MOTH - 104
Sep 13

a) This class is great.

b) " " Awesome

c) Fantastic.

d) a and b

e) all of above.

Tuesday 9:45 - 11:15 LSK 300C
Friday 9 - 10:30 LSK 300 C

Compounded Interest.

$ 1000

Anual Interest rate = 10%

2 years

Flat Interest rate (it is Not compounded)

$1000(1+0.1) = 1100$ After first year

$1000(1) + \underbrace{1000 \times 0.1} + 1000 \times 0.1 = 1200$ ↳ After second year.

$$1000(1 + \overbrace{0.1 + 0.1 + \dots + 0.1}^{t})$$ After t years

$$= 1000(1 + 0.1(t))$$

Componded Interest.

1) Compounded Annually

$1000(1+0.1) = 1100$ After first year.

$1100(1+0.1) = 1210$

2) Compounded Semi-annually

$1000(1+\frac{0.1}{2}) = 1050$ After first 6 months.

$1050(1+\frac{0.1}{2}) = 1102.5$ ↳ After first year

$1102.5(1+\frac{0.1}{2}) = 1157.6$ After 18 months

$1157.6(1+\frac{0.1}{2}) = 1215.5$ After two years

3) Compounded quarterly

$$1000(1+\frac{0.1}{4})^{8} = 1218.4$$

4) compounded monthly

$$1000(1+\frac{0.1}{12})^{24} =$$

5) Compounded daily

$$1000(1+\frac{0.1}{365})^{2\times365} = 1221.3$$

uncompounded

PV = Present value

FV = Future value

r = Annuall ~~Interest~~ Interest rate

t = number of years.

$$\boxed{\text{unCompounded.} \quad FV = PV(1 + rt)}$$

$$\boxed{\text{Compounded Frequency } n \quad FV = PV\left(1 + \frac{r}{n}\right)^{n \times t}}$$

Compounded continuously

$$FV = \lim_{n \to \infty} PV\left(1 + \frac{r}{n}\right)^{nt}$$

Bernoulli

$$\boxed{= PV \cdot e^{rt}}$$

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effective Interest Rate → compounded Annually

Example 1. You borrow 50000 from Nick, the Shark, charges you at Annual rate of r compounded continuously.

You pay him 100,000 after two year?

a) what was the Interest rate?

b) " " effective Interest rate?

$Fv = Pv \cdot e^{rt}$

$Fv = 100000$

$Pv = 50000$

$t =$ 2 years

$100000 = 50000 \, e^{2r} \Rightarrow$

$\frac{100000}{50000} = e^{2r} \Rightarrow$

$2 = e^{2r} \xrightarrow{\text{take } \ln}$

$\ln 2 = \cancel{\ln} (\cancel{e}^{2r}) = 2r$

$r = \frac{\ln 2}{2} = 34.7\%$

Laws of exponentials

$a^m \cdot a^n = a^{m+n}$

$a^m / a^n = a^{m-n}$

$(a^m)^n = a^{m \cdot n}$

$a^1 = a$

$a^0 = 1$

Laws of log

$\log(ab) = \log a + \log b$

$\log a^b = b \log a$

$\log(a/b) = \log a - \log b$

$\log 1 = 0$

$\log \rightsquigarrow \log_{10}$

$\ln \rightsquigarrow \log_e$

$\log_a^x / \log_b^x = \log_b^a$

You deposit \$3000 7% (Annual)

Compounded ~~&~~ Annually

Your friend \$2500 Interest 6.95%

Compound Continuously

Will her ~~money~~ balance ever reach yours?

Your money $FV = PV\left(1+\frac{r}{n}\right)^{nt}$

$$FV = 3000\left(1+\frac{0.07}{1}\right)^{t}$$

Your friend money

$$FV = PV\,e^{rt}$$

$$FV = 2500\,e^{0.0695t}$$

$$3000\,(1.07)^{t} = 2500\,e^{0.0695t}$$

$$\frac{3000}{2500}\,1.07^{t} = e^{0.0695t}$$

$$\left(\frac{6}{5}\right)1.07^{t} = e^{0.0695t} \longrightarrow \ln\left(\frac{6}{5}(1.07)^{t}\right) = \ln\left(e^{0.0695t}\right)$$

$$\ln\left(\frac{6}{5}\right) + \ln(1.07^{t}) = 0.0695t \rightarrow$$

$$\ln\left(\frac{6}{5}\right) + t\ln(1.07) = 0.0695t \qquad t = \frac{\ln\left(\frac{6}{5}\right)}{0.0695 - \ln 1.07}$$

$$\ln\frac{6}{5} = 0.0695t - t.\ln 1.07$$

$\cong 99$ years

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What continuously compounded rate is equivalent to 8% semi-annually

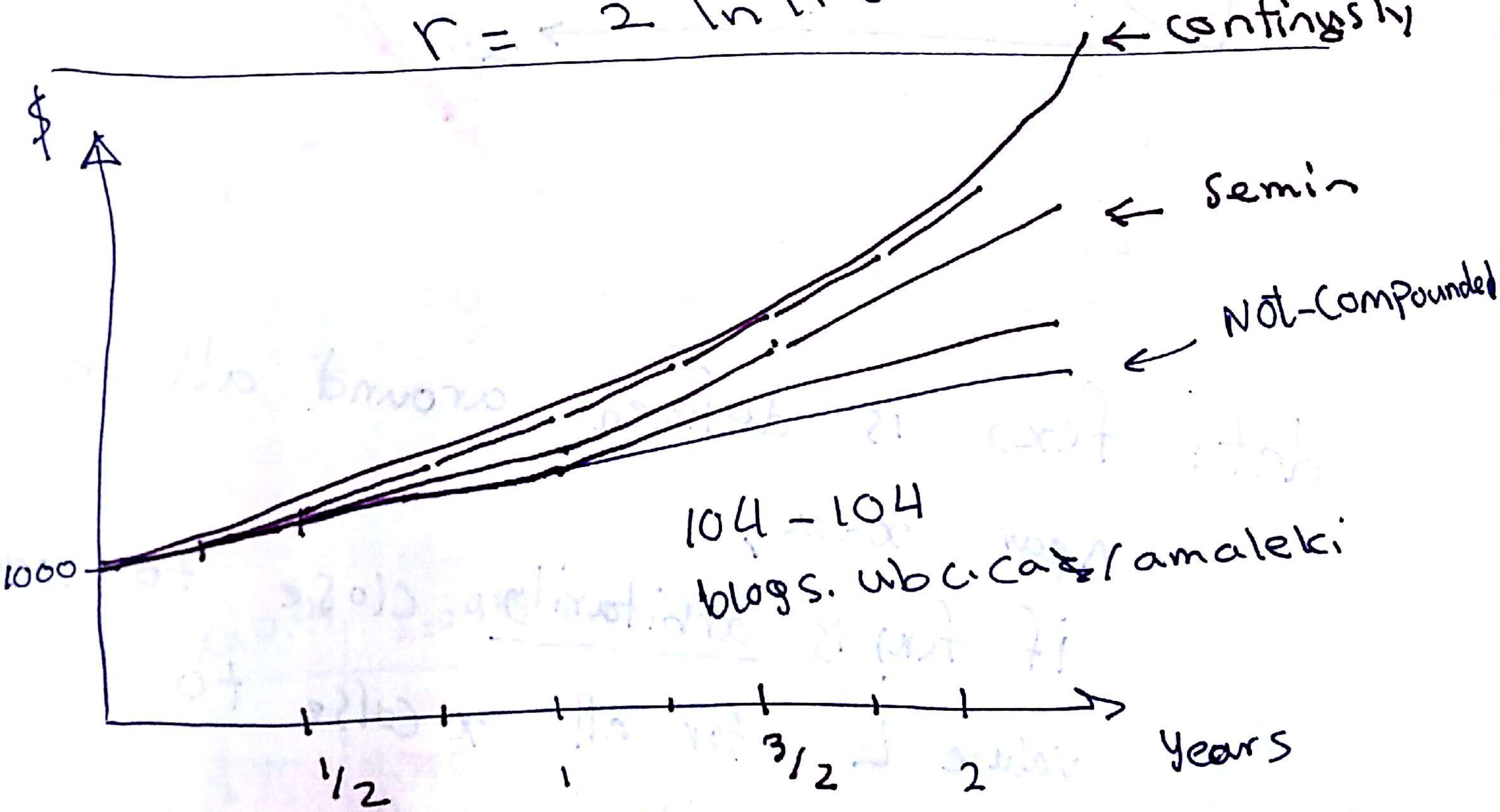
$$FV = \cancel{PV}\, e^{rt} = \cancel{PV}\left(1+\frac{0.08}{2}\right)^{2t}$$

$$e^{rt} = (1.04)^{2t} \longrightarrow$$

$$\cancel{\ln}(\cancel{e}^{rt}) = \ln(1.04^{2t})$$

$$r\cancel{t} = 2\cancel{t}\ln 1.04$$

$$r = 2\ln 1.04 = 7.84\%$$



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2/x

`$$\frac{2}{x}$$` ← LaTex

$$y = \frac{1}{2} g t^2$$

y → distance from ground

g → gravitational Acceleration $9.8\ m/s^2$

t → time.

$$y = 4.9 t^2$$

$$\bar{V} = \text{Average velocity} = \frac{\text{distance travelled}}{\text{time passed.}}$$

$$\bar{V}_{0-1} = \frac{y(1) - y(0)}{1 - 0} = \frac{4.9 - 0}{1} = 4.9 \frac{m}{s}$$

$$\bar{V}_{1-2} = \frac{y(2) - y(1)}{2 - 1} = \frac{4.9(2)^2 - 4.9}{1} = 14.7 \frac{m}{s}$$

$x=1$

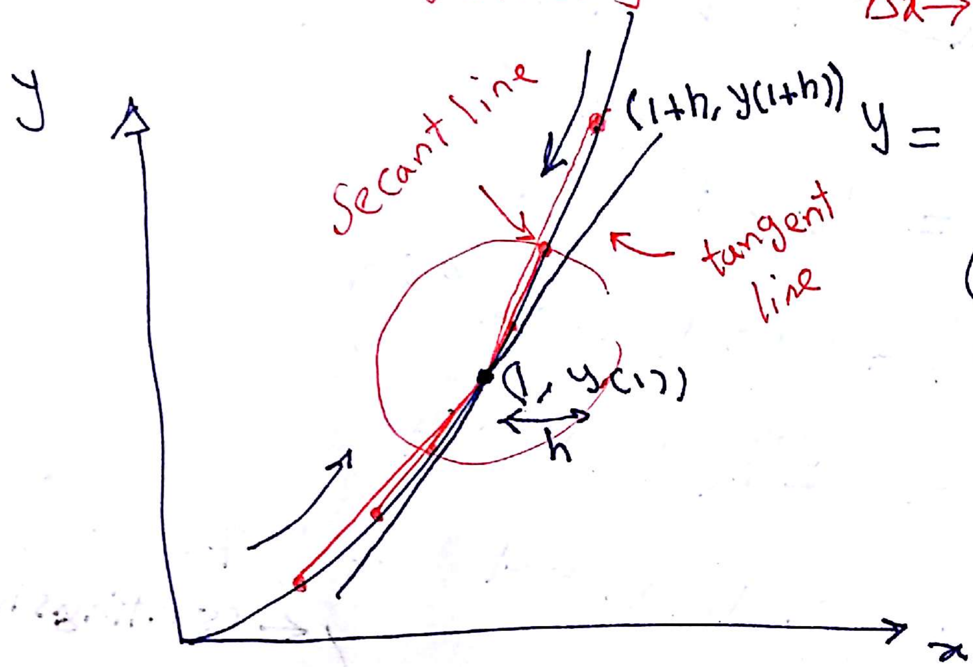
$x=0.5$	$x=0.9$	$x=0.99$	$x=0.999$	$x=1.001$	$x=1.01$	$x=1.1$	$x=1.5$	$x=2$
7.35	9.31	9.75	9.7951	9.8049	9.849	10.29	12.25	14.7

9.8 ← Instantaneous velocity

$$V_{.5-1} \quad V_{0.9-1} = \frac{y(1) - y(0.9)}{1 - 0.9} = \frac{4.9 - 4.9(0.9)^2}{0.1} = 9.31 \frac{m}{s}$$

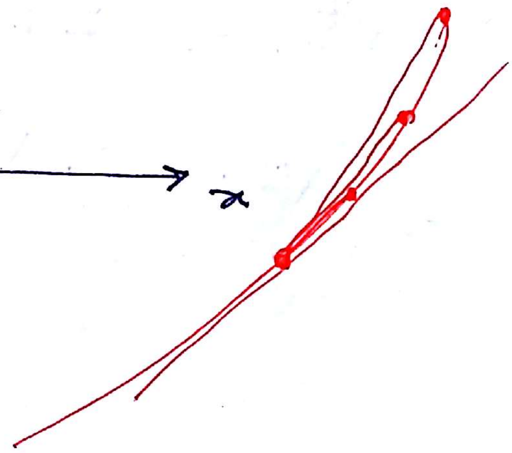
Average velocity $= \frac{\Delta y}{\Delta x}$

Instantaneous velocity $= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$



$y = 4.9x^2$

(x_1, y_1)
(x_2, y_2) $\rightarrow m = \frac{y_2 - y_1}{x_2 - x_1}$



def: $f(x)$ is defined around all x near $x=a$,

if $f(x)$ is <u>arbitarily</u> close to some value L for all x colse to (but not including) $x=a$

we then say

$$\lim_{x \to a} f(x) = L$$