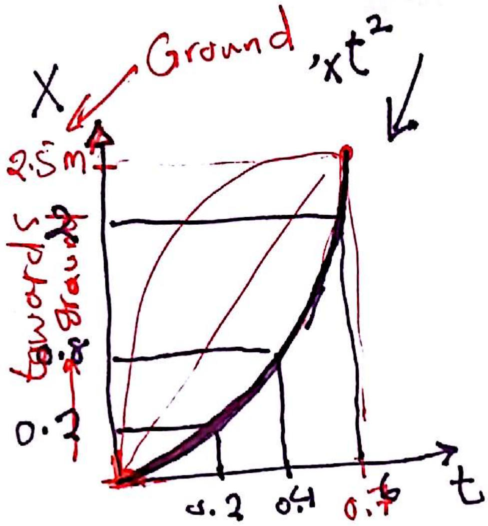
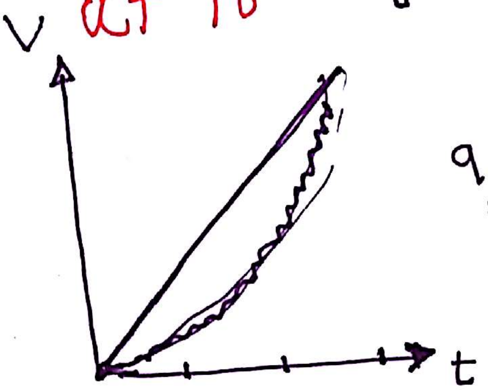
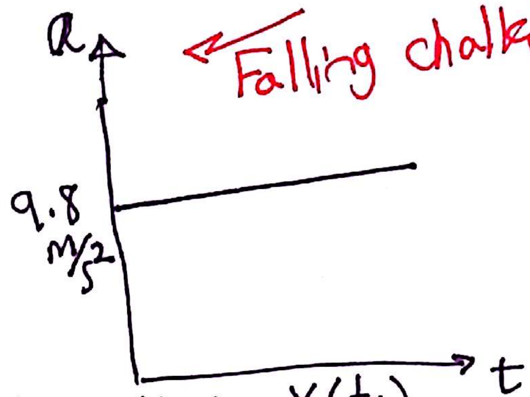


Oct 18





Falling chalk

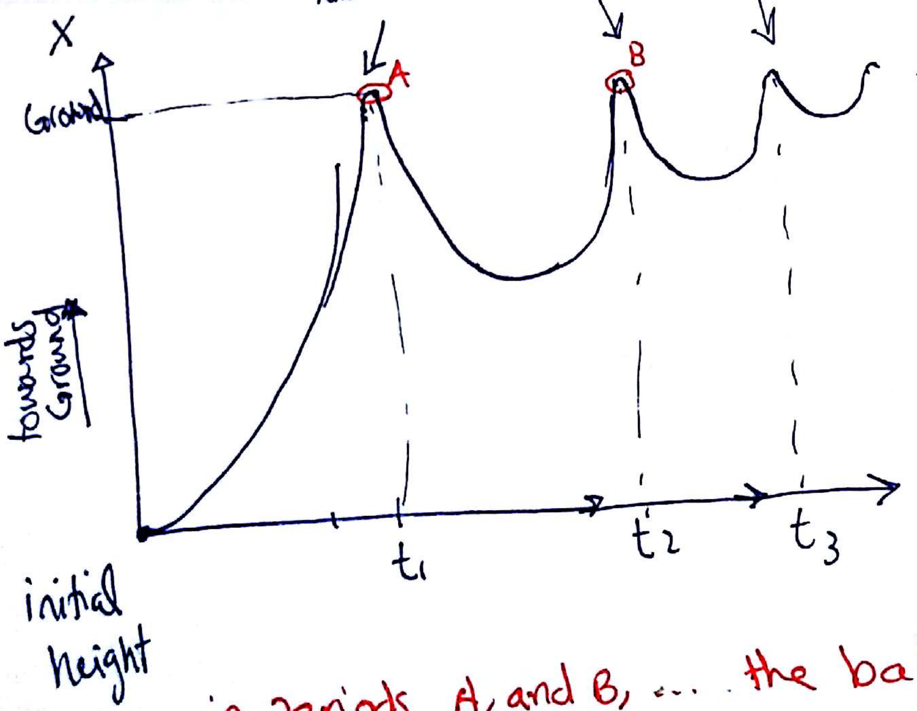
X(t) Position vs time → AVE. velocity $\frac{X(t_2)-X(t_1)}{t_2-t_1}$

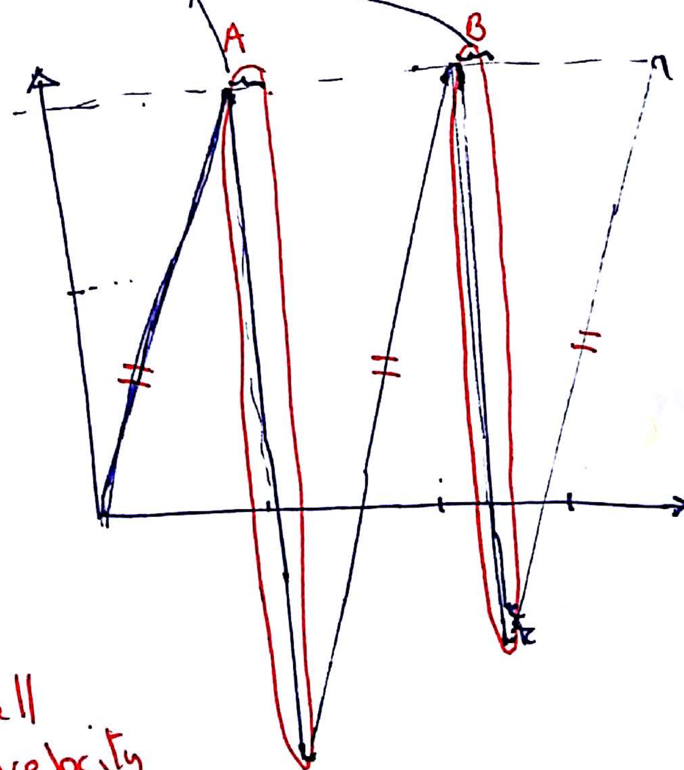
→ Instan. velocity $\frac{dX}{dt}$

V(t) velocity vs time → AVE acceleration $\frac{V(t_2)-V(t_1)}{t_2-t_1}$

→ Instan acceleration $\frac{dV}{dt}$

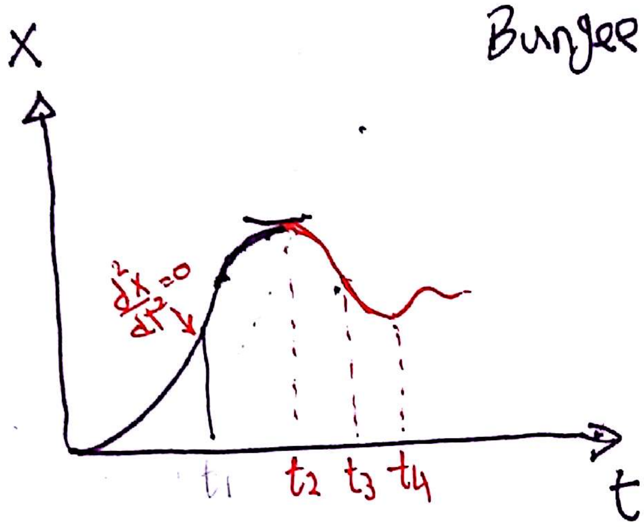
hits the ground, velocity suddenly becomes negative

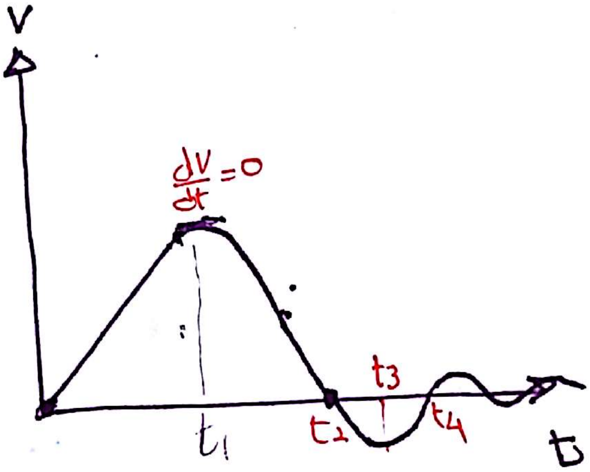




in periods A, and B, ... the ball hits the ground and suddenly velocity becomes negative. The three lines = are Parallel. why ?

Bungee jumping Example.

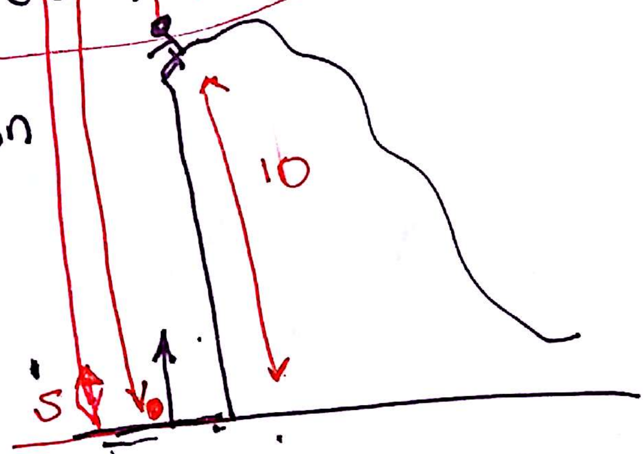




Example: We are standing at the edge of a clif $\boxed{10\,m}$ high and then I throw a stone upward from the clif with an initial velocity of $\boxed{20\frac{m}{s}}$. The Position of stone vs time is given by

$$S(t) = -5t^2 + 20t + 10$$

20m



$t=0 \Rightarrow S(0) = 10$

a) what is velocity as a function of time?

$$V = \frac{dS}{dt} = -10t + 20$$

$t=0 \Rightarrow V(0) = 20$

b) When does the stone reach its highest elevation? How High is that?

$\boxed{V=0}$ ← at highest Point

$-10t+20=0 \Rightarrow \boxed{t=2s} \Rightarrow S(2) = -5(2^2) + 20(2) + 10$

$= -20 + 40 + 10 = 30$

c) when does the stone hit the ground? at what velocity?

$s = 0 \rightarrow -5t^2 + 20t + 10 = 0$

$-5(t^2 - 4t - 2) = 0$

quadratic formula $t = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2}$

$= \dfrac{4 \pm \sqrt{16 + 8}}{2}$

$\sqrt{24} = \sqrt{4 \times 6}$
$= \sqrt{4} \cdot \sqrt{6} = 2\sqrt{6}$

$= \dfrac{4 \pm \sqrt{24}}{2} = \dfrac{4 \pm 2\sqrt{6}}{2}$

$= \boxed{2 \pm \sqrt{6}}$

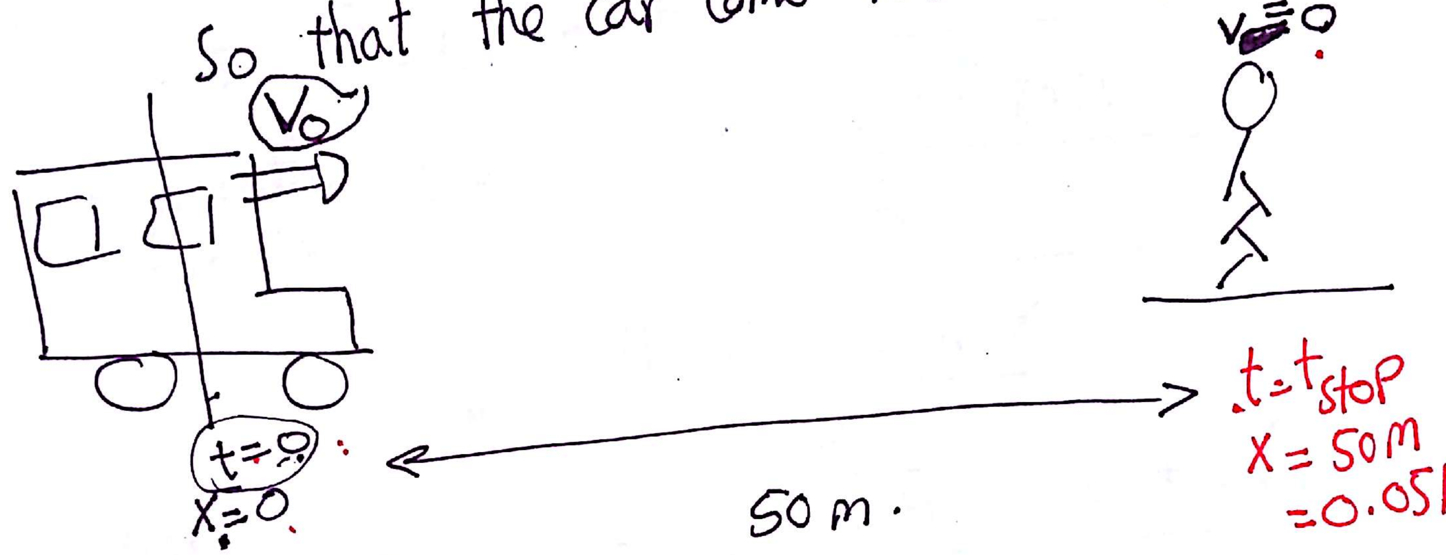
$t = 2 + \sqrt{6}$

$v(2+\sqrt{6}) = -10(2 + \sqrt{6}) + 20 = -20 - 10\sqrt{6} + 20$

$= -10\sqrt{6}$

downward.

A car's brake can decelerate the car at constant rate of $64000 \frac{km}{h^2}$.
What is the maximum velocity of the car so that the car come to stop within 50m?



$$a = -64000 \frac{km}{h^2}$$

$$\Downarrow$$

$$V = -64000\,t + C$$

$t=0 \Rightarrow V=V_0$

$$\Downarrow$$

$$V_0 = 0 + C \Rightarrow C = V_0$$

$$V = -64000\,t + V_0$$

t_{stop}, $V=0$

$$\Downarrow$$

$$-64000\,t_{stop} + V_0 = 0$$

$$\rightarrow t_{stop} = \frac{V_0}{64000}$$

derivative

anti-derivative

$$X = -32000\,t^2 + V_0 t + C'$$

$t=0 \Rightarrow X=0$

$$\Downarrow$$

$$0 = 0 + 0 + C' \Rightarrow C' = 0$$

$$X = -32000\,t^2 + V_0 t$$

$$0.05 = -32000\,(t_{stop})^2 + V_0(t_{stop}) \Rightarrow 0.05 = -32000\left(\frac{V_0}{64000}\right)^2 + V_0\left(\frac{V_0}{64000}\right)$$

$$0.05 = \frac{-32000}{64000^2} V_o^2 + \frac{V_o^2}{64000}$$

$$= V_o^2 \left(\frac{-32000}{64000^2} + \frac{1}{64000} \right)$$

$$= V_o^2 \left(\frac{-\cancel{32000}}{(64000)(\cancel{64000}_{2})} + \frac{1}{64000} \right)$$

$$= V_o^2 \left(\frac{-1}{2(64000)} + \frac{1}{64000} \right)$$

$$V_o^2 = \frac{0.05}{\frac{1}{64000} - \frac{1}{128000}}$$

$$V_o = \sqrt{\frac{0.05}{\frac{1}{64000} - \frac{1}{128000}}} \cong 80 \, km/h$$

truck \$20000 $\rightarrow$ \$21000 $\rightarrow \frac{1000}{20000} = 2\%$

Socks \$5 $\rightarrow$ \$6 $\rightarrow \frac{1}{5} = 20\%$

$$\boxed{\text{relative rate of change} = \frac{f'}{f}}$$

Example. find R.R.C. for population of bacteria given by $P(t) = 2000 e^{-0.2t}$.

$$\text{R.R.C} = \frac{P'}{P} = \frac{\cancel{2000}(-0.2)\cancel{e^{-0.2t}}}{\cancel{2000}\,\cancel{e^{-0.2t}}} = -0.2$$

$$\boxed{\frac{d}{dt}\left(\ln(f(t))\right) = \frac{f'(t)}{f(t)} = \text{R.R.C}}$$

$$\ln(P(t)) = \ln\left(2000 e^{-0.2t}\right)$$

$$\ln 2000 + \cancel{\ln}\left(\cancel{e}^{-0.2t}\right)$$

$$\ln 2000 - 0.2t$$

$$\frac{d}{dt}\left(\ln(P(t))\right) = -0.2$$

Business.

AVERAGE Cost $= \bar{C} = \dfrac{C(q)}{q}$

q = quantity

C = cost

$C(q) = \underline{10000 + 300\sqrt{q}}$ ← cost of producing q bicycles

$1 \longrightarrow C(1) = 10300$

$\bar{C}(1) = \dfrac{10300}{1} = 10300\$$

$9 \longrightarrow C(9) = 10000 + 300\sqrt{9} = 10900$

$\bar{C}(9) = \dfrac{10900}{9} \sim 1211\ \$$

$49 \longrightarrow C(49) = 10000 + 300\sqrt{49} = 12100$

$\bar{C}(49) = \dfrac{12100}{49} \cong 247\ \$$

AVE. Rate of change of cost

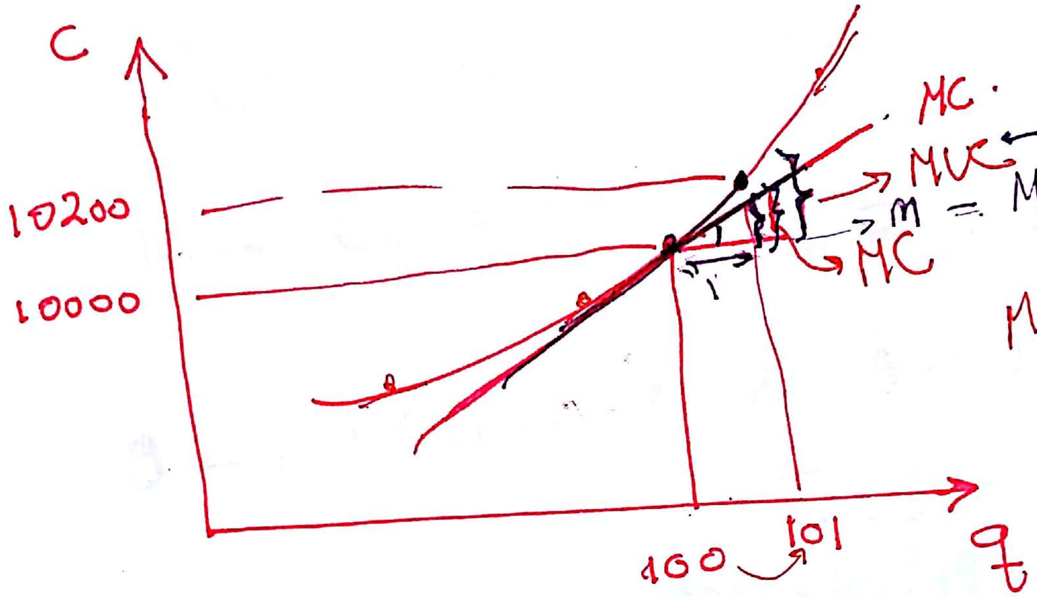
$$\frac{C(q_2) - C(q_1)}{q_2 - q_1}$$

Marginal cost $= C'(q_1) = \lim\limits_{q_2 \to q_1} \dfrac{C(q_2) - C(q_1)}{q_2 - q_1}$

$$C'(q) = MC = \frac{300}{2\sqrt{q}} = \frac{150}{\sqrt{q}}$$

Marginal unit cost =

$$MUC(n) = C(n+1) - C(n)$$

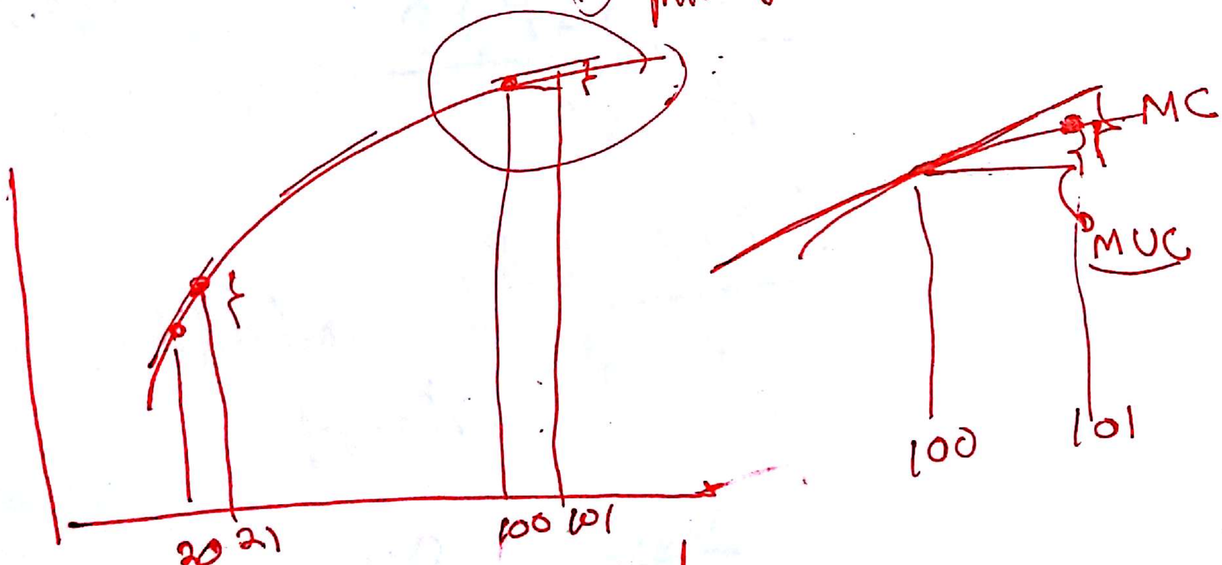


$MC(100) \approx MUC(100)$

$$MUC_{n=100} = C(101) - C(100) = 10200 - 10000 = 200 ₹$$

diseconomies of scale.

mining of companies



economies of scale.