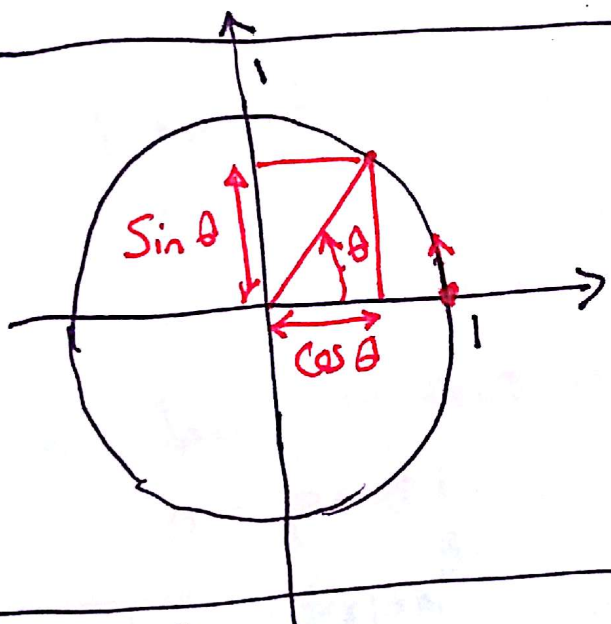
trigonometric function:



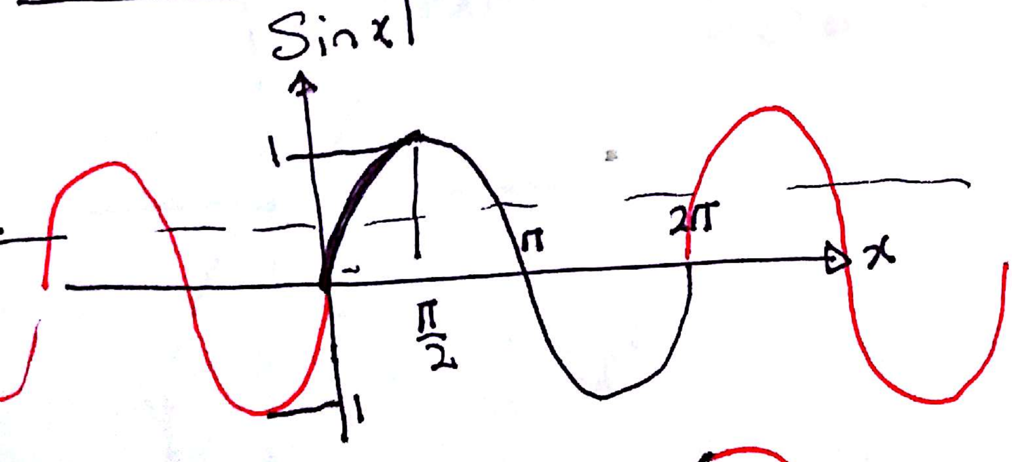
$$\sin\theta = \frac{OPP}{HYP}$$

$$\cos\theta = \frac{ADJ}{HYP}$$

$$\tan\theta = \frac{OPP}{ADJ} = \frac{\sin\theta}{\cos\theta}$$

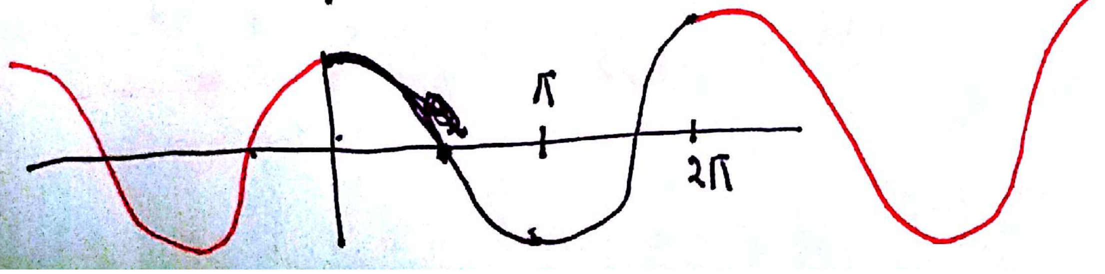


MATLAB



D = $\mathbb{R}$

R = [-1, 1]



D = $\mathbb{R}$

R = [-1, 1]

~~A function is invertab~~

$g(x)$ is called inverse of $f(x)$ $\left(g = f^{-1}(x)\right)$

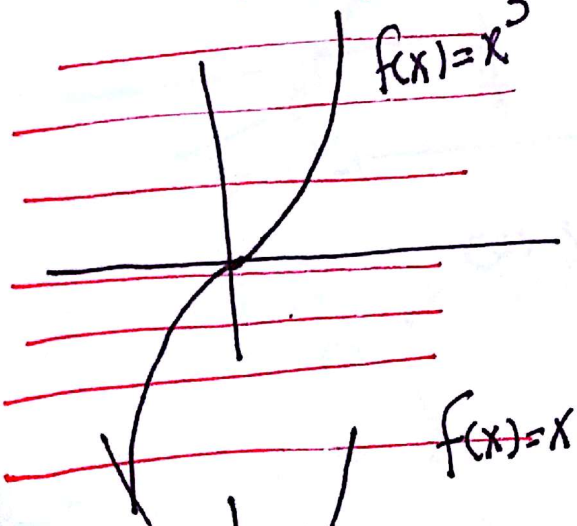
if and only if $f(g(x)) = g(f(x)) = x$

$f(x) = x^3 \rightarrow f^{-1}(x) = \sqrt[3]{x}$

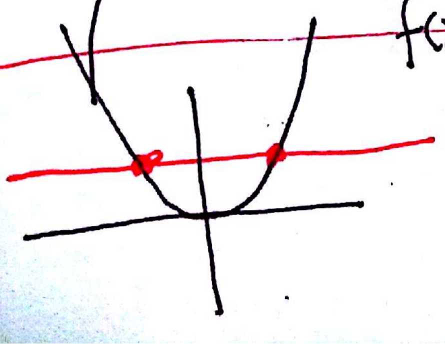
$f(f^{-1}(x)) = \sqrt[3]{(x^3)} = x$

A function is invertable only if it is 1-1

↳ A function is 1-1, only if No horizonta line intersect the function in more than one points.

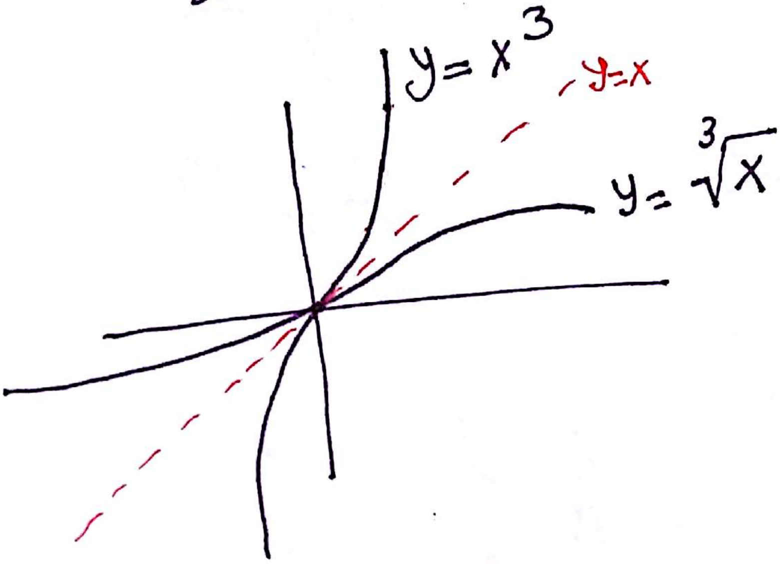


$f(x)$ is 1-1 $\Rightarrow$ invertable.

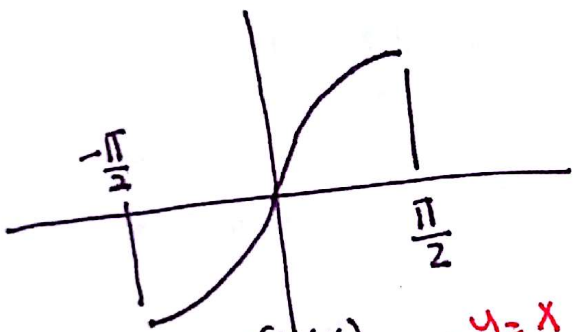


$f(x)$ is Not 1-1 $\Rightarrow$ Not invertable.

graphically mirror the function WRT $y=x$ so that you find graph of the inverse.



over $[-\frac{\pi}{2}, \frac{\pi}{2}]$ $\sin(x)$ is 1-1 $\Rightarrow$ invertable.



$\sin x$

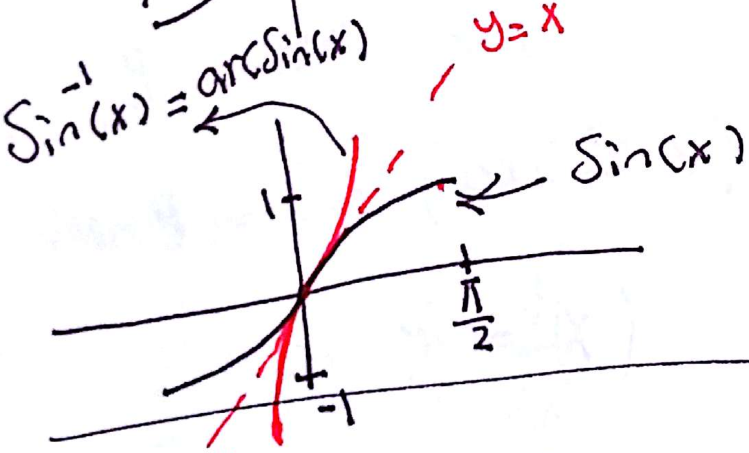
$D = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$R = [-1, 1]$

$\arcsin x$

$D = [-1, 1]$

$R = [-\frac{\pi}{2}, \frac{\pi}{2}]$



$[0, \pi]$ $\cos x$ is 1-1

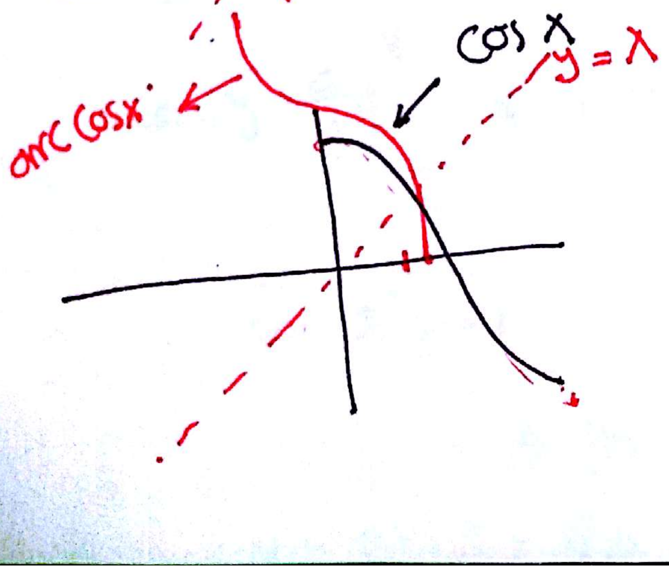
$\Rightarrow$ invertable.

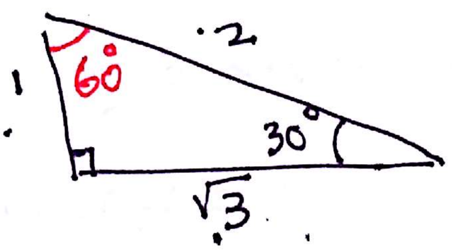
$\cos$ $D = [0, \pi]$

$R = [-1, 1]$

arc cos $D = [-1, 1]$

$R = [0, \pi]$

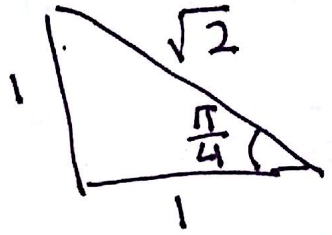




$30^\circ = \frac{\pi}{6}$ rad. $\quad 60^\circ = \frac{\pi}{3}$ rad.

$\sin\frac{\pi}{6} = \frac{1}{2} \qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{3} = \frac{1}{2}$



$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \qquad \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	2π
$\sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	1

Derivative of $\arcsin x$, $\arccos x$

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

$y = \arcsin x$

$\sin y = \sin(\arcsin x) = x$

$\frac{d}{dx}(\sin y) = \frac{d}{dx}(x)$

$\cos y \cdot \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

$\sin^2 y + \cos^2 y = 1$

$x^2 + \cos^2 y = 1 \Rightarrow \cos y = \sqrt{1-x^2}$

Scanned by CamScanner

$y = \arccos(x)$

$\cos(y) = x$

$-\sin(y) \cdot y' = 1.$

$y' = \dfrac{1}{-\sin(y)}$

$y' = -\dfrac{1}{\sqrt{1-x^2}}$

$\cos^2(y) + \sin^2(y) = 1$

$\sin(y) = \sqrt{1 - \cos^2(y)}$

$= \sqrt{1 - x^2}$

$\sin^2 y = 1 - \cos^2 y$

$\cancel{\cos}$ $y = \pm\sqrt{1 - \cos^2 y}$

sin

Find slope of tangent line at $x=0$ on the curv of $y = \text{Arc}\sin\left(x^2 + \frac{1}{2}\cos x\right)$

$y' = \dfrac{1}{\sqrt{1-\left(x^2+\frac{1}{2}\cos x\right)^2}} \cdot \left(2x - \frac{1}{2}\sin x\right)$

$x=0 \Rightarrow y' = \dfrac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}} \quad \left(2(0) - \frac{1}{2}\cancel{\sin(0)}^{\,0}\right) = 0$

differentiate $\arcsin(xy) = \frac{1}{2}x$ W.R.T x

$$\frac{d}{dx}\left(\arcsin(xy)\right) = \frac{d}{dx}\left(\frac{1}{2}x\right)$$

$$\frac{1}{\sqrt{1-x^2y^2}} \cdot \left(y + x\frac{dy}{dx}\right) = \frac{1}{2}$$

$$y + x\frac{dy}{dx} = \frac{1}{2}\sqrt{1-x^2y^2} \Rightarrow \frac{dy}{dx} = \frac{\frac{1}{2}\sqrt{1-x^2y^2} - y}{x}$$

$$\arcsin(xy) = \frac{1}{2}x$$

$$\cancel{\sin}(\cancel{\arcsin}\, xy) = \varepsilon\sin\left(\frac{x}{2}\right).$$

$$xy = \sin\frac{x}{2}$$

$$y = \frac{1}{x}\sin\frac{x}{2}$$

RATE of CHANGE

Suppose air is flowing into a spherical ballon. at the rate of $100 \frac{cm^3}{s}$, How fast is the Diameter changing when it is 50 cm ?

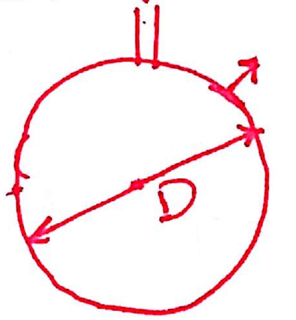
* Identify knowns an unknown.

Volume. = V =

Rate of change of volume. = $\frac{dV}{dt} = 100 \frac{cm^3}{s}$

Diameter = $D = 50$ cm

Rate of change of Diameter = $\frac{dD}{dt} = ?$



* Draw a diagram.

* Introduce notation

* Find an equation which relates unknown to the known.

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{4}{3}\pi\frac{D^3}{8} = \frac{\pi}{6}D^3$$

* Differentiate.

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{\pi}{6}D^3\right) \Rightarrow \frac{dV}{dt} = 3\frac{\pi}{6}D^2 \cdot \frac{dD}{dt}$$

$$\frac{dV}{dt} = \frac{\pi}{2}D^2\frac{dD}{dt}$$

$$\frac{d}{dx}\left((f(x))^n\right) = n f^{n-1} f'$$

* substitute $100 \frac{cm^3}{s} = \frac{\pi}{2}(50 cm)^2 \frac{dD}{dt} \to \frac{dD}{dt} = \frac{200}{2500\pi} \frac{cm}{s}$

An ice cube that is 3 cm on each side is melting ~~on~~ with the rate of 2 cubic cm per minute. How fast is length of each side decreasing?

volume $= V$

Rate of change volume $= \frac{dV}{dt} = 2\frac{cm^3}{min}$

side length $= a = 3cm$

Rate of change of side length $= \frac{da}{dt}$

$V = a^3$

$\frac{dV}{dt} = 3a^2 \frac{da}{dt} \Rightarrow -2\frac{cm^3}{min} = 3(3cm)^2 \cdot \frac{da}{dt}$

$\frac{da}{dt} = \frac{-2}{27} \frac{cm}{min}$

a is decreasing with a rate of $\frac{2}{27} \frac{cm}{min}$

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volume of sphere with Radius R

$V = \frac{4}{3}\pi R^3$

Surface area " " "

$A = 4\pi R^2$

~~Surface~~ Area of circle w/ Radius R

$$A = \pi R^2$$

Perimeter of circle "

$$P = 2\pi R$$

volume of cylinder with Radius R and height h

$$V = \pi R^2 \cdot h$$

Surface Area " " "

$$A = \underbrace{2\pi R} \cdot h$$

volume of a Cone with base radius R and height h

$$V = \frac{1}{3} \pi R^2 h.$$