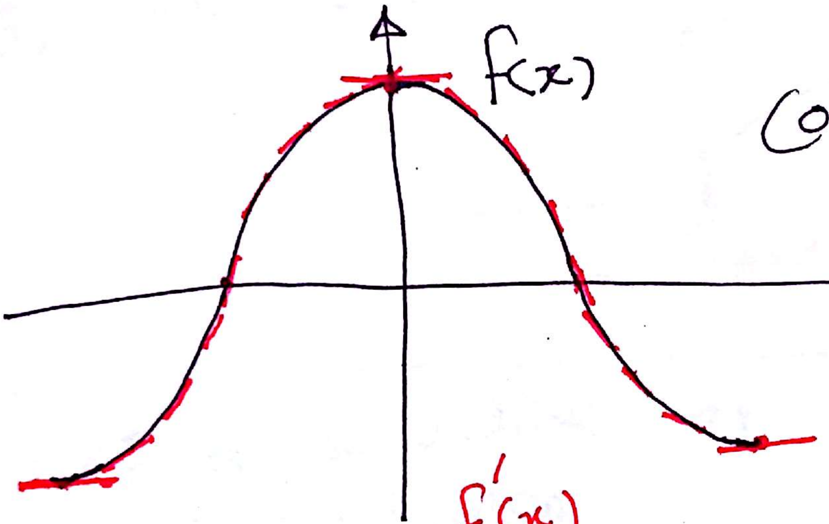
Sep 27.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad \frac{0}{0}$$

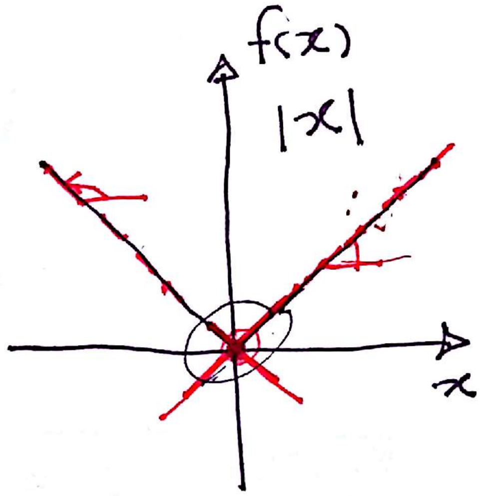
slope of tangent line.

instantaneous velocity (x time)

Rate of change.

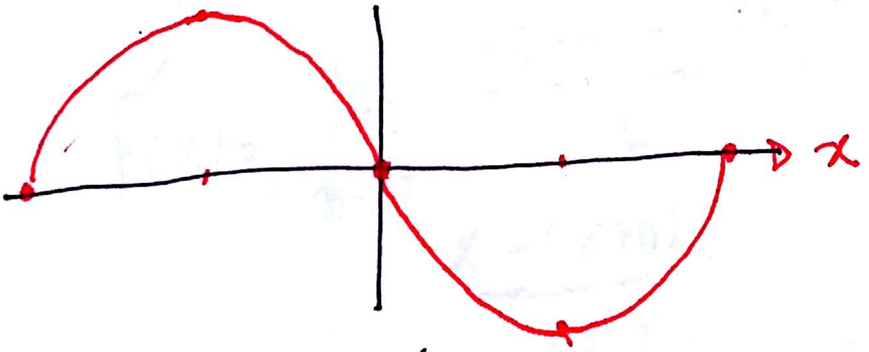


Cos(x)

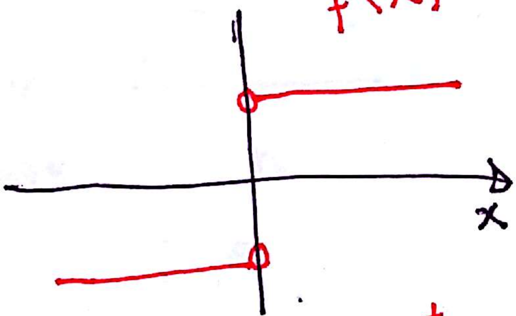


f'(x)

- Sin(x)

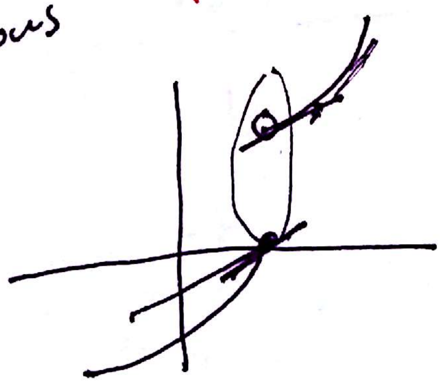


f'(x)



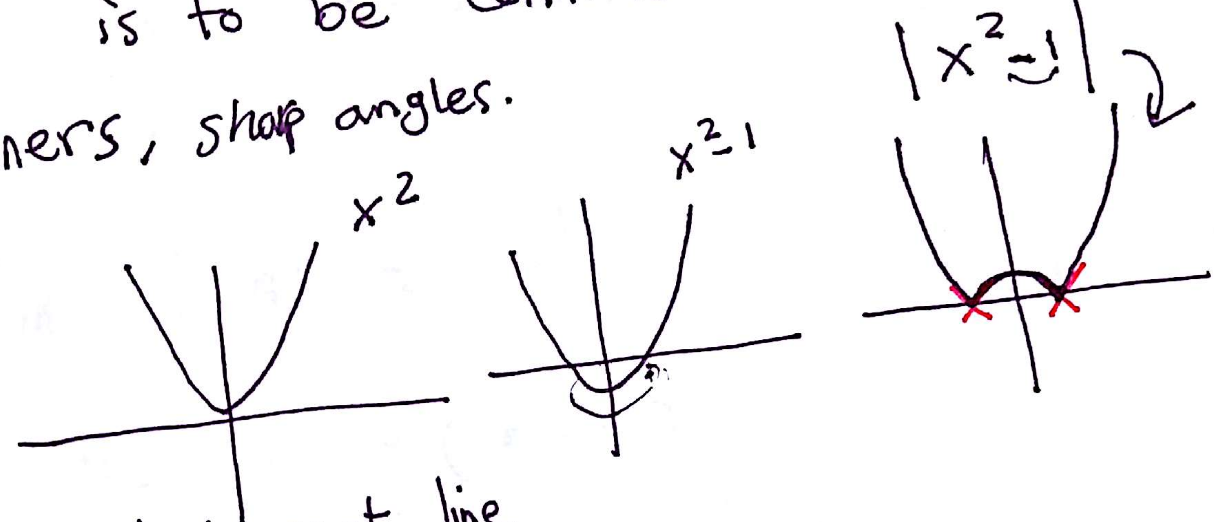
x = 0 is not differentiable

1) Not Continuous

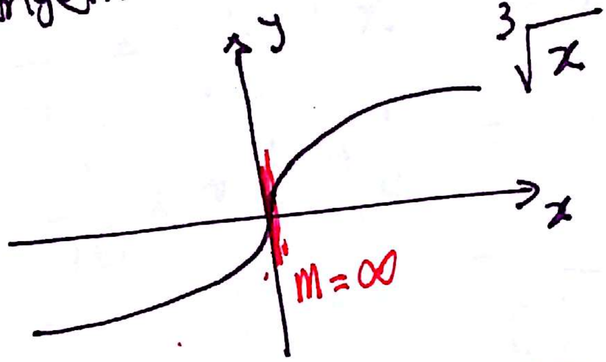


necessary condition for having derivative is to be continuous.

2) corners, sharp angles.



3) vertical tangent line



- Find derivative of $f(x) = \frac{1}{x}$ using limit definition

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x-(x+h)}{(x+h)\,x}}{h} = \lim_{h \to 0} \frac{\frac{-h}{(x+h)\,x}}{\frac{h}{1}} =$$

$$\lim_{h \to 0} \frac{-\cancel{h}}{(x+h)\,x\,\cancel{h}} = \lim_{h \to 0} \frac{-1}{(x+h)\,x} = \frac{-1}{x^2}$$

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find equation of tangent line at point $x = 4$ on $f(x) = \frac{1}{x}$

$\left(4, \frac{1}{4}\right)$;

$m = f'(4) = \frac{-1}{16}$

slope

$y - y_0 = m(x - x_0)$

$y - \frac{1}{4} = \frac{-1}{16}(x - 4) \Rightarrow$

$y = \frac{-1}{16}x + \frac{1}{4} + \frac{1}{4} = \frac{-1}{16}x + \frac{1}{2}$

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$1 + \tan^2 x$
e^x	e^x
a^x $(a>0)$	$a^x \ln(a)$

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laws of derivative:

if f' and g' exists

$$(f+g)' = f' + g'$$

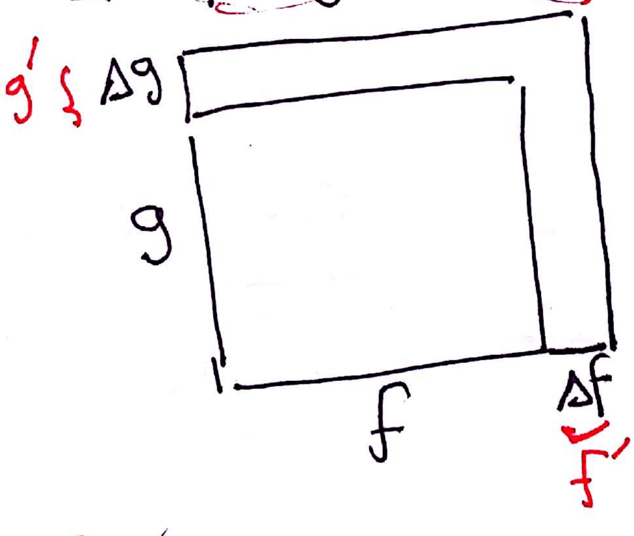
$$(f-g)' = f' - g'$$

$$(cf)' = c\,f'$$ c is a constant

Product rule =

$$(f\cdot g)' \neq f'\cdot g'$$

$$= f'\cdot g + f\cdot g'$$



$$S = f\cdot g$$

$$S_{New} = (f+\Delta f)(g+\Delta g) = \cancel{fg} + f\Delta g + \Delta F g + \Delta f \Delta g$$

$$\Delta S = S_{New} - S = f\Delta g + g\Delta f + \underbrace{\cancel{\Delta f \Delta g}}_{0.01\ 0.01}$$

0.0001

ONe order of mgnitude smaller

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Quotient rule:

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$f(x) = x|x|$. find derivative of $f(x)$ at $x = 0$

$$\cancel{f'(x) = x'|x| + x(|x|)'}$$

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} =$$

$$\lim_{h\to 0} \frac{\cancel{h}|h| - 0}{\cancel{h}} = \lim_{h\to 0} |h| = 0$$

Product rule does Not apply because derivative of $|x|$ does not exist. therefore take limit definition

Practice Problem Math 104,
Name: -------------------------------

Find the derivative of

a) $y = x^{45} - x^{-45}$

$(x^{45} - x^{-45})' = (x^{45})' - (x^{-45})' = 45x^{44} + 45x^{-46}$

$n=45 \quad n=-45$

b) $h(x) = e^x + \cos(x) - 2x\sqrt{x}$

$(e^x)' + (\cos(x))' - (2x\sqrt{x})' = e^x - \sin x - 3x^{1/2}$

c) $f(x) = x\sin x$

$u = x \quad u' = 1$

$v = \sin x \quad v' = \cos x$

$x\cos x + \sin x$

d) $f(x) = \left(2\sqrt{x} + \frac{3}{x}\right)\left(3\sqrt{x} - \frac{2}{x^2}\right)$

$\left(2\sqrt{x} + \frac{3}{x}\right)\left(3\sqrt{x} - \frac{2}{x^2}\right)' + \left(2\sqrt{x} + \frac{3}{x}\right)'\left(3\sqrt{x} - \frac{2}{x^2}\right)$

$\left(2\sqrt{x} + \frac{3}{x}\right)\left(\frac{3}{2\sqrt{x}} + 4x^{-3}\right) + \left(\frac{\cancel{2}}{\cancel{2}\sqrt{x}} + \frac{-3}{x^2}\right)\left(3\sqrt{x} - \frac{2}{x^2}\right)$

e) $g(x) = \frac{x\sqrt{x}-3}{x^2}$

$\sqrt{x} = x^{1/2}$

$(\sqrt{x})' = \frac{1}{2}x^{\frac{1}{2}-1}$

$= \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2x^{\frac{1}{2}}}$

$= \frac{1}{2\sqrt{x}}$

f) $f(t) = \frac{\pi}{2-\pi t}$

$\left(-\frac{2}{x^2}\right)' = (-2x^{-2})' = 4x^{-3}$

$\left(\frac{3}{x}\right)' = (3x^{-1})' = -3x^{-2}$

$= \frac{-3}{x^2}$

$(2\sqrt{x})' = (2x^{\frac{1}{2}})' =$

$2(x^{\frac{1}{2}})' = 2\,\frac{1}{2}\,x^{\frac{1}{2}-1} = x^{-1/2} = \frac{1}{\sqrt{x}}$

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Practice Problem Math 104,
Name: ---------------------------------

Find the derivative of

a) $y = x^{45} - x^{-45}$

b) $h(x) = e^x + \cos(x) - 2x\sqrt{x}$

c) $f(x) = x \sin x$

d) $f(x) = \left(2\sqrt{x} + \frac{3}{x}\right)\left(3\sqrt{x} - \frac{2}{x^2}\right)$

$$\frac{x\sqrt{x}-3}{x^2} = \frac{x\sqrt{x}}{x^2} - \frac{3}{x^2} = x^{-1/2} - 3x^{-2} \Rightarrow$$

$$f'(x) = -\frac{1}{2}x^{-3/2} + 6x^{-3} = \frac{6}{x^3} - \frac{1}{2x\sqrt{x}}$$

e) $g(x) = \frac{x\sqrt{x}-3}{x^2}$ $\quad u \quad v$ $\quad = \left(\frac{x^{\frac{3}{2}}-3}{x^2}\right)$

$$\frac{(x^{3/2}-3)'\,x^2 - (x^{3/2}-3)(x^2)'}{(x^2)^2} = \frac{\left[\left(\frac{3}{2}x^{\frac{1}{2}}\right)(x^2)\right] - \left[(2x)(x^{\frac{3}{2}}\right.}{x^4}$$

f) $f(t) = \frac{\pi}{2-\pi t}$ $= \frac{(\pi)'(2-\pi t) - (\pi)(2-\pi t)'}{(2-\pi t)^2}$

$$= \frac{0 - (\pi)(0-\pi)}{(2-\pi t)^2}$$

$$= \frac{\pi^2}{(2-\pi t)^2}$$

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Practice Problem Math 104,

Name: --------------------------------

Find the equation of tangent line and normal line to the curve of $y = \frac{2}{3-4\sqrt{x}}$ at point $x = 1$ on the curve.

$f(1) = \frac{2}{3-4\sqrt{1}}$

$\left(1, \frac{2}{3-4\sqrt{1}}\right)$

$y = 4(x-1) + \frac{2}{3-4\sqrt{1}}$

$y = 4(x-1) - 2$

$\cancel{f'(1)} =$

$f'(x) = \frac{(3-4\sqrt{x})(2)' - 2(3-4\sqrt{x})'}{(3-4\sqrt{x})^2}$ $\quad -4x^{\frac{1}{2}} \quad -2x^{-\frac{1}{2}}$

$f'(x) = \frac{0 - 2(-2x^{-\frac{1}{2}})}{(3-4\sqrt{x})^2}$

$f'(1) = \frac{-2(-2)}{(-1)^2}$

$f'(1) = 4$

Find the derivative of $f(x) = x|x|$ at $x=0$.

$(2x)' = 2$