Sep 20

$$\lim_{h\to 0} \frac{\frac{1}{8+h} - \frac{1}{8}}{h} = \frac{\frac{1}{8} - \frac{1}{8}}{0} = \frac{0}{0} \quad \text{Simplify}$$

$$\frac{1}{8+h} - \frac{1}{8} = \frac{8 - (8+h)}{8(8+h)} = \frac{-h}{(8+h)8}$$

$$= \lim_{h\to 0} \frac{\frac{-h}{8(8+h)}}{\frac{h}{1}} = \lim_{h\to 0} \frac{-\cancel{h}}{8(8+h)\cancel{h}} = \lim_{h\to 0} \frac{-1}{8(8+h)}$$

$$= \frac{-1}{64}$$

$$\lim_{x\to 0} \frac{\sqrt{x+1} - 1}{x} = \frac{1-1}{0} = \frac{0}{0} \quad \text{Simplify}$$

$$(\sqrt{x+1} - 1)(\sqrt{x+1} - 1) = (\sqrt{x+1})^2 + 1^2 - 2\sqrt{x+1}$$

does not work because still have $\sqrt{}$

$$(a+b)(a-b) = a^2$$

$$(\sqrt{x+1} - 1)(\sqrt{x+1} + 1) =$$

$$(\sqrt{x+1})^2 - 1^2 = x+1-1 = x$$

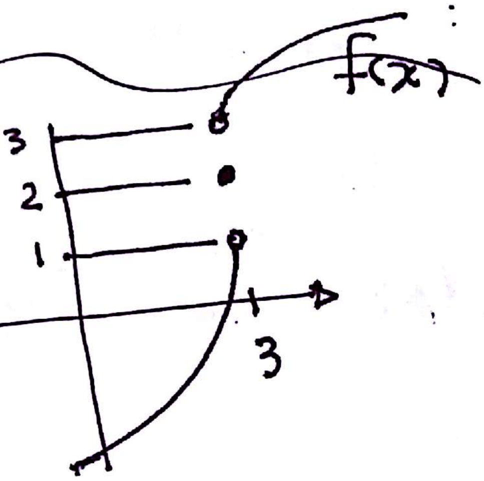
$$\sqrt{x+1}-1 = (\sqrt{x+1}-1)\frac{(\sqrt{x+1}+1)}{(\sqrt{x+1}+1)} = \frac{x}{\sqrt{x+1}+1}$$

$$\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x} = \lim_{x\to 0}\frac{\cancel{x}}{(\sqrt{x+1}+1)\cancel{x}} = \lim_{x\to 0}\frac{1}{\sqrt{x+1}+1} = \frac{1}{2}$$

left limit

$\lim_{x\to 3^-} f(x)$

approaching a number from all numbers on its left (smaller numbers)



righ lim

$\lim_{x\to a^+} f(x)$

limit ~~d~~f(x) when x approaching a from numbers that are on the right of a (larger than a)

$$\lim_{x\to 3^-} f(x) = 1 \qquad \lim_{x\to 3^+} f(x) = 3$$

limit of $\lim_{x\to a} f(x)$ exists if and only if

$$\lim_{x\to a^-} f(x) = \lim_{a^+} f(x)$$ exist

$$f(x) = \begin{cases} cx^3 - 2x & x > 3 \\ 2 & x = 3 \\ \dfrac{2x-6}{x^2-4x+3} & x < 3 \end{cases}$$

determine c such that $\lim\limits_{x \to 3} f(x)$ exists.

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3} cx^3 - 2x = 27c - 6$$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3} \frac{2x-6}{x^2-4x+3} = \frac{0}{0}$$

$$\lim_{x \to 3} \frac{2x-6}{x^2-4x+3} = \lim_{x \to 3} \frac{2\cancel{(x-3)}}{\cancel{(x-3)}(x-1)} = \frac{2}{3-1} = 1$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x) \Rightarrow 27c - 6 = 1$$

$$\boxed{c = \frac{7}{27}}$$

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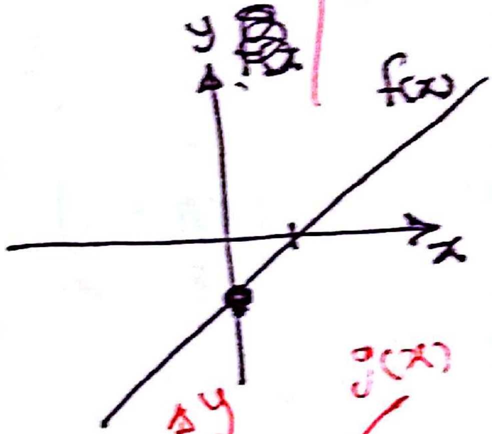
$$f(x) = \frac{x^3 - x^2}{x^2} \qquad D = \mathbb{R} - \{0\}$$

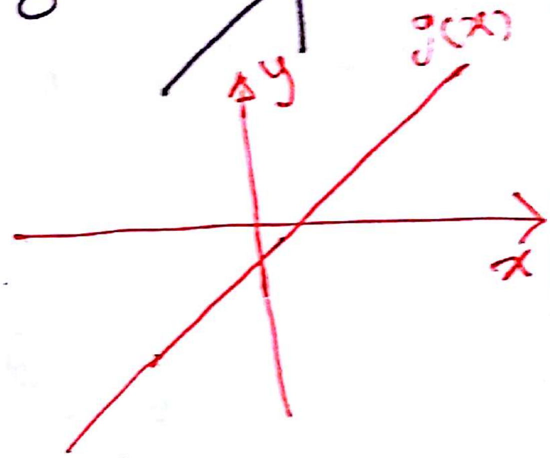
f is not continuous at $\boxed{x=0}$

$$= \frac{\cancel{x^2}(x-1)}{\cancel{x^2}} = x - 1$$

$$f(x) \begin{cases} x-1 & x \neq 0 \\ \text{undefined.} & x = 0 \end{cases}$$



$g(x) = x - 1$



Continuity

f(x) is continuous at $x = a$

if I) $\lim_{x \to a} f(x)$ exists

II) $f(a)$ exists

III) $\lim_{x \to a} f(x) = f(a)$

$\lim_{x \to a^-} f(x) = f(a)$ continuous from left

$\lim_{x \to a^+} f(x) = f(a)$ continuous from right necessary & sufficient condition for continuity is to have left and right continuity

$$f(x) = \begin{cases} x^2 - 2a & x < 1 \\ 1 & x = 1 \\ 2\sqrt{x} - c\sin\frac{\pi}{2}x & x > 1 \end{cases}$$

determine a & c, such that function is continuous at $x=1$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} x^2 - 2a = 1 - 2a$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} 2\sqrt{x} - c\sin\frac{\pi}{2}x = 2 - c$$

functions are continuous in their domain

1) Polynomial

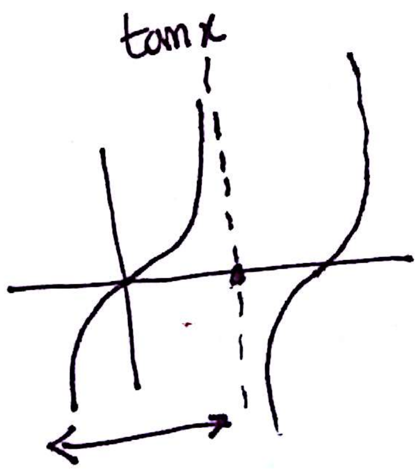
2) e^x

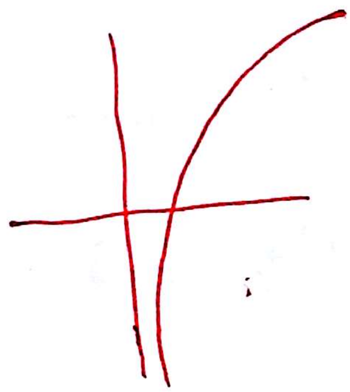
3) a^x, $a > 0$

4) $\sin x$, $\cos x$

5) $\tan x$, $\cot x$

6) $\ln x$





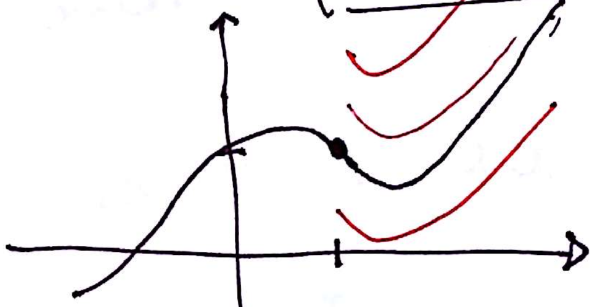
$f(1) = 1$

left Continuity $\lim_{x \to 1^-} f(x) = f(1)$

$1 - 2a = 1 \Rightarrow \boxed{a = 0}$

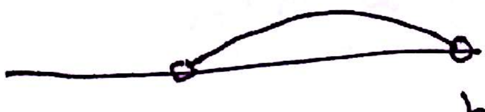
right Continuity $\lim_{x \to 1^+} f(x) = f(1)$

$2 - c = 1 \Rightarrow \boxed{c = 1}$

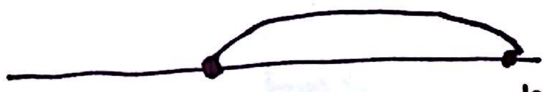


Continuity on Interval.

(a, b)



$[a, b]$



function $f(x)$ is continuous on (a, b) if $f(x)$ is Continuous for all x $a < x < b$

function f is continuous on $[a,b]$ if f is continuous on all x $a<x<b$, and f is continuous from right at $x=a$, f is continuous from left at $x=b$

$(a,b]$

$[a,b)$

Laws of continuity

$f(x)$ and $g(x)$ continuous at $x=a$

I) $f \cdot g$ is also continuous at $x=a$

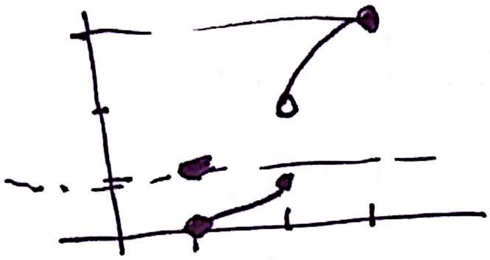
II) $f+g$ " "

III) $f-g$ " "

IV) f/g " " if $g(a) \neq 0$

$f(x)$ is continuous at $x=a$, $g(x)$ is continuous at $x=f(a)$, then ~~$f(g(x))$~~ $g(f(x))$ is continuous at $x=a$.

$h(x) = \sin(x^2-1)$ $\longrightarrow$ $f(x) = x^2-1$, $g(x) = \sin(x)$, $h(x) = g(f(x))$ $\rightarrow$ $h(x)$ is continuous every where draw a function

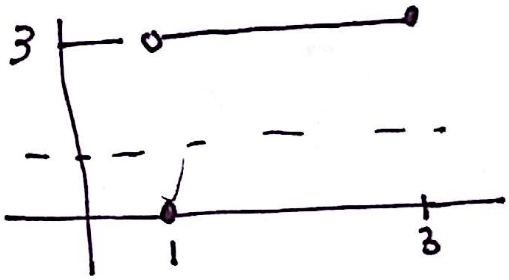


$f(1) = 0$

$f(3) = 3$

$f(x) = 1$ does not have a solution

make continuous $(1, 3)$.



make continuous $[1, 3]$

No answer.

by Intermediate value theorem