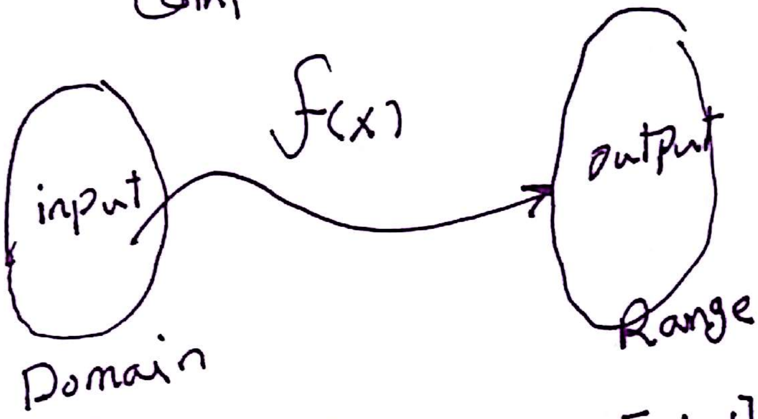
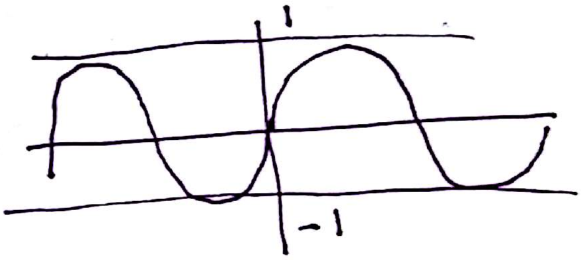
Oct 4, 2016

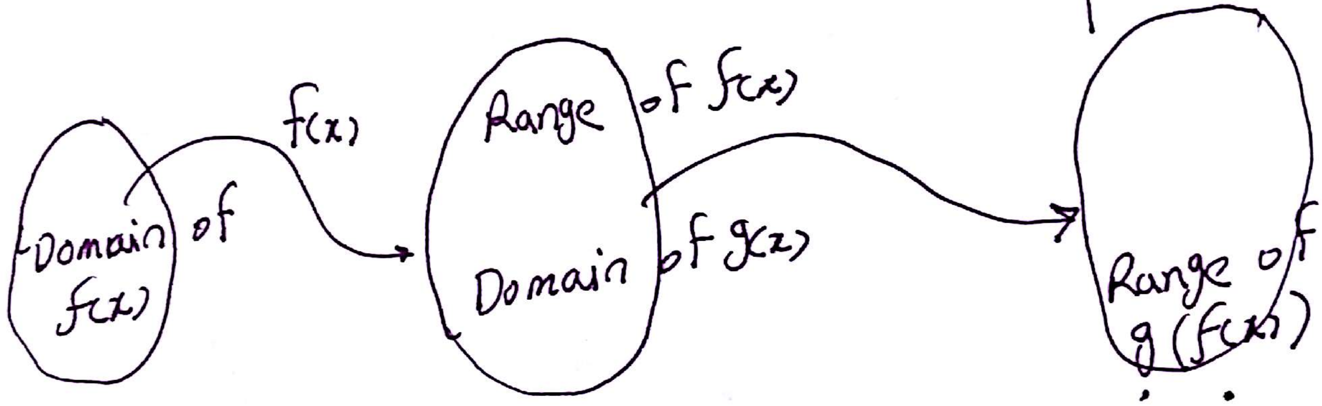
chain rule:

Composition of function Review.



Domain		Range
$\mathbb{R}$	$\sin(x)$	$[-1,1]$
$[0,\infty)$	$\sqrt{x}$	$[0,\infty)$





$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$$

$$f(x) = x^2 + \sin x$$

$$g(x) = e^x$$

$$f(g(x)) = f(e^x) = (e^x)^2 + \sin(e^x) = e^{2x} + \sin(e^x)$$

$$g(f(x)) = g(x^2 + \sin x) = e^{x^2 + \sin x}$$

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$$f(f(x)) = f(x^2+\sin x) =$$
$$(x^2+\sin x)^2 + \sin(x^2+\sin x)$$

chain rule:

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}[f(g(x))] = \frac{df}{dx}\Big|_{g(x)} \cdot \frac{dg}{dx}\Big|_{x}$$

$$\frac{du}{dx} = \frac{du}{\cancel{dy}} \cdot \frac{\cancel{dy}}{dx}$$

$u = u(y)$
$y = y(x)$

Leibniz Notation

$$\frac{du}{dx} = \frac{dy}{\cancel{dz}} \frac{\cancel{dz}}{\cancel{dw}} \frac{\cancel{dw}}{dx}$$

$$y(x) = (2x+1)^{99}$$

$u = 2x+1$

$y = u^{99}$

$$y(u(x)) = y(2x+1) = (2x+1)^{99}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 99u^{98} \cdot 2 = 198(2x+1)^{98}$$

$$(2x+1)^2 = 4x^2+4x+1$$

^{3}

if we were to find $(2x+1)^{99}$, it would be 100 terms, almost impossible to calculate. see the last page. This calculation is done using Maple

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

derivative of outer func evaluted at Inner function.

derivative of Inner func.

$$y(x) = (2x+1)^{99}$$

$$y'(x) = 99(2x+1)^{98} \, 2 = 198(2x+1)^{98}$$

$$y = f(x) = \sqrt{13x^2 - 5x + 8}$$

$$u = 13x^2 - 5x + 8$$

$$y = \sqrt{u}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} =$$

$$\frac{1}{2\sqrt{u}} \cdot (26x - 5) = \frac{26x - 5}{2 \cdot \sqrt{13x^2 - 5x + 8}}$$

derivative of Inner function · der. outer func. evaluated at inner func

$$\frac{26x^{2} - 5}{2\sqrt{13x^2 - 5x + 8}}$$

$f(x) = \sin(5x) \Rightarrow$

$f'(x) = 5\cos(5x)$

$f(x) = \sin(\underbrace{5\ln(x^2)})$

$f'(x) = (2x)(5\frac{1}{x^2})\cos(5\ln(x^2)) = \frac{10}{x}\cos(5\ln(x^2))$

or more clearly, we could first simplify f(x) and then differentiate.

~~$f(x) = \tan(\sqrt{x})$~~

$f(x) = \sin(\underbrace{10\ln(x)})$

$f'(x) = \frac{10}{x}\cos(10\ln(x))$

$f(x) = \tan\sqrt{x}$

$f'(x) = \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

$(\tan(x))' = 1+\tan^2 x$

$= \sec^2 x$

$= (\frac{1}{\cos x})^2$

$f(x) = \sqrt{\sin(e^{\tan x})}$

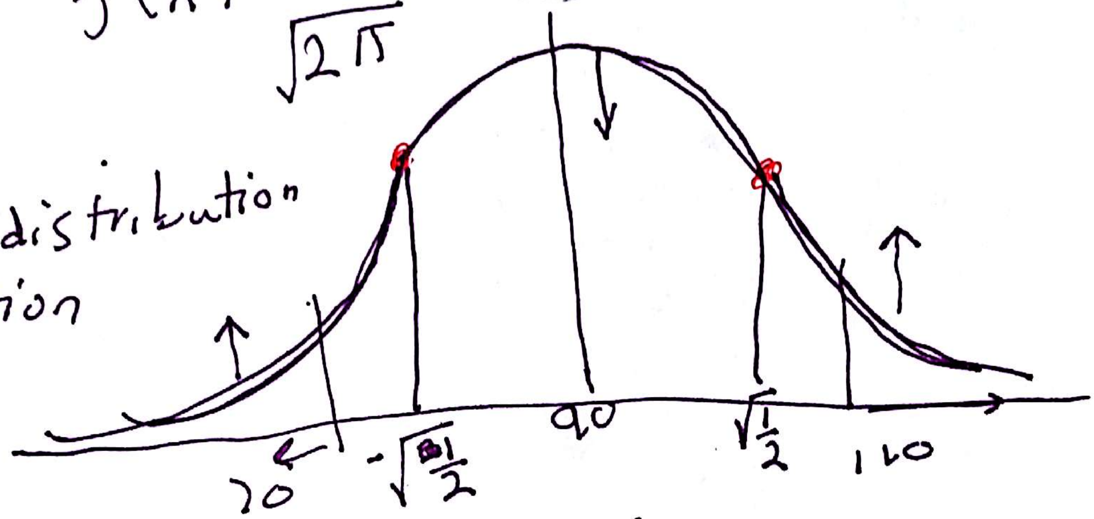
$f'(x) = \sec^2 x \cdot e^{\tan x} \cdot \cos(e^{\tan x}) \cdot \frac{1}{2\sqrt{\sin(e^{\tan x})}}$

$$y = \ln(\ln(\ln(\cos x)))$$

$$y' = -\sin x \cdot \frac{1}{\cos x} \cdot \frac{1}{\ln(\cos x)} \cdot \frac{1}{\ln(\ln(\cos x))}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2}$$

gaussian distribution function



find where $f''(x)$ is positive

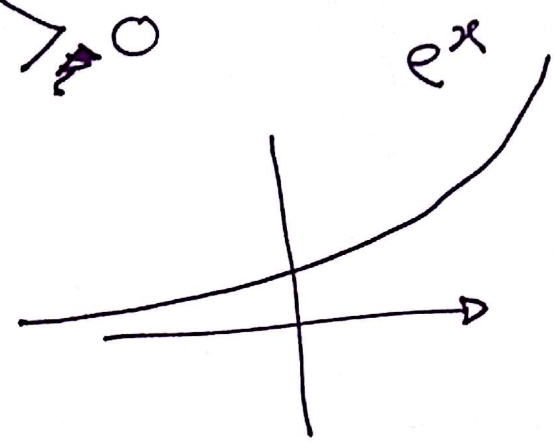
$$f'(x) = \frac{(-2x)}{\sqrt{2\pi}} e^{-\cdots}$$

$$f'' = \frac{1}{\sqrt{2\pi}}\left[(-2x)' e^{-x^2} + (-2x)(e^{-x^2})'\right] =$$

$$\frac{1}{\sqrt{2\pi}}\left[-2e^{-x^2} - (2x)(-2x)e^{-x^2}\right] =$$

$$= \frac{1}{\sqrt{2\pi}} e^{-x^2}\left(4x^2 - 2\right)$$

$$\frac{1}{\sqrt{2\pi}} \underbrace{e^{-x^2}}_{>0} \underbrace{(4x^2-2)}_{>0} > 0$$



$4x^2 - 2 > 0 \Rightarrow$

$4x^2 > 2 \Rightarrow x^2 > \frac{1}{2} \longrightarrow$

$\sqrt{x^2} = |x| > \sqrt{\frac{1}{2}} \longrightarrow$ $x > \sqrt{\frac{1}{2}}$

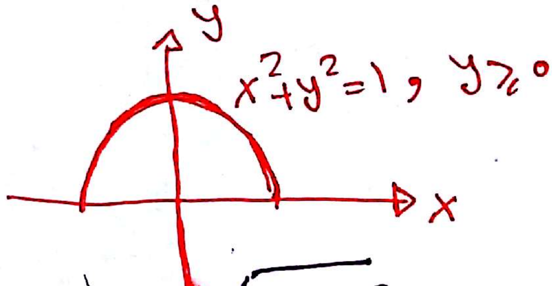
$x < -\sqrt{\frac{1}{2}}$

$x^2 + y^2 = 1$

$x^2 + y^2 = R^2 \longrightarrow$ circle with Radius R centered at origin

for $y \geq 0$ this graph is half circle.

find tangent line ~~at~~ and normal line at $x = \frac{\sqrt{2}}{2}$ on the curve



$y^2 + x^2 = 1 \xrightarrow{\text{Solve for } y}$

$y^2 = 1 - x^2 \rightarrow \sqrt{y^2} = \sqrt{1-x^2} \rightarrow y = \cancel{\pm}\sqrt{1-x^2}$

$y = +\sqrt{1-x^2} \Rightarrow$ slope of tangent line $= \frac{dy}{dx} = \frac{-2x}{2\sqrt{1-x^2}}$

Slope of tangent line at $x = \frac{\sqrt{2}}{2}$ $= \dfrac{-2 \cdot \frac{\sqrt{2}}{2}}{2\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}} =$

$$= \frac{-\sqrt{2}}{2\sqrt{1-\frac{1}{2}}} = \frac{-\sqrt{2}}{2\sqrt{\frac{1}{2}}} = \frac{-\frac{\sqrt{2}}{1}}{2\,\frac{1}{\sqrt{2}}} = \frac{-\sqrt{2}\,\sqrt{2}}{2} = -1$$

$$\left.\begin{array}{l} y = \sqrt{1-x^2} \\ x = \frac{1}{\sqrt{2}} \end{array}\right\} \rightarrow y = \sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $\longrightarrow$ tangent line equation $\quad y - \frac{1}{\sqrt{2}} = -\left(x - \frac{1}{\sqrt{2}}\right)$

$m = -1$

$$y = -x + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = -x + \frac{2}{\sqrt{2}} = -x + \sqrt{2}$$

$$\text{Slope of normal line} = \frac{-1}{\text{Slope of tangent line}}$$

$$= \frac{-1}{-1} = 1$$

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $\longrightarrow$ normal line equation $\quad y - \frac{1}{\sqrt{2}} = x - \frac{1}{\sqrt{2}}$

$m = 1$

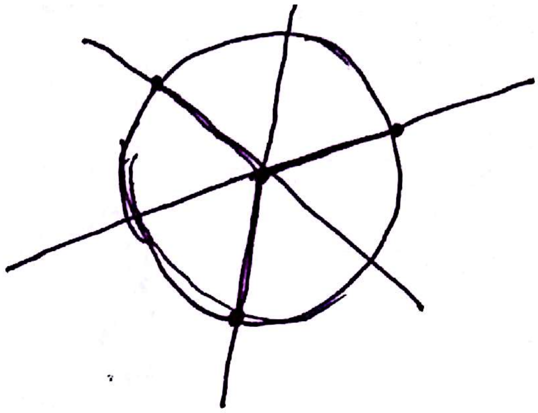
$$\Downarrow$$

$$y = x$$

This agrees well with our geometry course. In geometry, we know radius is always perpendicular to the circle. in this case, y=x is the radius of the circle that is passing through (1/sqrt(2), 1/sqrt(2)) .

Implicit differentiation.

$x^2+y^2=1$



y is give <u>Implicitly</u> as a function of x

$$x^2+(y(x))^2=1$$

$$\left(x^2+(y(x))^2\right)'=(1)'=0$$

$$(x^2)'+\left((y(x))^2\right)'=0$$

$$2x \quad + \quad 2y(x)\cdot y'(x)=0$$

$$\frac{d}{dx}\left(x^2+(y(x))^2\right)=\frac{d}{dx}1=0$$

$$\frac{d}{dx}(x^2)+\frac{d}{dx}\left((y(x))^2\right)=0$$

$$2x \quad + \quad 2y\cdot y'=0$$

$$2yy'=-2x \longrightarrow y'=\frac{-2x}{2y}=\frac{-x}{y}$$

Generalized differentiation.

$$\frac{d}{dx}\left(f(x)^{n}\right) = n \cdot f(x)^{n-1} \cdot f'(x)$$

$$\frac{d}{dx}\left(\sin(f(x))\right) = \cos(f(x)) \cdot f'(x)$$

$$\frac{d}{dx}\left(e^{f(x)}\right) = e^{f(x)} \cdot f'(x)$$

$$\frac{d}{dx}\left(\ln(f(x))\right) = \frac{1}{f(x)} \cdot f'(x)$$

$$\frac{d}{dx}\left(\sqrt[n]{f(x)}\right) = \frac{d}{dx}\left((f(x))^{\frac{1}{n}}\right) = \frac{1}{n} f(x)^{\frac{1}{n}-1} \cdot f'(x)$$

$$x^4 + xy^3 + yx^3 = 1$$

$$\Downarrow$$

$$\frac{d}{dx}\left(x^4 + xy^3 + yx^3\right) = \frac{d}{dx}(1) = 0$$

$$\frac{d}{dx}(x^4) + \frac{d}{dx}(xy^3) + \frac{d}{dx}(yx^3) = 0$$

$$4x^3 \qquad \frac{d}{dx}(x) \cdot y^3 + x\frac{d}{dx}(y^3) \qquad \frac{d}{dx}(y) \cdot x^3 + y\frac{d}{dx}(x^3) = 0$$

$$4x^3 + 1 \cdot y^3 + \underline{x(3y^2y')} + \underline{y'x^3} + y(3x^2) = 0$$

$$y'(3xy^2 + x^3) = -(4x^3 + y^3 + 3yx^2)$$

$$y' = -\frac{4x^3 + y^3 + 3yx^2}{3xy^2 + x^3}$$