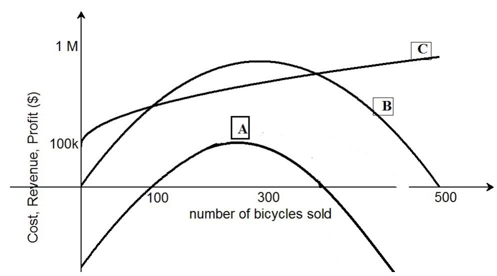
First Name: ______________ Last Name: ______________

Student-No: ______________ Section: ______________

Short answer questions

1. [2 marks] Each part is worth 1 mark.

   (a) A company manufactures bicycles. The figure below shows Cost, Revenue and Profit as a function of number of bikes sold. Mark these functions appropriately (for example, What does graph A represent? Cost, Revenue or Profit? and so on for graph B and C).



Answer: A= Profit, B= Revenue and C = Cost

Solution: Cost and Revenue cannot be negative. Cost is the only function that can positive when $q = 0$.

   (b) Compute $\lim_{t\to 0} \frac{\sqrt[3]{-1-t^2}}{t^3 - e^t}$. If limit does not exist, write DNE.

Answer: 1

Solution: $\lim_{t\to 0} \frac{\sqrt[3]{-1-t^2}}{t^3 - e^t} = \frac{\sqrt[3]{-1}}{0-1} = \frac{-1}{-1} = 1$

Long answer questions — you must show your work

2. [2 marks] Amir has an offer from his bank to change his current investment plan with annual interest rate of 12% compounded continuously to a new plan with annual interest rate of r compounded semi-annually. What should the minimum r be so that he at least makes the same amount of money?

Answer: $i = 2\left(e^{0.06} - 1\right)$

Solution: We use $FV = PVe^{0.12t} = PV(1 + r/2)^{2t}$ to get

$$e^{0.12t} = (1 + \frac{r}{2})^{2t} \Rightarrow 0.12t = 2t\ln(1 + \frac{r}{2}) \Rightarrow$$
$$0.06 = \ln(1 + \frac{r}{2}) \Rightarrow e^{0.06} = 1 + \frac{r}{2} \Rightarrow$$
$$r = 2\left(e^{0.06} - 1\right)$$

3. 2 marks Compute the limit $\lim_{x \to 1} \frac{x^4 - 1}{2x^2 + 4x - 6}$

Answer: $\frac{1}{2}$

Solution: Direct substitution yields 0/0, so we simplify first:

$$\frac{x^4 - 1}{2x^2 + 4x - 6} = \frac{(x^2 - 1)(x^2 + 1)}{2(x+3)(x-1)} = \frac{(x-1)(x+1)(x^2+1)}{2(x+3)(x-1)} = \frac{(x+1)(x^2+1)}{2(x+3)}$$

Hence the limit is $\lim_{x \to 1} \frac{(x+1)(x^2+1)}{2(x+3)} = \boxed{1/2}$.

4. EatPumpkin is a new chain restaurant with its famous pumpkin appetizer. It is found that if the price of pumpkin appetizer is \$8 each, an average of 60 people order the dish each day. When it drops the price of the appetizer to \$5, the number ordering it rises to 75. Assume that the demand q is a linear function of the price p and each appetizer costs the restaurant \$3 (neglect constant cost).

(a) 2 marks Find the linear demand equation as a function of price (p)

Answer:

$$p = \frac{-1}{5}q + 20$$

Solution: A data point is (q, p) = (quantity, price). So the demand curve will be given by $p = mq + b$ and will have slope $m = \frac{rise}{run} = \frac{\Delta p}{\Delta q} = \frac{-3}{15} = \frac{-1}{5}$. So the equation becomes

$$p = \frac{-1}{5}q + b.$$

Substituting in the point $q = 60$ and $p = 8$ yields:

$$\begin{aligned} p &= \frac{-1}{5} \cdot q + b \\ 8 &= \frac{-1}{5} \cdot 60 + b \\ 8 &= -12 + b \\ 20 &= b \end{aligned}$$

Therefore,

$$p = \frac{-1}{5}q + 20$$

(b) 2 marks Find the weekly profit function $P(q)$.

Answer: $P(q) = \frac{-7}{5}q^2 + 119q$

Solution: We'll need Cost': $C(q) = 3q$
and Revenue: $R = p \cdot q = (\frac{-1}{5}q + 20)q = \frac{-1}{5}q^2 + 20q$
From which we see that:
$P(q) = R(q) - C(q) = \frac{-1}{5}q^2 + 20q - 3q = \frac{-1}{5}q^2 + 17q$ so weekly profit is $\frac{-7}{5}q^2 + 119q$

(c) bonus 1 marks What is the optimized price for the pumpkin appetizer?

Answer: $p =$\$11.5

Solution: $P(q) = \frac{-1}{5}q^2 + 17q = -\frac{1}{5}q(q - 85)$
Because of symmetry of quadratic function, we know $P(q)$ is maximized in between the two roots.The two roots are at $q = 0$ and $q = 85 \Rightarrow q_{max} = 42.5$ which is equivalent to $p =$\$11.5.