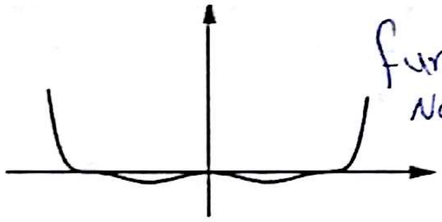
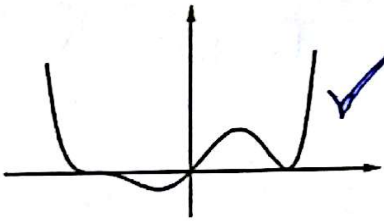
1. Which graph can be the plot of function

$$f(x) = x(x-1)^2(x+1)^3\ ?$$

$f(x)=0 \rightarrow x=0, 1, -1$



function is Not even X $f(-x) \neq f(x)$



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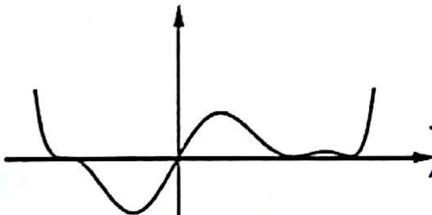
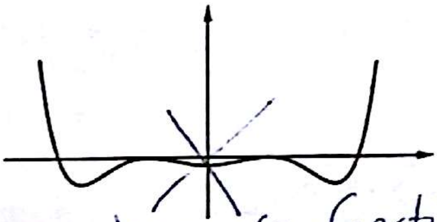


X function has only three x intercepts



because function Passing though origin $f(0)=0$

2. For what values of a, function $f(x) = x^3 + ax^2 + x$ is increasing *everywhere?*

$f'(x)$ must be Positive because $f'(x)$ exists everywhere.

$$f'(x) = 3x^2 + 2ax + 1$$

for $f'(x)$ to be Positive everywher, it should not have any root $\Rightarrow$ $\Delta = b^2 - 4ac \leq 0 \rightarrow$

$(2a)^2 - 4(3)(1) \leq 0 \rightarrow 4a^2 \leq 12$

$\rightarrow a^2 < 3 \rightarrow -\sqrt{3} \leq a \leq \sqrt{3}$

Note because 3 > 0, the function $f'(x)$ is always positive

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3. Find the interval(s) over which $f(x) = (x-1)^3(x+1)$ is decreasing.

$$f'(x) = 3(x-1)^2(x+1) + (x-1)^3 = (x-1)^2(3x+3+x-1) = (x-1)^2(4x+2)$$

$f'(x)=0 \to x=1$

$x = -\frac{1}{2}$

	$-\infty$ to $-\frac{1}{2}$	$-\frac{1}{2}$ to 1	1 to ∞
$4x+2$	$-$	$+$	$+$
$(x-1)^2$	$+$	$+$	$+$
f'	$-$	$+$	$+$

on $(-\infty, -\frac{1}{2})$, f' is decreasing

4. Function $f(x)$ is positive and *strictly* decreasing everywhere. Which one of these functions is increasing? $\to f(x) > 0 \longrightarrow f'(x) < 0$

a. $\frac{1}{f(x)}$ b. $\sqrt{f(x)}$ c. $f^3(x)$ d. $f(x^2)$

a) $g = \frac{1}{f(x)} \to g' = -\frac{f'(x)}{f^2} = -\frac{<0}{>0} \Rightarrow g' > 0 \to g$ is increasing.

b) $g = \sqrt{f(x)} \to g' = \frac{f'}{2\sqrt{f}} = \frac{<0}{>0} \Rightarrow g' < 0 \to g$ is decreasing

c) $g = f^3 \to g' = 3f^2 f' < 0 \Rightarrow g' < 0 \longrightarrow$ " "

d) $g = f(x^2) \to g' = 2x f'(x^2) \longrightarrow$ for $x < 0 \to g' > 0$ increasing
for $x > 0 \to g' < 0$ decreasing

5. For what values of a the function

$$f(x) = \frac{ax-5}{x+a-6}$$

is increasing for all $x > 1$?

$$f'(x) = \frac{a(x+a-6) - (ax-5)}{(x+a-6)^2} = \frac{ax+a^2-6a-ax+5}{(x+a-6)^2} = \frac{a^2-6a+5}{(x+a-6)^2}$$

denom is always positive $\to$ We determine sign of Numer. for different values of a. first find where $f' = 0$

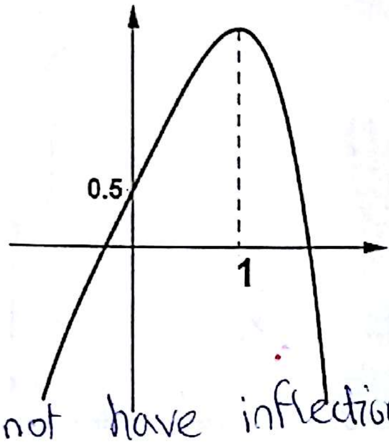
$a^2 - 6a + 5 = 0 \to (a-5)(a-1) = 0 \to a = 1, a = 5$

a	$-\infty$ to 1	1 to 5	5 to ∞
	$+$	$-$	$+$

for $a < 1$ or $a > 5$ the function ~~is increasing~~

'Note function will have a vertical asymptote at $x_p = 6-a$ and for $a < 1$ $x_p = 6 - a > 5 \Rightarrow$ the function is not increasing for all values of $x > 1 \to$ only accept $a > 5$

6. The figure below is the graph of function $f(x) = ax^4 + bx^3 + 2x + c$. Find the coefficients a, b and c and determine the value of local maximum.



y intercept at $y=0.5 \rightarrow$

$x=0,\ y=0.5 \rightarrow \boxed{C=0.5}$

local max at $x=1 \rightarrow f'(1)=0$

$\cancel{4ax^3+3bx}$ $\quad f' = 4ax^3+3bx^2+2$

$f'(1) = 4a+3b+2=0$

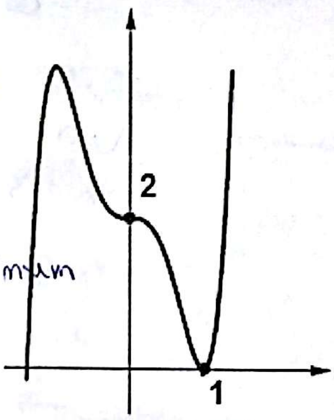
$\rightarrow 4a+3b+2=0$

f(x) does not have inflection point $\Rightarrow f''>0$ or $f''<0$

$f'' = 12ax^2+6bx = (12ax+6b)x$. f'' should not chang sign $\rightarrow$ we must have $6b=0 \rightarrow f''=12ax^2$ never changes sign

$\boxed{b=0} \rightarrow 4a+2=0 \rightarrow \boxed{a=-\frac{1}{2}}$

7. The graph below is function of $f(x) = 3x^5 + ax^3 + bx^2 + cx + d$. Find all coefficients a, b, c and d.



$f(0)=2 \rightarrow \boxed{d=2}$

$f(1)=0 \rightarrow 3+a+b+c+d=0$

$\rightarrow 3+a+b+c+2=0$

$a+b+c=-5$

$f'(1)=0 \Leftrightarrow x=1$ local minimum

$f'(x)=15x^4+3ax^2+2bx+c$

$f'(1)=15+3a+2b+c=0$

$x=0$ is inflection point $\Rightarrow$

$f''(0)=0$

$f''=60x^3+6ax+2b$

$f''(0)=2b=0 \rightarrow \boxed{b=0}$

$\left.\begin{array}{l} a+c=-5 \\ 3a+c=-15 \end{array}\right\} \Rightarrow 2a=-10$

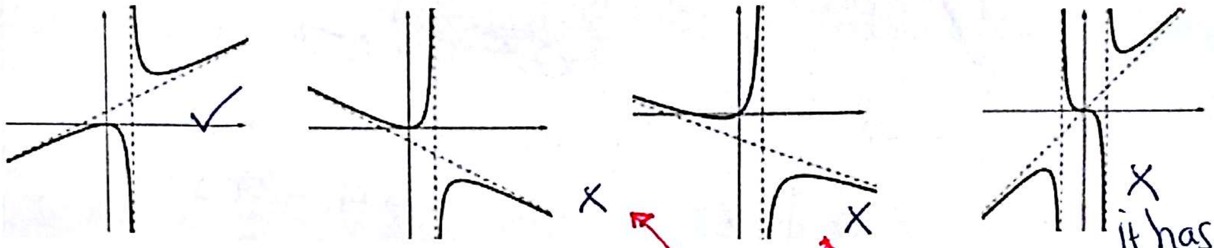
$\Downarrow$

$\boxed{a=-5}$

$\boxed{c=0}$

8. Which graph can belong to the function

$$f(x)=\frac{x^2}{2x-4}$$



it has only one vert. Asympt

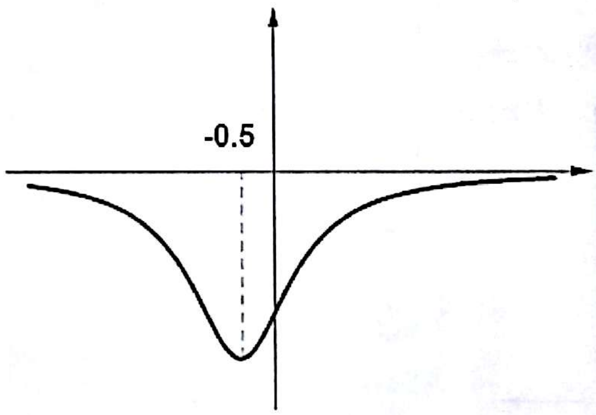
vertical asymptote at $x=2$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x^2}{2x-4} = \frac{4}{0^+} = \infty$

9. The graph of function

$$f(x)=\frac{ax^2+bx-2}{x^2+cx+1}$$

is shown in figure below. Find the values of a, b and c.



from figure, Horizontal asymptote $y=0$

$\Rightarrow \lim_{x\to\infty} \frac{ax^2+bx-2}{x^2+cx-1}=0$

$\to \lim_{x\to\infty} \frac{ax^2}{x^2}=0$

$\boxed{a=0}$ $\to$

No x-intercept $\to$ $\boxed{b=0}$ $\to$ if $b\neq 0$, there will x-intercept at $x=\frac{2}{b}$

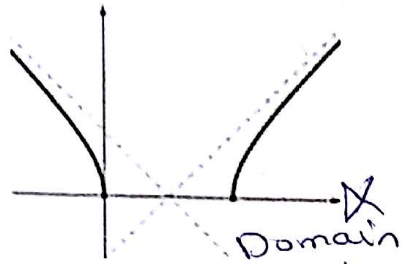
$\to f(x)=\frac{-2}{x^2+cx+1}$ $\quad f'(x)=\frac{2(2x+c)}{(x^2+cx+1)^2}$ $\quad f'(-0.5)=0$

local min at $x=-0.5 \to f'(-0.5)=0 \to$

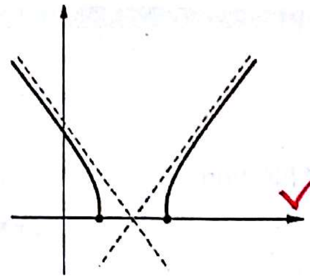
$\to -1+c=0 \to \boxed{c=1}$

10. Which one can be the graph of function $f(x) = \sqrt{x^2 - 3x + 2}$

Domain
$x^2 - 3x + 2 = (x-1)(x+2) \geq 0$
$\Downarrow$
$x \leq 1$ or $x \geq 2$
(use table if necessary)



X
Domain Not correct



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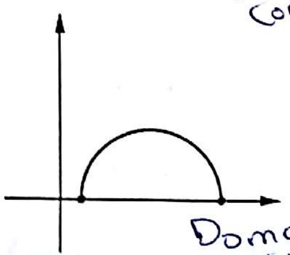
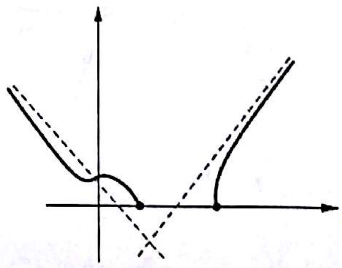
$f'(x) = \dfrac{2x-3}{2\sqrt{x^2-3x+2}}$



X
Domain Not correct



X

$f'(x) = 0 \rightarrow x = 1.5$
$\downarrow$
Not in domain
$\rightarrow$ No local max
local min

11. Sketch the curve of function

$f(x) = e^{2x-x^2}$

Domain $\mathbb{R}$

$f(-x) = e^{-2x-x^2} \neq f(x)$ or $-f(x) \rightarrow$ No Symmetry

No vertical asymptote

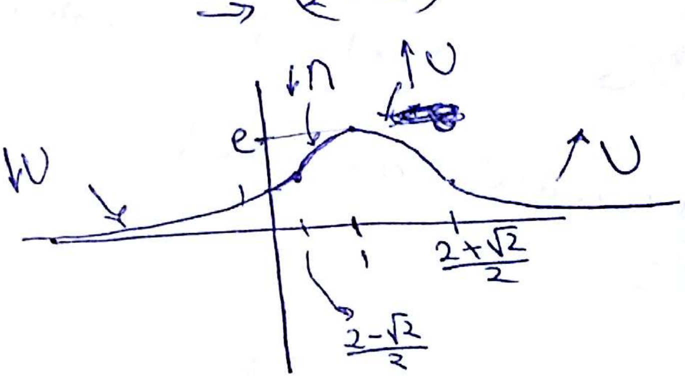
$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} e^{2x-x^2} = e^{-\infty} = 0 \rightarrow$ $y = 0$ Horizontal asymptote.

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} e^{2x-x^2} = e^{-\infty} = 0 \rightarrow$

$f'(x) = (2-2x)e^{2x-x^2} = 0 \rightarrow x = 1$ $\rightarrow$ f': $+$ $\nearrow$ | 1 | $-$ $\searrow$

$f''(x) = -2e^{2x-x^2} + (2-2x)^2 e^{2x-x^2} = (e^{2x-x^2})((2-2x)^2 - 2) = 0$

$\rightarrow (2-2x)^2 - 2 = 0 \rightarrow 2-2x = \pm\sqrt{2} \rightarrow x = \dfrac{2 \pm \sqrt{2}}{2}$
inflection points



f'': $+$ $\cup$ | $\frac{2-\sqrt{2}}{2}$ | $-$ $\cap$ | $\frac{2+\sqrt{2}}{2}$ | $+$ $\cup$

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12. Sketch the curve of function

$$f(x) = \ln(\frac{x}{x-1})$$

Domain: $\frac{x}{x-1} > 0$

		0		1	
x	$-$		$+$		$+$
$x-1$	$-$		$-$		$+$
$\frac{x}{x-1}$	$+$	0	$-$		$+$

$\Rightarrow$ Domain $x < 0$ or $x > 1$

No Symmetry

$x = 0$ is not in domain $\to$ No y-intercept

$f(x) = 0 \to \frac{x}{x-1} = 1 \to x = x-1 \to 0 = -1$ Not Possible

No x-intercept

$x = 1$ can be Vertical asymptote because it makes denom $= 0$

$x = 0$ " " " it makes $\log = 0$

see domain

$\lim_{x \to 1^+} \ln \frac{x}{x-1} = \lim \ln \frac{1}{0^+} = \ln(\infty) = \infty$

$\lim_{x \to 0^-} \ln \frac{x}{x-1} = \ln \frac{0^-}{-1} = \ln(0^+) = -\infty$

$\lim_{x \to \infty} \ln \frac{x}{x-1} = \lim_{x \to \infty} \ln \frac{x}{x} = \ln 1 = 0$

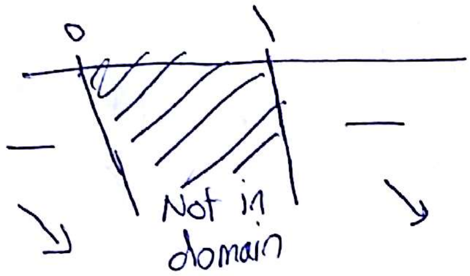
$\lim_{x \to -\infty} \ln \frac{x}{x-1} = \lim_{x \to -\infty} \ln \frac{x}{x} = \ln 1 = 0$

$x = 1$ and $x = 0$ vertical asymptotes

$y = 0$. Horizontal asymptote

$$f'(x) = \frac{\frac{(x-1)-x}{(x-1)^2}}{\frac{x}{x-1}} = \frac{\frac{-1}{(x-1)^2}}{\frac{x}{x-1}} = \frac{-1}{x(x-1)} =$$

f'

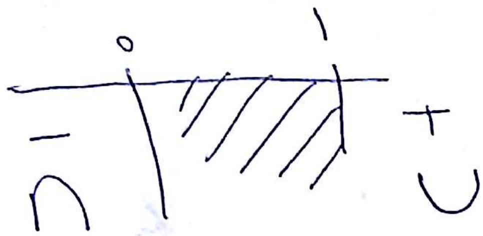


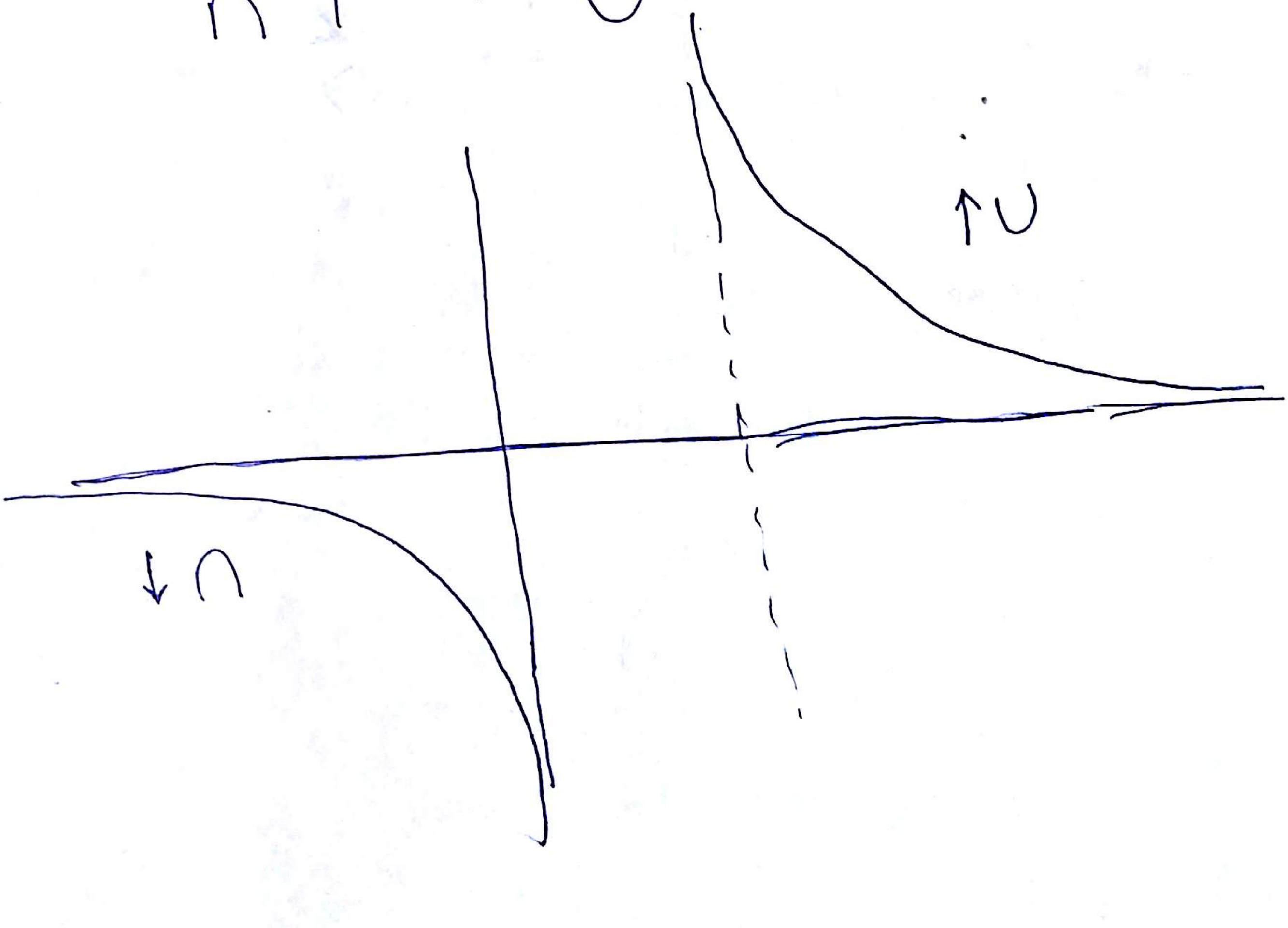
f is ~~decr~~ decreasing in its domain

$$f'' = \frac{+2x-1}{(x^2-x)^2} = 0 \rightarrow x = \frac{1}{2} \text{ Not in domain}$$

$x^2 - x$

f''

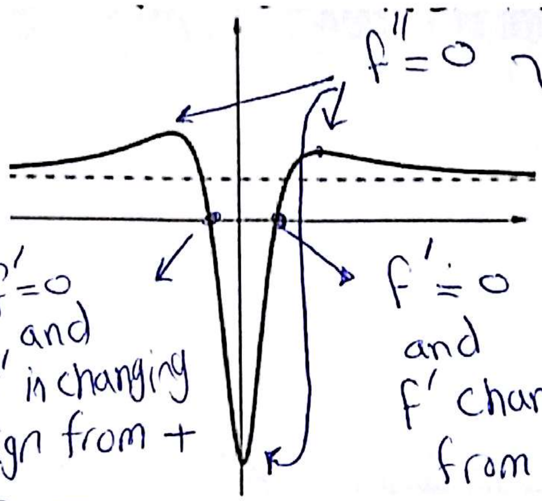




13. Figure below shows the graph of $f'(x)$. How many local maxima, local minima and inflection point does function $f(x)$ have?

$f'(x)=0$

$f''=0 \rightsquigarrow f''$ changing sign (because it is a local max for f') $\Rightarrow$ inflection points



$f'=0$ and f' in changing sign from + to − $\Downarrow$ localmax

$f'=0$ and f' changing sign from − to + $\Downarrow$ local min

1 local max

1 local min

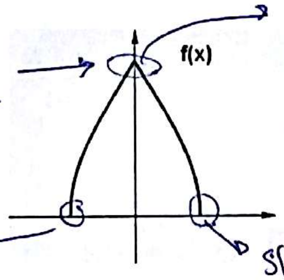
3 inflection points

14. Which figure can be the graph of derivative of function $f(x)$.

$f'(0^+) < 0$

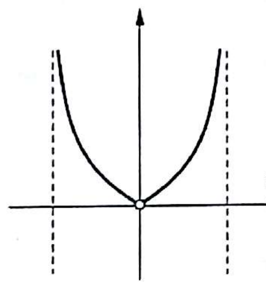
$f'(0^-) > 0$

f' doesont exist here

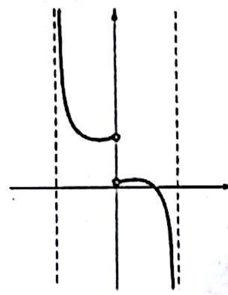


$f'=\infty$

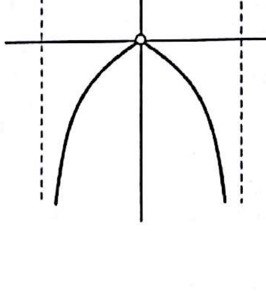
slope of tangent line $= -\infty$ $\Downarrow$ $f'=-\infty$



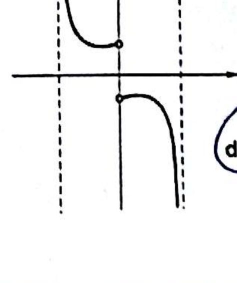
a)



b)



c)



d)