Tailoring Bandgaps and Vibroacoustic Response of Periodic Materials and Structures

AmeriMech SYMPOSIUM ON
DYNAMICS OF PERIODIC MATERIALS AND STRUCTURES
ATLANTA, GA, USA

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Outline

- Periodic materials and structures
- Bandgap analysis and tailoring (Part 1)
- Acoustic response tailoring (Part 2)
- Prospects

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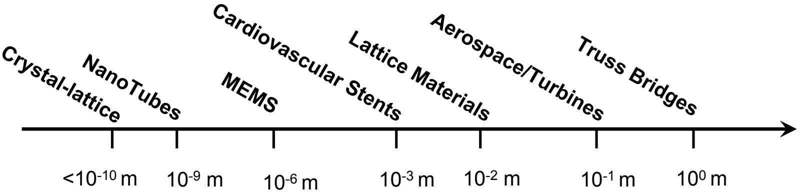

Material vs. Structure

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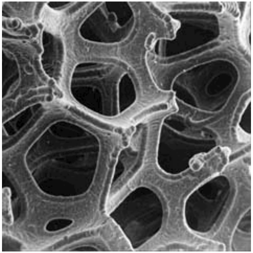
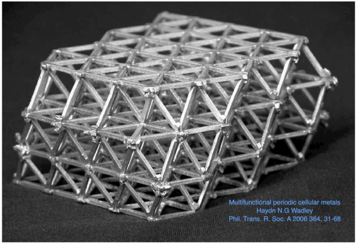


B) Multilayered tetrahedral truss lattice



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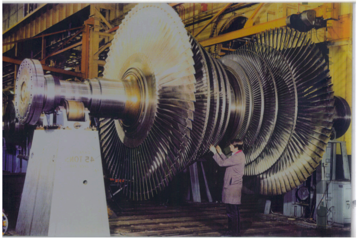

Shape+Size+Scale = Material Property

Structures made of materials vs. materials with structure.

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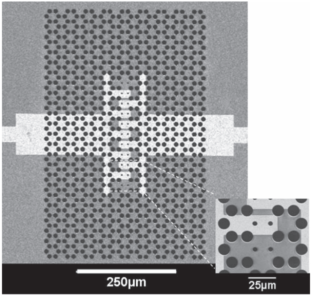
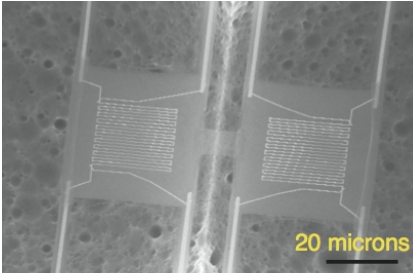
Why Study Bandgaps?

MEMS Phononic Crystals



VHF (140 MHz) Bandpass MEMS filter based on Phononic Crystal (Mohammadi et al., JMEMS, 2012)

Si-Nanomesh Structure



Si Nanomesh to reduce thermal conductivity (Nature Nanotech, Vol.5, 2010)

Bandgaps can be used for signal filtering in MEMS/NEMS and tailor thermoelectric properties of nano materials

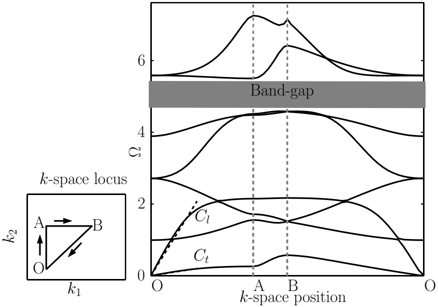
(Hopkins et al., ACS, Nano Letters, 11, 2011).

Passive vibroacoustic isolation.

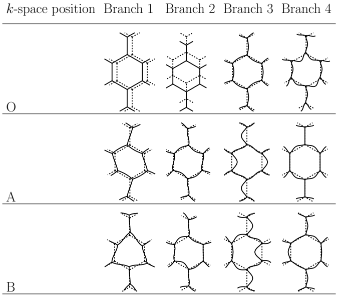
Bandgaps

Frequency intervals over which wave propagation is forbidden.

Band structure



Cell deformation



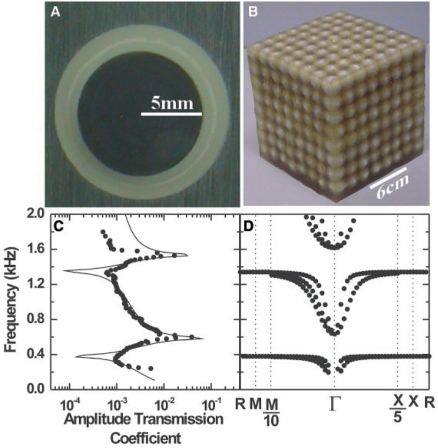
Finite element +Bloch theory ⇒ Solve $\boldsymbol{K}(\boldsymbol{k})\boldsymbol{q} = \omega^2\boldsymbol{M}(\boldsymbol{k})\boldsymbol{q}$
⇒ Band structure

Phani, A.S., Woodhouse, J., Fleck, N.A., 2006, "Wave Propagation in Two-dimensional Periodic Lattices," Journal of the Acoustical Society of America, 119(4), pp. 1995-2005.
Phani, A.S., and Fleck, N.A., 2008, "Elastic Boundary Layers in Isotropic Periodic Lattices," ASME: Journal of Applied Mechanics, 75 (2), pp. 021020-021027

Bragg and sub-Bragg bandgaps

1. Bandgaps arise from Bragg scattering mechanism. Limitation: wavelengths ≈ cell size(lattice constant)

2. Internal resonance mechanism allows for sub-Bragg bandgaps. Sonic crystals, metamaterials exploit this.



Liu et al., Science, 2000

L. Liu and M. I. Hussein, 2012, "Wave motion in periodic flexural beams and characterization of the transition between Bragg scattering and local resonance," J. Appl. Mech. 79, 011003.

Bandgap Analysis

For periodic materials and structures with symmetric unitcell band gaps can be inferred WITHOUT Finite element +Bloch theory $\Rightarrow$ Solve $\boldsymbol{K}(\boldsymbol{k})\boldsymbol{q} = \omega^2 \boldsymbol{M}(\boldsymbol{k})\boldsymbol{q}$.

Locked and free unitcell resonances are band edges of symmetric systems.

Lord Rayleigh, 1887, "On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure," Philos. Mag. 24, 145–159 .

L. Brillouin, *Wave Propagation in Periodic Structures*, 2nd ed. (Dover Publications, New York, 2003), pp. 1–68.

Mead, D.J.,1996,"Wave propagation in continuous periodic structures: Research contributions from Southampton, 1964–1995," Journal of Sound and Vibration, 190, pp. 495-524.

Raghavan, L., and Phani, A.S., 2013, "Local resonance bandgaps in periodic media: theory and experiment," Journal of the Acoustical Society of America, Vol.134, Issue.3, pp. 1950-9159

Unitcell resonances vs. Bandstructure

1. Unitcell resonance analysis will only reveal the edges but not the dispersion structure.

2. Then why unitcell resonance analysis?

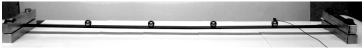
3. Relatively mature field of inverse structural dynamics

4. Given the resonances of a finite unit cell what is the corresponding configuration/design of the structure(s)?

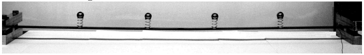
5. Band gap tailoring = inverse structural dynamics problem

Illustrative Periodic Structures

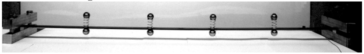
Beam with periodic masses



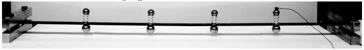
Beam with periodic resonators

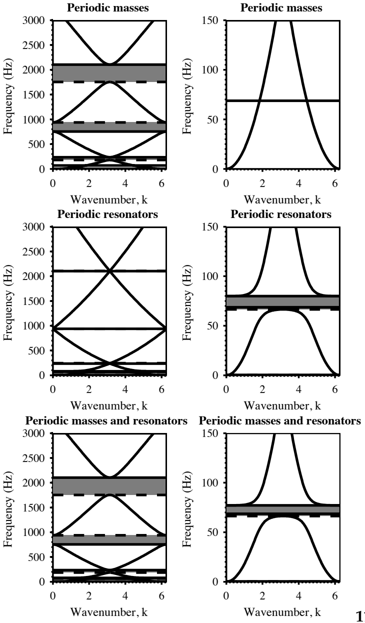


Beam with periodic masses and resonators



Beam with heavy periodic masses and resonators





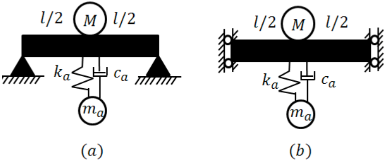
Unitcell Resonances Are Important

1. Band edges are decided by the resonant frequencies of a symmetric unit cell under *locked* and *free* boundary conditions (Mead, JSV, Vol.40 1975)

   (Xiao *et al.* Phys.Lett.-A, 2011)

2. Determining the natural frequencies of the unit cell is sufficient!

Unitcell BCs for band edges

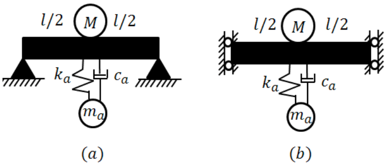


(a) locked and (b) free boundary conditions.

How can we find the natural frequencies of the unitcell? FE, PDEs ....?

Receptance coupling from Structural Dynamics.

Dynamic Receptance Analysis



Beam (modal form) $$H_b \approx \sum_{r=1}^{n} \frac{\Phi_r^2(x)}{a_r(\omega_r^2 - \omega^2 + i2\zeta_r\omega\omega_r)}$$

Resonator $$H_a = -\frac{k_a - m_a\omega^2 + ic_a\omega}{(k_a + ic_a\omega)m_a\omega^2}$$

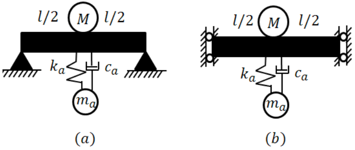
Mass $$H_m = -\frac{1}{M\omega^2}$$

Parallel coupling $$\frac{1}{H} = \frac{1}{H_b} + \frac{1}{H_a} + \frac{1}{H_m}$$

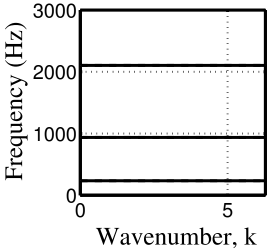
Unitcell resonances are the poles of H. H_b depends on the normal modes (ω_r, $\phi_r(x)$) of the beam unit cell (background medium).

Beam Without Periodicity

$M = 0, m_a = 0, k_a = 0, c_a = 0$



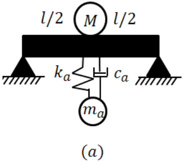
Unitcell resonances are identical for both pinned and guided unit cells ⇒ Zero bandgap width. Guided beam has a zero frequency (rigid body) mode.

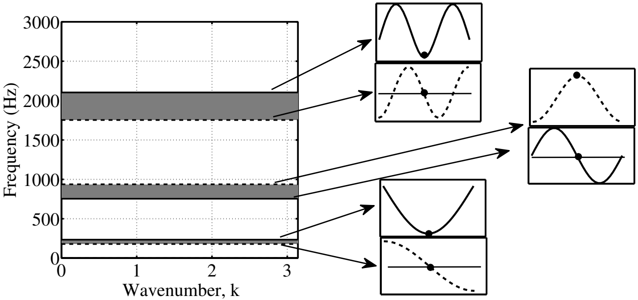


Solid lines—locked unit cell resonances; Dashed lines— free unit cell resonances

Beam With Periodic Masses

$M \neq 0, m_a = 0, k_a = 0, c_a = 0$



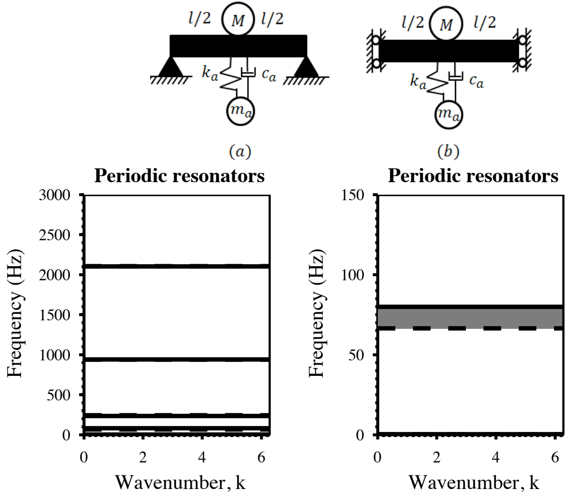


Solid lines—locked unit cell resonances; Dashed lines— free unit cell resonances

Separation of symmetric and antisymmetric modes gives band gap.

Beam With Periodic Resonators

$M = 0, m_a \neq 0, k_a \neq 0, c_a \neq 0$



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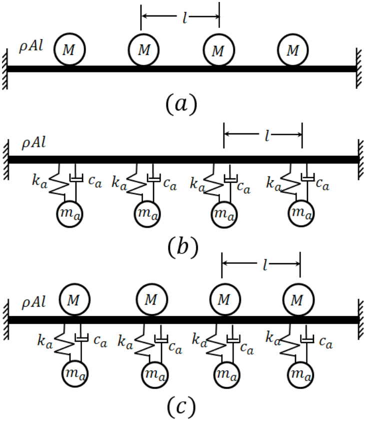
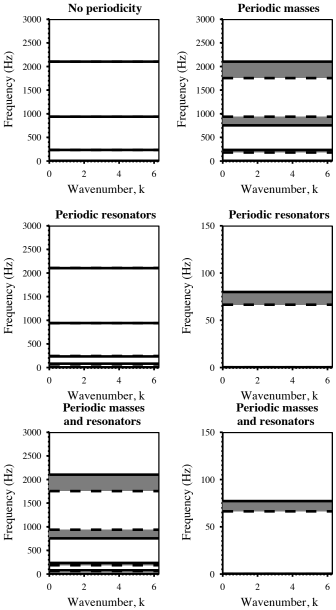
Sub-Bragg bandgap mechanism: rigid mode of guided unit cell

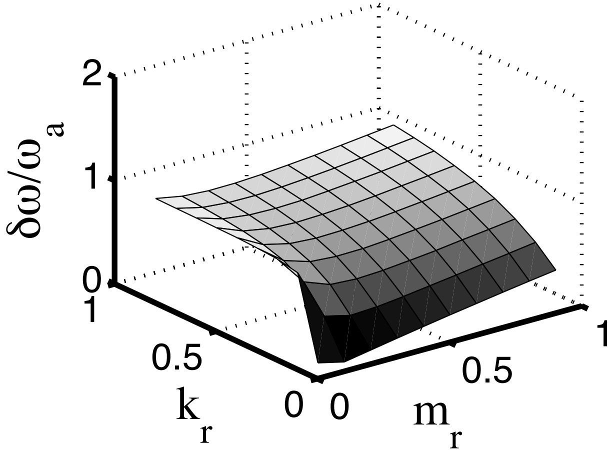
Full Picture



Two-fold periodicity is better.



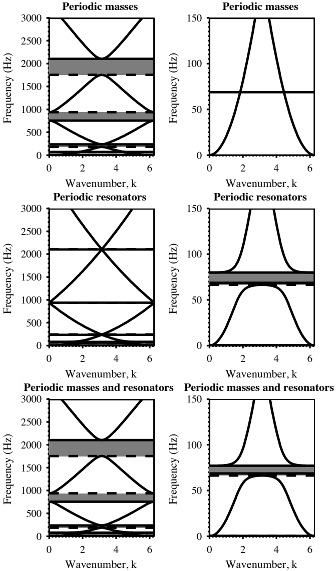
Design Chart for Sub-Bragg Bandgap Width

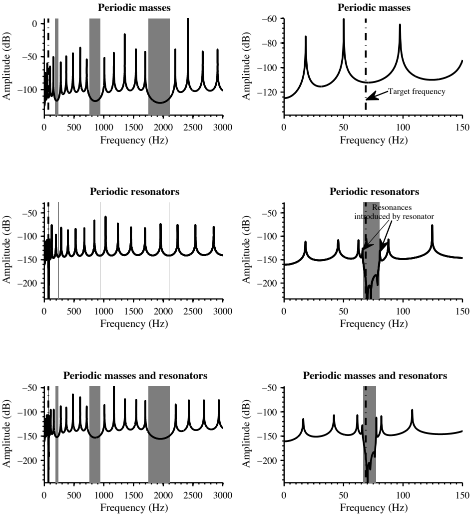


$\delta\omega$ = Bandgapwidth, $\omega_a = \sqrt{\frac{k_a}{m_a}}$, k_r = stiffness ratio, m_r = mass ratio.

Stronger coupling and heavier resonators enhance sub-Bragg bandgap width.

Bandgaps in Frequency Response





Bragg and Sub-Bragg Bandgaps are minima in frequency response.

Material vs. Structure

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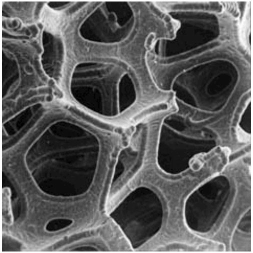

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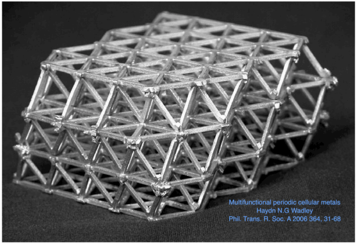

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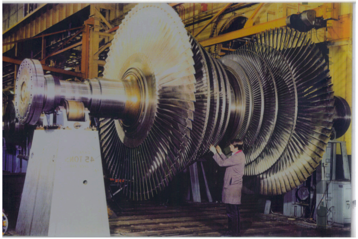


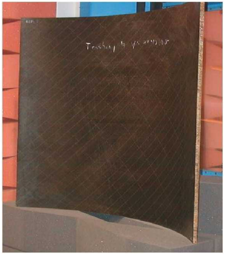
Shape+Size+Scale = Material Property

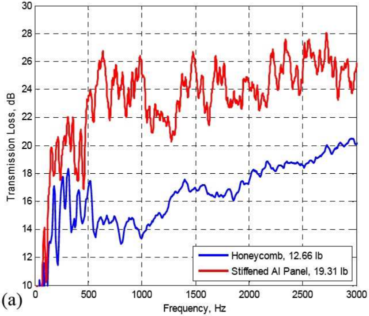
Structures made of materials vs. materials with structure.

Multifunctional Applications–Acoustics — # 1

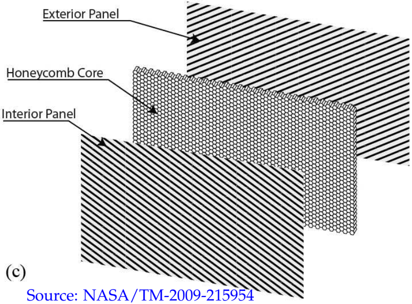
Do lighter, stronger and stiffer materials sound better?

Multifunctional Applications–Acoustics — # 2





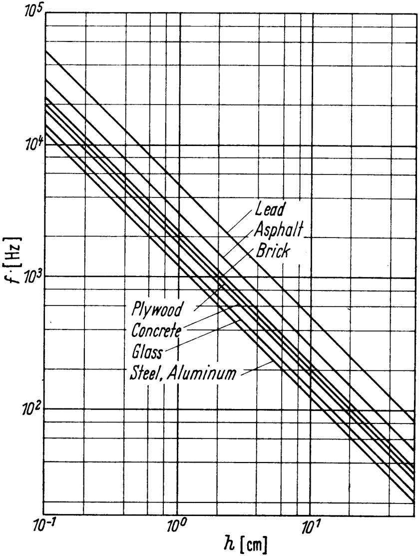
(a)



(c)

Source: NASA/TM-2009-215954

Multifunctional Applications–Acoustics — # 3



1. Structural requirements are in conflict with acoustic demands.
2. Light and stiff structures are acoustically poor.
3. Sound Radiation

$$\sigma \approx \frac{P\lambda_c}{\pi^2 S}\sqrt{\frac{f}{f_c}}, \quad f << f_c \tag{1}$$

$$\approx 0.45\frac{P}{\lambda_c}, \quad f = f_c \tag{2}$$

$$\approx 1, \quad f >> f_c \tag{3}$$

(4)

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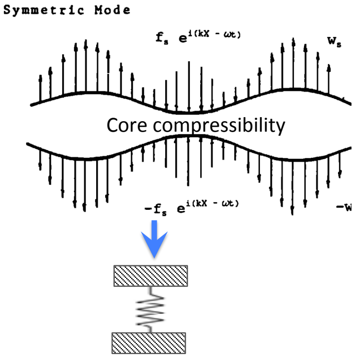
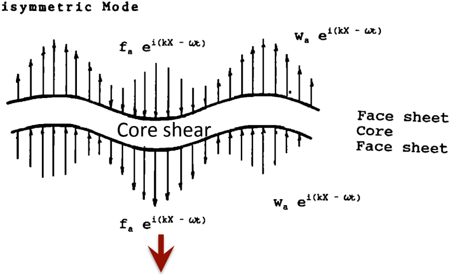
Wave Response of Symmetric Sandwich Panels

An ideal core should maintain constant distance between face sheets without allowing any relative sliding.



Double-Wall-Resonance
(High transmission)



Coincidence (Wavenumber resonance): Wave speeds in the panel match sound wave speed in the fluid (High transmission)

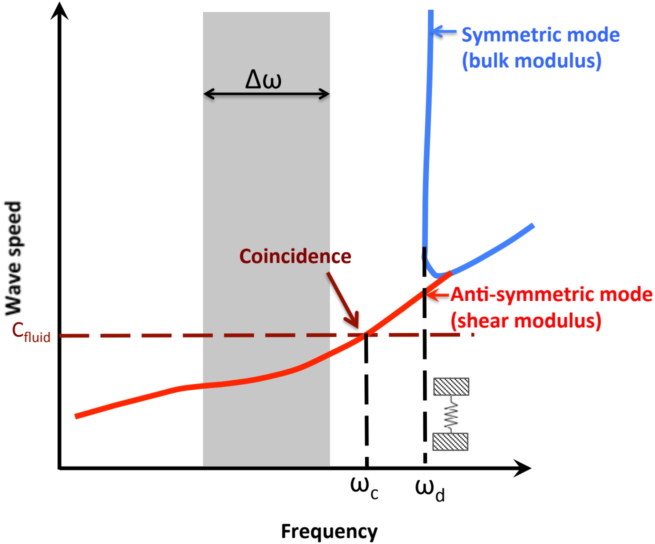
Adapted from Moore & Lyon, JASA 1991

Periodic Materials Can Help?

Can truss lattice materials can offer a potential solution?

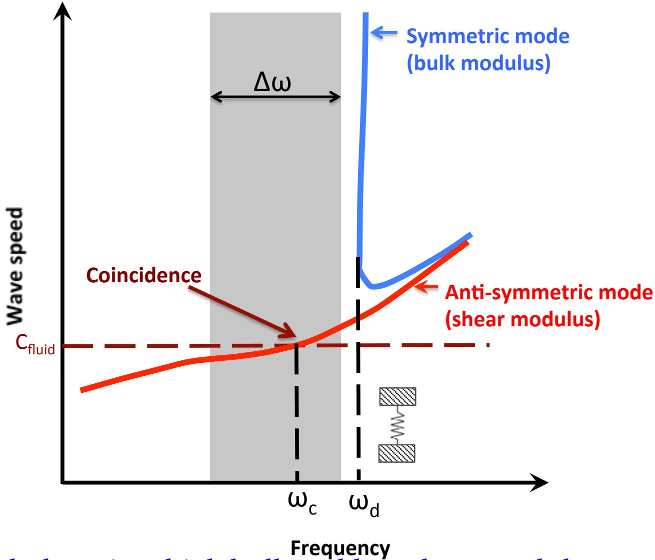
Provided core material properties are tailored in a sandwich design.

Shear Panel (Kurtze & Watters, 1956)



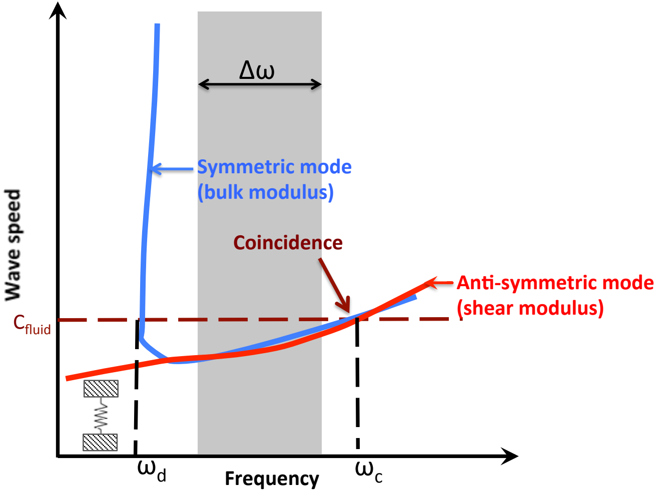
High bulk modulus and low shear modulus (Metal Rubber?).

Coincidence Panel (Warnaka, Holmers, 1969)



High damping, high bulk and low shear modulus are required, weight penalty?

Mode Cancelling Panel (Moore & Lyon, 1990)



Low bulk and low shear modulus are required, stiffness penalty?

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Concluding Remarks

1. Structural approaches provide powerful tools to calculate the width and location of Bragg and sub-Bragg bandgaps induced by a local resonator.

2. Bandgap tailoring can be posed as an inverse structural dynamics problems.

3. Fundamental conflict between structural (light and stiff) and acoustic requirement.

4. Truss core materials are promising provided their dynamic effective properties (shear and bulk modulus) are tailored.

5. *Transformational acoustics ideas (Metal water from A.N. Norris) may be applied in the core material designs?

Thank You!

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a place of mind UBC

Graduate Students: P.Chopra, L. Raghavan, & E.Mehr

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