Oct 25, 2016

Review Quiz 3

$(\arctan x)'$

$y = \arcsin x^{\arccos x} =$

$\Downarrow$

$\ln y = \underbrace{\arccos x}\ \ln \arcsin x$

arc S $\quad \arcsin x = \underbrace{\sin^{-1}} x \neq \frac{1}{\sin x}$

$f^{-1}(x) \neq \frac{1}{f(x)}$

$\swarrow$

inverse of $f(x)$.

Price elasticity of demand.

$$E_D = \frac{P}{q}\frac{dq}{dP} \qquad \frac{dR}{dP} = q(1+E_D)$$

* $\frac{dq}{dP} < 0$ because demand law

  $\Downarrow$

  $E_D < 0$

* when finding E_D, you're assuming $q = q(P)$

* $|E_D| < 1 \Rightarrow E_D > -1 \Rightarrow \frac{dR}{dP} > 0$

  $\Rightarrow$ inelastic $\Rightarrow P\uparrow, R\uparrow$

* $|E_D| > 1 \Rightarrow E_D < -1 \Rightarrow \frac{dR}{dP} < 0$

  $\Rightarrow$ elastic $\Rightarrow P\downarrow, R\uparrow$

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suppose demand function for a certain metal is $q = 100 - 2P$

q = quantity (Pound)

P = Price (1000 \$)

a) what quantity can be sold at $P = 30$ [redacted] \$ thousand dallars.

$$q = 100 - 2(30) = 40 \text{ Pounds}$$

b) determine function $E_D(P)$

$$E_D = \frac{P}{q} \cdot \frac{dq}{dP} = \frac{P}{100-2P} \cdot (-2)$$

$$= \frac{-2P}{100-2P}$$

c) determine E_D at $P = 30$, and interpret it

$$E_D(30) = \frac{-2(30)}{100-2(30)} = \frac{-60}{40} = \frac{-3}{2}$$

$\boxed{|E_D| = \frac{3}{2} > 1.}$

$\Downarrow$

elastic

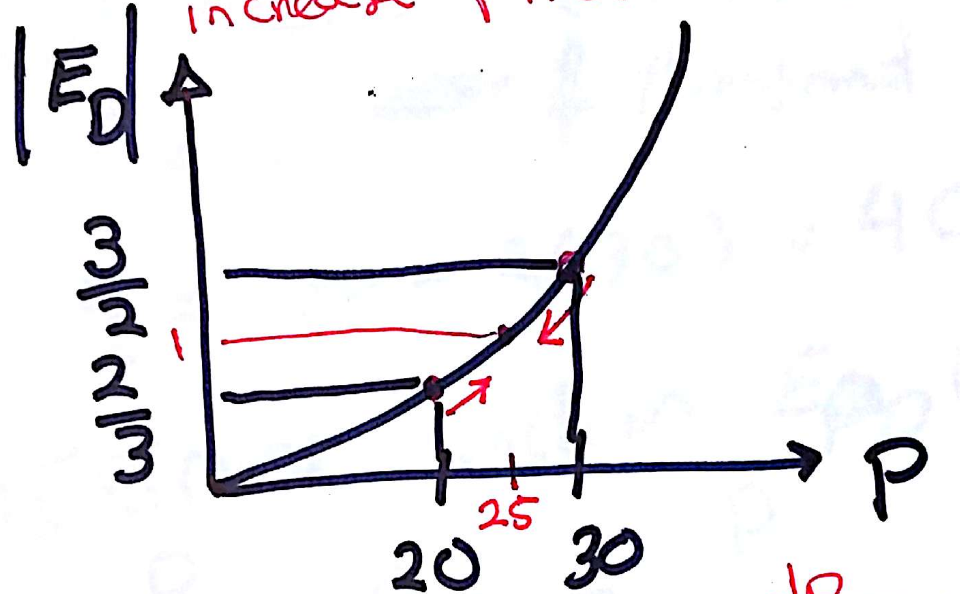
$\Downarrow$

Lower Price to raise Revenue

d) Find elasticity at $P = 20$ and interpret it.

$$E_D(20) = \frac{-2(20)}{100-2(20)} = \frac{-40}{60} = \frac{-2}{3}$$

↳ $|E_D| < 1 \Rightarrow$ inelastic $\Rightarrow$ increase price to increase Revenue.



$|E_D| = 1 \Rightarrow E_D = -1 \Rightarrow \frac{dR}{dP} = 0$

$R = R(p) \longrightarrow$ take derivative
Set it to zero

e) find the optimum price to optimise Revenue

$|E_D| = 1 \rightarrow E_D = -1 \Rightarrow \frac{-2P}{100-2P} = -1 \rightarrow -2P = 2P - 100$

$\Downarrow$

$100 = 4P$

$P = 25$

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for what values of P, is the demand function $q = 6000\, e^{-0.5P}$ elastic?

$$E_D = \frac{P}{q} \cdot \underbrace{\frac{dq}{dP}}_{} = \frac{P}{\cancel{6000\, e^{-0.5P}}} \left(\cancel{6000} (-0.5) \cancel{e^{-0.5P}} \right)$$

$$= -0.5P = -\frac{P}{2}$$

elastic

$$|E_D| > 1 \Rightarrow \underline{E_D < -1} \rightarrow \frac{-P}{2} < -1$$

$$\Downarrow$$

$$-P < -2$$

$$\xleftarrow{(-1)} \Downarrow$$

$$\boxed{P > 2}$$

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[12] 4. A cell phone supplier has determined that demand for its newest cell phone model is given by

$$qp + 30p + 50q = 8500,$$

where q is the number of cell phones the supplier can sell at a price of p dollars per phone. You may find it useful in this problem to know that elasticity of demand is defined to be $E(p) = +pf'(p)/f(p)$ for the demand function $q = f(p)$.

(a) If the current price is \$150 per phone, will revenue increase or decrease if the price is lowered slightly?

Answer:

$$q(p+50) = 8500 - 30p$$

$$q = \frac{8500 - 30p}{p+50}$$

method 1

$$E_D = \frac{p}{q}\frac{dq}{dp}$$

$$\frac{dq}{dp} = \frac{-30(p+50) - 8500 + 30p}{(p+50)^2}$$

$$\boxed{E_D = \frac{p}{\frac{8500-30p}{p+50}} \cdot \frac{-30(p+50) - 8500 + 30p}{(p+50)^2}}$$

$P = 150$

$150q + 30(150) + 50q = 8500$

$200q = 8500 - 4500 = 4000$

$\Rightarrow \boxed{q = 20}$

(b) ~~What price should the cell phone supplier set for this cell phone to maximize its revenue from sales of the phone?~~

Answer:

Methode 2

$$qP + 30P + 50q = 8500$$

$$\frac{d}{dP}(qP + 30P + 50q) = \frac{d}{dP}(8500) = 0$$

$$q + P\frac{dq}{dP} + 30 + 50\frac{dq}{dP} = 0 \rightarrow \frac{dq}{dP} = -\frac{q+30}{P+50}$$

$$E_D = \frac{P}{q}\frac{dq}{dP} = \boxed{-\frac{P}{q}\frac{(q+30)}{(P+50)}}$$

$$-\frac{150}{20}\frac{(20+30)}{150+50}$$

$$-\frac{(150)\cancel{50}^{1}}{20(\cancel{200})_4} = -\frac{15}{8}$$

Part b). find the price to maximize Revenue.

$|E_D| = 1 \to E_D = -1 \Rightarrow$

$+\frac{P}{q}\,\frac{q+30}{P+50} = +1 \Rightarrow P(q+30) = q(P+50)$

$\cancel{Pq} + 30P = \cancel{qP} + 50q \to q = \frac{30P}{50} = \frac{3P}{5}$

substitute back in equation

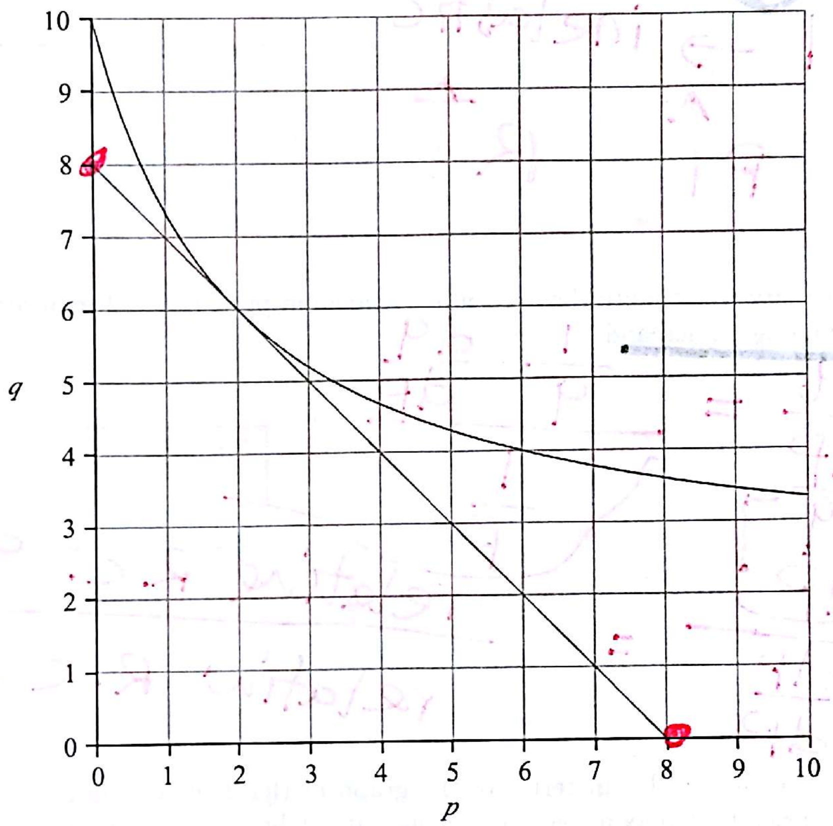
$\left(\frac{3P}{5}\right)\cdot P + 30P + \overset{10}{\cancel{50}}\left(\frac{3P}{\cancel{5}}\right) = 8500$

$\frac{3}{5}P^2 + 30P + 30P = 8500.$

$\frac{3}{5}P^2 + 60P - 8500 = 0$

find by quad. form.

[10] 5. In the following figure, the curved graph is a demand curve where q is the demand and p is the price in dollars. The straight line is the tangent line to this demand curve at the point $(p, q) = (2, 6)$. Recall that the price elasticity of demand is given by $\epsilon(p) = \frac{p}{q}\frac{dq}{dp}$.



(a) Compute $\epsilon(2)$, the elasticity at price \$2.

Answer:

$y = -x + 8$
↳ -1 ⇒ slope

$E_p = \frac{p}{q} \cdot \frac{dq}{dp} = \frac{2}{6}(-1) = -\frac{2}{6} = -\frac{1}{3}$

slope of tangent line $= \frac{\text{rise}}{\text{run}} = \frac{-8}{8} = -1$

(b) In order to increase revenue, should the price be raised or lowered from $2?

$E_{D.} = \frac{-1}{3}$

$|E_D| < 1 \rightarrow$ inelastic

$P \uparrow \quad R \uparrow$

Answer:

(c) Use elasticity to estimate the percentage change in price from $2 required to result in a 5% increase in demand.

$E_D = \frac{P}{q}\frac{dq}{dp} = \frac{\frac{1}{q}\frac{dq}{dp}}{\frac{1}{p}}$

$= \frac{\boxed{\frac{1}{q}\frac{dq}{dp}}}{\frac{1}{p}\cdot\frac{dp}{dp}} = \frac{\text{relative R.C. } q}{\text{relative R.C. } P} = \frac{\%\text{ change } q}{\%\text{ change of } P}$

Answer:

(d) Describe geometrically (in terms of the graph of the demand curve) how you would find the price that maximizes the revenue. It might be useful to note that the slope of a line connecting a point on this demand curve to the origin is q/p. Restrict your answer to the box below.

$E_D = \frac{\%\text{ change of } q}{\%\text{ " } P}$

$E_D = \frac{-1}{3}$

$\%\text{ change in } q = 5\% = 0.05$

$$E_D = \frac{-1}{3} = \frac{0.05}{\%\text{ change in } P}$$

$$\%\text{ change} P = \frac{0.05}{-1/3} = -0.15$$

$$= -15\%$$

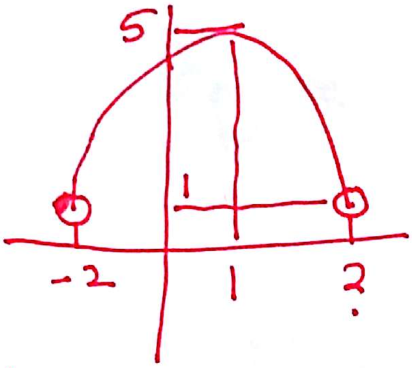
Extreme values.

let $f(x)$ be defined on an interval I containing $x = c$, $f(c)$ is minimum (absolute minimum, Global minimum) if of all x in I

$$f(c) \leq f(x)$$

$f(c)$ in maximum (absolute maximum global maximum) if for all x in I

$$f(c) \geq f(x)$$



(-2,2)

Maximum (-2,2)

minimum :

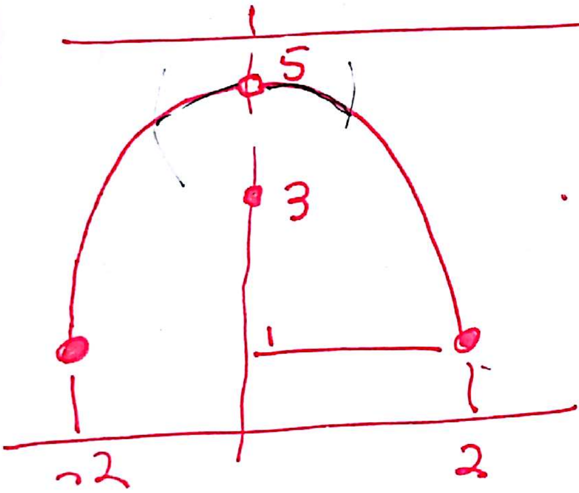
Max is $\underline{\underline{5}}$

~~min is $\underline{\underline{1}}$~~

~~min~~ there is no minimum

|| $\boxed{x=2}$

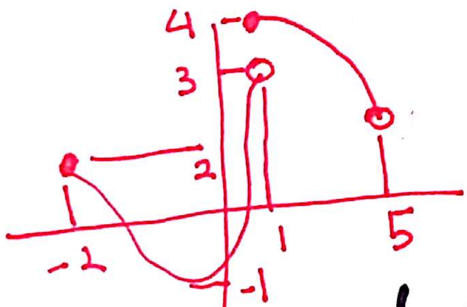
because is Not in domain



Min = 1

Max = No maximum.

Not continuous



max = 4

min = -1

Extreme value theorem (EVT)

let f(x) be continuous on a $\underline{\underline{\text{close}}}$ interval then f has a max & a min in the interval.