Long answer question — you must show your work

3. [4 marks] Consider the function $f(x) = \sqrt{x+9}$.

   (a) Find the 1st order Taylor polynomial $T_1(x)$ of $f(x)$ about $x = 0$.

   (b) Use $T_1(x)$ to approximate $\sqrt{10}$.

   (c) Find a bound for the absolute value of the error $|R_1|$ in the approximation. Justify your answer.

   (Note: You may leave your answers in calculator ready form.)

Name: _____ Student-No: _____

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

   (a) Suppose a particle's position is given by $s(t) = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t + 2016$. Over what time interval is the particle moving in the negative direction?

   Answer:

   (b) Estimate $\tan^2(0.01) + 0.01$ using a linear approximation.

   Answer:

Long answer question — you must show your work

3. [4 marks] You place a cone-shaped bottle on the table. It is 6cm high. Its circular base (which is resting on the table) has radius 3cm and its volume is $18\pi cm^3$. You fill it with water at rate of $1cm^3$ per second. What is the rate at which the depth of the water changes when it is 3cm deep?
   HINT: Think about the shape of the air in bottle.

Name: _____ Student-No: _____

Very short answer questions

1. [2 marks] Each part is worth 1 marks. Please write your answers in the boxes.

   (a) Evaluate the following limit by interpreting it as a derivative:

   $$\lim_{x \to \pi/4} \left(\frac{\tan(x) - 1}{x - \pi/4} \right)$$

   You may use the derivative rules we have learned so far in class — but do not use L'Hopital's rule.

   Answer:

   (b) Let $y = \arctan(\log(x))$. Compute $\frac{\mathrm{d}y}{\mathrm{d}x}$.
   (Recall: $\log x = \log_e x = \ln x$.)

   Answer:

Name: _____ Student-No: _____ Section: _____

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

   (a) Find $f'(x)$ if $f(x) = (x^2 + 1)^{\sin(x)}$.

   (b) Let $f(x)$ be a function differentiable at $x = 3$ and let $g(x) = x \cdot f(x)$. The line tangent to the curve $y = f(x)$ at $x = 3$ has slope 2 while the line tangent to the curve $y = g(x)$ at $x = 3$ has slope 5. What is $f(3)$?

Long answer question — you must show your work

3. [4 marks] If $x^2y + 4 = 4y^2e^x$, then find $\frac{dy}{dx}$ at all points where $x = 0$. You must justify your answer.

Name: _____ Student-No: _____ Section: _____

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

(a) Find the equation of the tangent line to the graph of $y = \tan(x)$ at $x = \dfrac{\pi}{3}$.

Answer:

(b) Find all values of c so that the following function is continuous at $x = 0$:

$$f(x) = \begin{cases} x^2 - c & x \geq 0 \\ \sin cx & x < 0 \end{cases}$$

Justify your answer using the definition of continuity.

Answer:

Long answer question — you must show your work

3. [4 marks] Use the definition of the derivative to determine whether the following function

$$f(x) = \begin{cases} \dfrac{x}{\cos x} & \text{if } x \leq 0 \\ \sqrt{1+2x} - 1 & \text{if } x > 0 \end{cases}$$

is differentiable at $x = 0$. You must justify your answer.

Very short answer questions

1. [2 marks] Each part is worth 1 marks. Please write your answers in the boxes.

   (a) Compute $\lim_{x \to 1} \dfrac{2x}{\sqrt{3x^2 + 7}}$.

   Answer:

   (b) Compute the limit $\lim_{x \to -3} \dfrac{x^2 + 5x + 6}{x + 3}$

   Answer:

Name: _____ Student-No: _____

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

   (a) Find the left-hand and right-hand limits of $\dfrac{x}{\sqrt{x^2}}$ as $x \to 0$.

   Answer:

   (b) Evaluate $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2+5} - x}{3x+5}$

   Answer:

Long answer question — you must show your work

3. [4 marks] Compute the limit $\lim_{x \to -1} \dfrac{\sqrt{x^2+3}-2}{2x+2}$.

Answer:

Long Problems. In questions 2 - 6, show your work. No credit will be given for the answer without the correct accompanying work.

[14] **2**. Consider the function

$$f(x) = \frac{x^2}{x^2 - 4}.$$

Its first and second derivatives are given by

$$f'(x) = -\frac{8x}{(x^2-4)^2}, \qquad f''(x) = \frac{8(3x^2+4)}{(x^2-4)^3}.$$

(a) Find all x such that $f'(x) = 0$ or $f'(x)$ does not exist.

(b) Find all x such that $f''(x) = 0$ or $f''(x)$ does not exist.

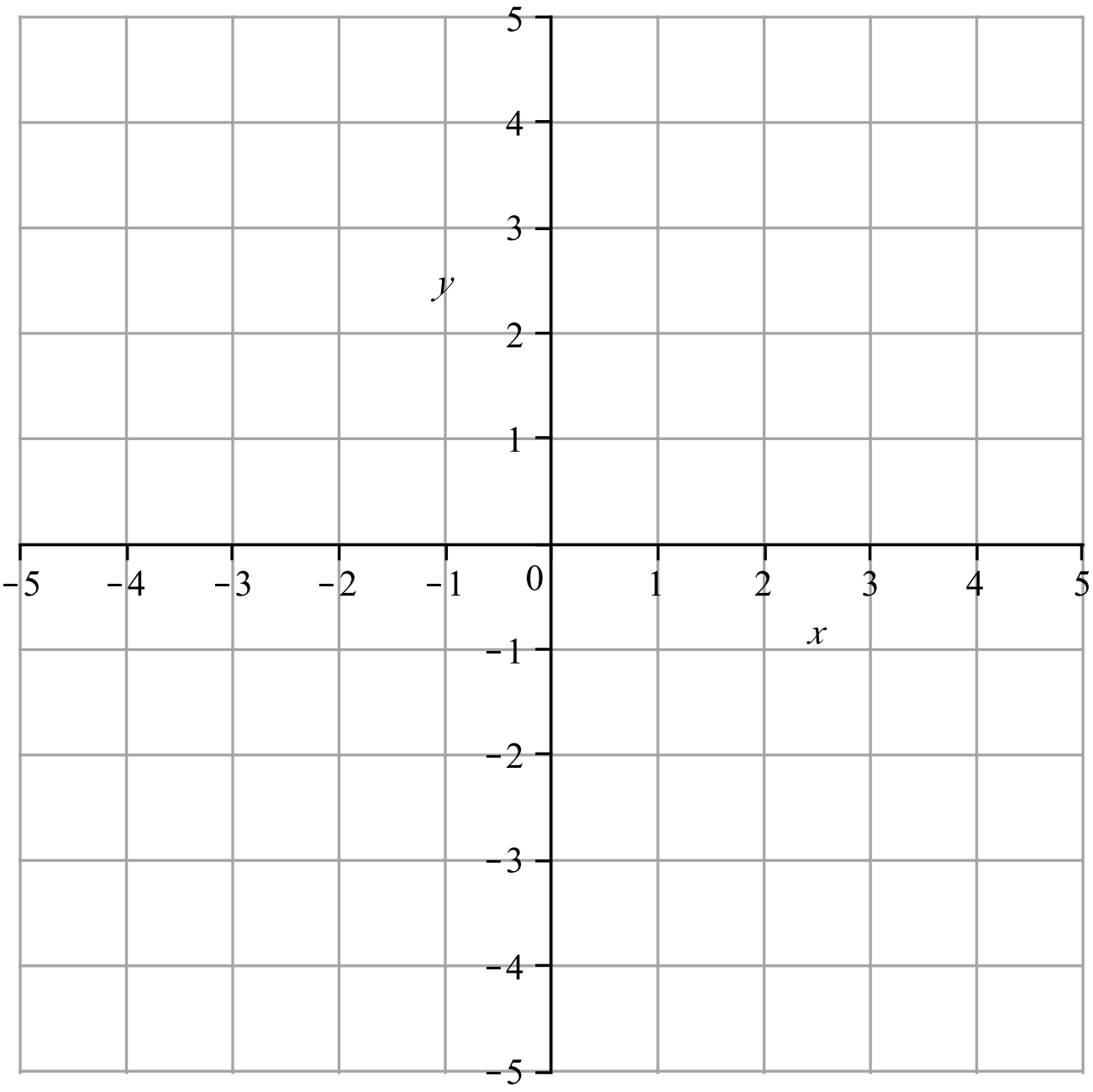
(c) On which intervals is $f(x)$ increasing? On which intervals is $f(x)$ decreasing?

(d) On which intervals is $f(x)$ concave up? On which intervals is $f(x)$ concave down?

(e) Find the coordinates of all local maxima, local minima, and inflection points. Be sure to indicate which is which.

(f) Find any horizontal and vertical asymptotes of the function $f(x)$ and write their equations.

(g) Draw the graph of $f(x)$ on the graph provided. Accurately place all critical points and inflection points, indicate all asymptotes, and make sure your graph correctly shows where $f(x)$ is increasing and decreasing and correctly shows its concavity.



[10] **3**. Two cylindrical tanks are being filled simultaneously at exactly the same rate. The smaller tank has a radius of 5 metres, and the water rises at a rate of 0.5 metres per minute. The larger tank has a radius of 8 metres. How fast is the water rising in the larger tank? It may be helpful to know that the volume of a cylinder of radius R and height H is $V = \pi R^2 H$.

Answer:

[12] **4**. A rectangular storage container with an open top is to have volume of 8 cubic metres. The length of its base is twice the width. Material for the base costs \$4.50 per square metre, and material for the sides costs \$6 per square metre. Find the cost of the material for the cheapest such container.

Answer:

[10] **5**. Currently 1800 people ride a commuter passenger ferry each day and pay \$4 for a ticket. The number of people q willing to ride the ferry at price p is determined by the relationship

$$p = \left(\frac{q - 3000}{600}\right)^2.$$

The company would like to increase its revenue. Use the price elasticity of demand ϵ to give advice to management on whether it should increase or decrease its price from \$4 per passenger. Recall that $\epsilon = \dfrac{p}{q}\dfrac{dq}{dp}$.

You borrow 10 thousand dollars from Nick the Shark, who charges you at a fixed rate r that is compounded continuously. If you pay Nick 100 thousand dollars 2 years later, what was the annual rate of interest that he charged? (A calculator-ready form will suffice.)

If a function f is continuous for all x and if f has a local maximum at $(-1, 4)$ and a local minimum at $(3, -2)$, which of the following statements *must* be true?

(A) The graph of f has an inflection point somewhere between $x = -1$ and $x = 3$.

(B) $f'(-1) = 0$.

(C) The graph of f has a horizontal asymptote.

(D) The graph of f has a horizontal tangent line at $x = 3$.

(E) The graph of f intersects both axes.

Answer:

) At $x = 0$, which of the following is true for the function $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.

(B) f is decreasing.

(C) f is discontinuous.

(D) f has a local minimum.

(E) f has a local maximum.

[8 pts] An architect is designing a house in the form of a cylinder covered by a roof in the shape of half a sphere (extending above the cylinder). Suppose the material used to build the cylindrical wall is half the price of the material that is used to build the roof per unit area. If the total volume of the house is fixed, what ratio between the height of the wall and the radius of the roof will minimize the cost?

If you put money in an account that pays 6% interest, compounded continuously, how long will it take for your money to triple?

[12] **3**. Suppose that when a busy restaurant charges \$7 for its tomato appetizer, an average of 60 people order the dish each night. When it drops the price of the appetizer to \$5, the number ordering it rises to 66. Assume that the demand q is a linear function of the price p. If each appetizer costs the restaurant \$3 to make, use calculus to find the price it should charge to maximize its profit from the appetizer. You do not need to justify that your answer provides the maximum profit.

Very short answer questions

1. [2 marks] Each part is worth 1 marks. Please write your answers in the boxes.

   Consider the function $h(x) = x^3 - 12x + 5$.

   (a) At which point x does $h(x)$ have a **local** maximum?

   Answer:

   (b) At which point x does $h(x)$ have a **local** minimum?

   Answer:

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

   (a) Find the interval(s) where $f(x) = \dfrac{\sqrt{x}}{x+1}$ is increasing.

   Answer:

   (b) Your friend uses the first order Taylor polynomial of $\log(x)$ about $x = 1$ to approximate $\log(1.2)$. Find a bound for the absolute value of the error $|R_1|$ in their approximation. Justify your answer. (Note: $\log(x) = \ln(x) = \log_e(x)$)